

Predictive Sliding Mode Control for perturbed discrete delay time systems: Robustness Analysis

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Abstract—This paper presents a control strategy for perturbed discrete system with time delay, using Sliding Mode Control (SMC) and Model based Predictive Control (MPC). A predictive sliding mode control strategy is proposed and a discrete-time reaching law is improved. By applying a predictive sliding surface and a reference trajectory, combining with the state feedback correction and rolling optimization method in the predictive control strategy, a predictive sliding mode controller, for perturbed discrete system with time delay, is synthesized. The combination of SMC and MPC improves the performance of these two control laws. A robustness analysis proves that the designed control strategy has stronger robustness and chattering reduction property. Finally, a numerical example is given to illustrate the effectiveness of the proposed theory.

Index Terms—Discrete time delay systems, Sliding mode control, Model predictive control, Robustness, Stability, Chattering.

I. INTRODUCTION

Systems with delays frequently appear in engineering. Typical examples of time-delay systems are communication networks, chemical processes, tele-operation systems, biosystems, underwater vehicles and so on. The presence of delays makes system analysis and control design much more complicated. In fact, due to the delay, the output of the system could not follow the control input in time. So controlling this kind of systems can be a challenging task, especially in the presence of uncertainties and parameters variations. Many control methodologies are proposed to control such systems [1]–[3].

Sliding Mode Control is one of the powerful control methods for systems containing uncertainties and unknown disturbances. For large class of systems, this kind of control is particularly interesting due to its ability to deal with non linearities, uncertainties, modelling error and disturbances [7], [8].

Sliding mode control design is composed of two steps. At the first step, a sliding surface must be designed. At the second step, a feedback control law is designed to provide convergence of a system trajectory to the sliding surface; thus, the sliding surface should be reached in a finite time. The system's motion on the sliding surface is called the sliding mode.

In addition, this design approach enjoys the advantage of the

reduction of the systems order. This advantage motivates the application of the sliding mode techniques to time delay systems. It has been extensively used because of its robustness to the uncertainties, disturbances and the delay on the continuous time delay systems [1], [3], [4] and the discrete time delay systems [5], [6].

However, it is well known that chattering is the flaw for SMC, but reducing chattering has, certainly, as a result, decreasing robustness of the closed loop systems. Many approaches have been proposed to solve this problem such as high order sliding mode control with classical sliding function [9]–[11] or with dynamic sliding function [12]. But the problem of the chattering phenomenon still exist in the case of perturbed large time delay systems.

To overcome this challenge, a Predictive Sliding Mode Control strategy is proposed, in this paper. In Model Predictive Control, a model of the system is used to predict the future behavior of the system within a prediction horizon. The principle of receding horizon control is employed to compute an optimal future input sequence, such that a certain cost is minimized. This concept can improve the performance in a reaching mode and this is one of the goals that Predictive Sliding Mode control strategy achieve [13]–[17]. Another one is the capability of controlling processes with time delays [19]–[21].

This work deals with a predictive sliding mode control for perturbed discrete delay time systems. This approach is designed to force the state trajectories to reach the surface in finite time and keep them on it, and in the same time to reject disturbances and parameters variation.

The paper is organized as follows: Section 2 describes the basic concepts by giving the system description and preliminaries and by giving the synthesis of the Predictive Sliding Mode Controller. A stability analysis of the proposed control is presented, in section 3. In section 4, the advantages of the presented controller are verified by a numerical simulation example in the presence of disturbances and parameters variation. Finally, section 5 draws conclusions of the paper.

II. BASIC CONCEPTS

A. System description and preliminaries

Consider the following discrete time delay system [1]:

$$\begin{cases} x(k+1) = (A + \Delta A)x(k) + (A_d + \Delta A_d)x(k-d) \\ \quad + (B + \Delta B)u(k) + v(k) \\ x(k) = \phi(k), k = -d, -d+1, \dots, 0. \end{cases} \quad (1)$$

Where $x(k) \in R^n$ is the state vector and $u(k) \in R$ is the scalar input control. The matrices A and $A_d \in R^{n \times n}$ and $B \in R^n$ are relative to a nominal model of the system. The parametric uncertainties are given by ΔA , ΔA_d and ΔB . $v(k) \in R^n$ is a vector containing the disturbance input. ϕ is the initial condition.

The system (1) can be written in the following form:

$$x(k+1) = Ax(k) + A_dx(k-d) + Bu(k) + w(k) \quad (2)$$

with

$$w(k) = \Delta Ax(k) + \Delta Bu(k) + \Delta A_dx(k-d) + v(k) \quad (3)$$

B. Predictive sliding mode control with classical surface

In this section, we analyze the Predictive sliding mode control with classical surface. Define a sliding mode function as follows:

$$s(k) = Cx(k) \quad (4)$$

where $C \in R^{1 \times n}$ which is determined by LMI in [12]. For the system (1), the sliding condition is described as:

$$|s(k+1)| - |s(k)| < 0 \quad (5)$$

According to the sliding mode function (4), the sliding mode value at $(k+1)$ can be obtained:

$$\begin{aligned} s(k+1) &= Cx(k+1) \\ &= C[Ax(k) + A_dx(k-d) + Bu(k) + w(k)] \\ &= C[Ax(k) + A_dx(k-d) + Bu(k)] + Cw(k) \end{aligned} \quad (6)$$

If there are no disturbances, the sliding function (4) value at time $(k+p)$ is:

$$\begin{aligned} s(k+p) &= CA^p x(k) + \sum_{j=1}^p CA^{j-1} Bu(k+p-j) \\ &\quad + \sum_{s=1}^p CA^{s-1} A_d x(k-d+p-s) \end{aligned} \quad (7)$$

where $k \in Z$ and $p \in Z$. Having the purpose to obtain some desired performances, such as strong-robustness, fast convergence and chattering elimination, we introduce a reaching law to ensure the convergence of the sliding function $s(k)$ to zero. To ensure a quasi-sliding mode, the sliding mode function must verify the reaching law [18]:

$$s(k+1) = s(k) - m \operatorname{sgn}(s(k)) \quad (8)$$

with m is the discontinuous term magnitude.

We consider, now, the sliding mode control problem for system (1), taking the reaching law (8) as a reference sliding mode trajectory, the following equations are obtained:

$$\begin{cases} s_r(k+p) = s_r(k+p-1) - m \operatorname{sgn}(s_r(k+p-1)) \\ s_r(k) = s(k) \end{cases} \quad (9)$$

Therefore, the predictive sliding mode value of time k on time $(k-p)$ can be deduced:

$$\begin{aligned} s(k/k-p) &= CA^p x(k-p) + \sum_{j=1}^p CA^{j-1} Bu(k-j) \\ &\quad + \sum_{s=1}^p CA^{s-1} A_d x(k-d-s) \end{aligned} \quad (10)$$

Equation (10) can be described in vector form as follows:

$$S_p(k+1) = \Gamma x(k) + \Omega U(k) + ZX_d(k) \quad (11)$$

where

$$S_p(k+1) = [s(k+1), s(k+2), \dots, s(k+N)]^T$$

$$S_p(k) = [s(k), s(k+1), \dots, s(k+N-1)]^T$$

$$U(k) = [u(k), u(k+1), \dots, u(k+N-1)]^T$$

$$\Gamma = [(CA)^T (CA^2)^T \dots (CA^N)^T]^T$$

$$X_d(k) = [x(k-d), x(k-d+1), \dots, x(k-d+N-1)]^T$$

$$\Omega = \begin{bmatrix} CB & 0 & \dots & \dots & 0 \\ CAB & CB & \dots & \dots & 0 \\ \vdots & \vdots & \dots & \dots & \vdots \\ CA^{M-1}B & CA^{M-2}B & \dots & CAB & CB \\ CA^{N-2}B & CA^{N-3}B & \dots & CA^{N-M}B & CA^{N-M-1}B \\ CA^{N-1}B & CA^{N-2}B & \dots & CA^{N-M+1}B & CA^{N-M}B \\ CA_d & 00 & \dots & \dots & 00 \\ CAA_d & CA_d & \dots & \dots & 00 \\ \vdots & \vdots & \dots & \dots & \vdots \\ \vdots & \vdots & \dots & \dots & \vdots \\ \vdots & \vdots & \dots & \dots & \vdots \\ CA^{N-1}A_d & CA^{N-2}A_d & \dots & CA^{N-N+1}A_d & CA_d \end{bmatrix}$$

With N is prediction horizon, M is control horizon and the minimum costing horizon N' is chosen equal to 1.

In practice, to make correction to the future predictive sliding mode value $s(k+p)$, we introduce the error between practical sliding mode value $s(k)$ and predictive sliding mode value $s(k/k-p)$. Therefore, the output of sliding mode prediction $\tilde{s}_p(k+p)$ is given as follows:

$$\begin{aligned} \tilde{s}(k+p) &= s(k+p) + h_p e(k) \\ \tilde{s}(k+p) &= CA^p x(k) + \sum_{j=1}^p CA^{j-1} Bu(k+p-j) \\ &\quad + \sum_{s=1}^p CA^{s-1} A_d x(k-d+p-s) + h_p e(k) \end{aligned} \quad (12)$$

Where $e(k) = s(k) - s(k/k-p)$ and h_p is a correct coefficient.

Rewrite Equation (12) in vector form:

$$\tilde{S}_p(k+1) = S_p(k+1) + H_p E(k) \quad (13)$$

where

$$\tilde{S}_p(k+1) = [\tilde{s}_p(k+1), \tilde{s}_p(k+2), \dots, \tilde{s}_p(k+N)]^T$$

$$H_p = \text{diag}[h_1, h_2, \dots, h_N]$$

$$E(k) = S(k) - S_{mp}(k)$$

$$S(k) = [s(k), s(k), \dots, s(k)]_{1 \times N}$$

$$S_{mp}(k) = [s(k/k-1), s(k/k-2), \dots, s(k/k-N)]^T$$

The following corresponding optimization cost function is defined [16]:

$$j_p = \sum_{j=1}^N q_j [\tilde{s}_p(k+1) - s_r(k+j)]^2 + \sum_{l=1}^M g_l [u(k+l-1)]^2 \quad (14)$$

where $s_r(k+1)$ is the sliding mode reference trajectory, q_j and g_l are weight coefficients.

In order to simplify the synthesis of the controller, we consider ($q_j = 1$) and $g_l = g$. So, The following corresponding optimization cost function is written by:

$$j_p = \sum_{j=1}^N [\tilde{s}_p(k+i) - s_r(k+j)]^2 + \sum_{l=1}^M g [u(k+l-1)]^2 \quad (15)$$

Rewrite Equation(15) in vector form:

$$j_p = \left\| \tilde{S}_p(k+1) - S_r(k+1) \right\|^2 + \|U(k)\|_G^2 = [\Gamma x(k) + \Omega U(k) + Z X_d(k) + H_p E(k) - S_r(k+1)]^T [\Gamma x(k) + \Omega U(k) + Z X_d(k) + H_p E(k) - S_r(k+1)] + U(k)^T G U(k) \quad (16)$$

where

$$S_r(k+1) = [s_r(k+1), s_r(k+2), \dots, s_r(k+N)]^T$$

$$G = [g, g, \dots, g]$$

$$X_d(k) = [x(k-d), x(k-d+1), \dots, x(k-d+N-1)]^T$$

Let $\frac{\partial j_p}{\partial U(k)} = 0$, the optimal law can be obtained:

$$U(k) = -(\Omega^T \Omega + G)^{-1} \Omega^T [\Gamma x(k) + Z X_d(k) + H_p E(k) - S_r(k+1)] \quad (17)$$

Because of rolling optimization, only the present control input signal is implemented, the next time control signal $u(k+1)$ will be calculated recursively by the control law.

III. ROBUSTNESS ANALYSIS

We consider the system (1) and the sliding mode function (4). The following is given:

$$\begin{aligned} s(k+1) &= Cx(k+1) \\ &= C[Ax(k) + A_d x(k-d) + Bu(k) + w(k)] \\ &= C[Ax(k) + A_d x(k-d) + Bu(k)] \\ &+ C[\Delta A x(k) + \Delta B u(k) + \Delta A_d x(k-d) + v(k)] \end{aligned} \quad (18)$$

The sliding function value at time ($k+p$) is:

$$\begin{aligned} s(k+p) &= C A^p x(k) + \sum_{j=1}^p C A^{j-1} B u(k+p-j) \\ &+ \sum_{s=1}^p C A^{s-1} A_d x(k-d+p-s) + \sum_{i=1}^p C A^{i-1} w(k+p-i) \end{aligned} \quad (19)$$

Equation (13) can be described in vector form as follows:

$$S_p(k+1) = \Gamma x(k) + \Omega U(k) + Z X_d(k) + K W \quad (20)$$

with:

$$K = \begin{bmatrix} C & 0 & \dots & 0 \\ CA & C & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{N-1} & CA^{N-2} & \dots & C \end{bmatrix}$$

and $W = [w(k), w(k+1), \dots, w(k+N-1)]^T$

Or, we have:

$$U(k) = -(\Omega^T \Omega + G)^{-1} \Omega^T [\Gamma x(k) + Z X_d(k) + H_p E(k) - S_r(k+1)]$$

So:

$$\begin{aligned} S_p(k+1) &= \Gamma x(k) + \Omega U(k) + Z X_d(k) + K W \\ &= \Gamma x(k) + \Omega [-(\Omega^T \Omega + G)^{-1} \Omega^T [\Gamma x(k) + Z X_d(k) + H_p E(k)] \\ &- S_r(k+1) + Z X_d(k) + K W] \end{aligned}$$

The action of the weight coefficient G is used to limit the control input U . So, we can suppose that $G = 0$, i.e., there is no limitation for control input U .

$$\begin{aligned} S_p(k+1) &= \Gamma x(k) - [\Gamma x(k) + Z X_d(k) + H_p E(k) - S_r(k+1)] \\ &+ Z X_d(k) + K W \\ &= S_r(k+1) - H_p E(k) + K W \end{aligned} \quad (21)$$

where:

$$E(k) = S(k) - S_{mp}(k)$$

$$S(k) = [s(k), s(k), \dots, s(k)]_{1 \times N}$$

$$S_{mp}(k) = [s(k/k-1), s(k/k-2), \dots, s(k/k-N)]^T$$

$$s(k) = s((k-p)+p)$$

$$s(k) = C A^p x(k-p) + \sum_{j=1}^p C A^{j-1} B u(k-j)$$

$$+ \sum_{s=1}^p C A^{s-1} A_d x(k-d-s) + \sum_{i=1}^p C A^{i-1} w(k-i)$$

$$s(k/k-p) = C A^p x(k-p) + \sum_{j=1}^p C A^{j-1} B u(k-j) +$$

$$\sum_{s=1}^p C A^{s-1} A_d x(k-d-s) \quad e(k) = s(k) - s(k/k-p) =$$

$$\sum_{i=1}^p C A^{i-1} w(k-i) \quad E(k) = S(k) - S_{mp}(k) = \tilde{K} \tilde{W}$$

$$\tilde{K} = \begin{bmatrix} C & 0 & \dots & 0 \\ C & CA & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C & CA & \dots & CA^{N-1} \end{bmatrix}$$

$$\tilde{W} = [w(k-1), w(k-2), \dots, w(k-N)]^T$$

Because of rolling optimization, only the present control input signal is implemented. The practical sliding mode function can be described as follows:

$$s(k+1) = [1, 0 \dots 0] \left[S_r - H_p \tilde{K} \tilde{W} + KW \right] \quad (22)$$

Equation(22) can be re-written to:

$$s(k+1) = s_r(k+1) + C[w(k) - h_1 w(k-1)] \quad (23)$$

From the viewpoint of practice, usually, we choose $h_1 = 1$ [20].

Then equation(22) can be re-written to:

$$s(k+1) = s_r(k+1) + C[w(k) - w(k-1)] \quad (24)$$

Or, we have:

$$\begin{aligned} s_r(k+p) &= s_r(k+p-1) - m \operatorname{sgn}(s_r(k+p-1)) \\ s_r(k) &= s(k) \end{aligned}$$

That's why:

$$s(k+1) = s_r(k) - m \operatorname{sign}(s_r(k)) + C[w(k) - w(k-1)] \quad (25)$$

So, we have:

$$s(k+1) = s(k) - m \operatorname{sign}(s(k)) + C[w(k) - w(k-1)] \quad (26)$$

In discrete time sliding mode control, a quasi sliding mode is considered in the vicinity of the sliding surface, such that $|s(k)| < \varepsilon$ where $s(k)$ is the sliding function and ε is a positive constant called the quasi sliding mode band width [22].

Bartoszewicz in [23], gave the following sufficient and necessary condition for a system to satisfy a convergent quasi sliding mode:

$$\begin{cases} s(k) > \varepsilon \Rightarrow -\varepsilon \leq s(k+1) < s(k) \\ s(k) < -\varepsilon \Rightarrow s(k) < s(k+1) \leq \varepsilon \\ |s(k)| < \varepsilon \Rightarrow |s(k+1)| \leq \varepsilon \end{cases} \quad (27)$$

We suppose that $\forall k$, $C[w(k) - w(k-1)]$ is bounded such that:

$$|C(w(k) - w(k-1))| < w_0 \quad (28)$$

With w_0 a positive constant.

Theorem 1. Consider the system (1), to which the discrete predictive sliding mode control is applied (17). This system verifies a convergent quasi sliding mode if the discontinuous term amplitude m is chosen such that:

$$m > w_0 \quad (29)$$

Where w_0 is the external disturbances and system's parameter's variation bound given by (28).

Proof. ε is chosen equal to $m + w_0$. To prove the convergence of the proposed control technique, we must, then, check the following three conditions:

$$s(k) > m + w_0 \Rightarrow -(m + w_0) \leq s(k+1) < s(k) \quad (30)$$

$$s(k) < -(m + w_0) \Rightarrow s(k) < s(k+1) \leq m + w_0 \quad (31)$$

$$|s(k)| < m + w_0 \Rightarrow |s(k+1)| \leq m + w_0 \quad (32)$$

1. Let's begin by the condition (30):

$$s(k) > m + w_0 \Rightarrow -(m + w_0) \leq s(k+1) < s(k)$$

* The inequality

$$s(k+1) < s(k)$$

can be written as follows:

$$s(k) + C(w(k) - w(k-1)) - m \operatorname{sign}(s(k)) < s(k) \quad (33)$$

Knowing that $s(k) > 0$, the inequality (33) becomes:

$$s(k) + C(w(k) - w(k-1)) - m < s(k) \quad (34)$$

By subtracting $s(k)$ from both sides of this last inequality, we obtain:

$$C(w(k) - w(k-1)) - m < 0 \quad (35)$$

This last inequality is true because m is chosen that $m > w_0$.

* The inequality

$$-(m + w_0) \leq s(k+1)$$

can be written as follows:

$$-m - w_0 < s(k) + C(w(k) - w(k-1)) - m \quad (36)$$

Which gives:

$$-w_0 - C(w(k) - w(k-1)) < s(k) \quad (37)$$

This last inequality is true, knowing that $s(k) > m + w_0 > 0$ and $-w_0 - C(w(k) - w(k-1)) < 0$.

2. Let's consider condition (31):

$$s(k) < -(m + w_0) \Rightarrow s(k) < s(k+1) \leq m + w_0$$

By replacing $s(k+1)$ by its expression, we obtain:

$$s(k) < s(k) + C(w(k) - w(k-1)) + m \leq m + w_0 \quad (38)$$

* The inequality

$$s(k) + C(w(k) - w(k-1)) + m \leq m + w_0 \quad (39)$$

can be written as follows:

$$s(k) + C(w(k) - w(k-1)) \leq w_0 \quad (40)$$

Which gives:

$$s(k) \leq w_0 - C(w(k) - w(k-1)) \quad (41)$$

This last inequality is true because

$$w_0 - C(w(k) - w(k-1)) > 0 \text{ and } s(k) < 0.$$

* The inequality

$$s(k) < s(k) + C(w(k) - w(k-1)) + m \quad (42)$$

is evident, knowing that:

$$m > w_0 > C(w(k) - w(k-1)) \quad (43)$$

3. Let's consider condition (32):

$$|s(k)| < m + w_0 \Rightarrow |s(k+1)| \leq m + w_0$$

* If $s(k) > 0$, then, the inequality

$$|s(k)| < m + w_0 \quad (44)$$

becomes:

$$0 < s(k) < m + w_0 \quad (45)$$

which gives:

$$\begin{aligned} & C(w(k) - w(k-1)) - m < s(k) + C(w(k) - w(k-1)) - m \\ & < m + w_0 + C(w(k) - w(k-1)) - m \\ & \Rightarrow -w_0 - m < s(k) + C(w(k) - w(k-1)) - m \\ & < m + w_0 + w_0 - m \\ & \Rightarrow -w_0 - m < s(k) + C(w(k) - w(k-1)) - m \\ & < w_0 + w_0 \\ & \Rightarrow -w_0 - m < s(k) + C(w(k) - w(k-1)) - m \\ & < m + w_0 \\ & \Rightarrow -w_0 - m < s(k+1) < m + w_0 \end{aligned}$$

Then,

$$|s(k+1)| < m + w_0 \quad (46)$$

* If $s(k) < 0$, then, the inequality

$$|s(k)| < m + w_0 \quad (47)$$

becomes:

$$-m - w_0 < s(k) < 0 \quad (48)$$

which gives:

$$\begin{aligned} & C(w(k) - w(k-1)) + m - m - w_0 \\ & < s(k) + C(w(k) - w(k-1)) + m \\ & < C(w(k) - w(k-1)) + m \\ & \Rightarrow -w_0 - w_0 < s(k) + C(w(k) - w(k-1)) + m \\ & < m + w_0 \\ & \Rightarrow -w_0 - m < s(k) + C(w(k) - w(k-1)) + m \\ & < m + w_0 \\ & \Rightarrow -w_0 - m < s(k+1) < m + w_0 \end{aligned}$$

So,

$$|s(k+1)| < m + w_0 \quad (49)$$

The verification of the three conditions (30), (31) and (32) proves the existence of the convergent quasi sliding mode. Therefore, the controller given by (17) is stable.

IV. SIMULATION RESULTS

We consider a discrete time delay system (1), described by the following equation:

$$\begin{cases} x(k+1) = (A + \Delta A)x(k) + (A_d + \Delta A_d)x(k-d) + (B + \Delta B)u(k) + v(k) \\ x(k) = \varphi(k), \quad k = -d, -d+1, \dots, 0. \end{cases}$$

Where:

$$A = \begin{bmatrix} 1.2 & 0.1 \\ 1 & 0.6 \end{bmatrix}; A_d = \begin{bmatrix} -0.2 & 0 \\ 1 & -1 \end{bmatrix}; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\phi(k)^T = \begin{bmatrix} -0.7 & 0.5 \end{bmatrix}, \quad k = \{-10, \dots, 0\}, d = 10.$$

The sampling period T is chosen, according to the system's dynamics, equal to $0.01s$.

The parameters variation are given by:

$$\begin{aligned} \Delta A &= 0.01 \begin{bmatrix} 5 \sin(-\frac{2k\pi}{100}) & 6 \sin(-\frac{2k\pi}{100}) \\ 3 \sin(-\frac{2k\pi}{100}) & 3 \sin(-\frac{2k\pi}{100}) \end{bmatrix}; \\ \Delta A_d &= 0.01 \begin{bmatrix} 5 \sin(-\frac{2k\pi}{100}) & 6 \sin(-\frac{2k\pi}{100}) \\ 3 \sin(-\frac{2k\pi}{100}) & 3 \sin(-\frac{2k\pi}{100}) \end{bmatrix}; \\ \Delta B &= 0.01 \begin{bmatrix} 2 \sin(-\frac{2k\pi}{100}) \\ 3 \sin(-\frac{2k\pi}{100}) \end{bmatrix} \end{aligned}$$

The evolution of disturbances is given in figure(1):

For the Predictive Sliding Mode controller, choosing

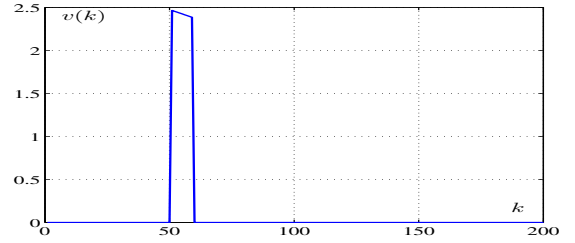


Fig. 1. Evolutions of disturbances

$G = 0.001 * I_{M \times M}$. C is designed as $C = \begin{bmatrix} 2 & 0.3 \end{bmatrix}$. Select the predictive horizon $N = 10$, the control horizon $M = 5$ and the correct coefficient matrix:

$H_p = \text{diag} \begin{bmatrix} 1 & 0.8 & 0.6 & 0.5 & 0.3 & 0.2 & 0.2 & 0.4 & 0.1 & 0.5 \end{bmatrix}$
Satisfying the robustness condition, the discontinuous term magnitude m is chosen as follow:

$$\begin{cases} m = 3 & \text{if } 48 < k < 62 \\ m = 0.02 & \text{else} \end{cases}$$

For the version of Predictive Sliding Mode controller, with poor choice of m , we kept the same coefficients and parameters, but m is chosen as: $m = 0.02$.

For the dynamic second sliding mode controller, C is designed as $C = \begin{bmatrix} 2 & 0.3 \end{bmatrix}$, $m = 0.02$ and $\beta = 0.1$.

The states response, sliding mode function, control input, with predictive sliding mode controller (with good or poor choice of the discontinuous term magnitude m) and dynamic second sliding mode controller, are given in Figures 2 to 5, respectively.

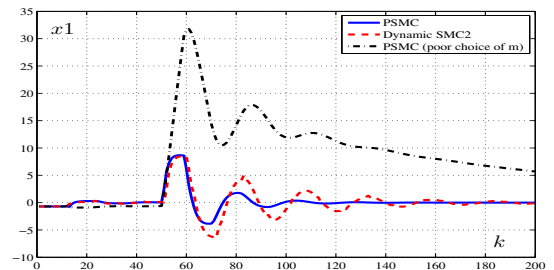


Fig. 2. Evolutions of the state $x_1(k)$.

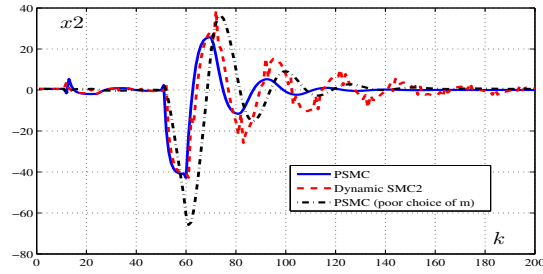


Fig. 3. Evolutions of the state $x_2(k)$

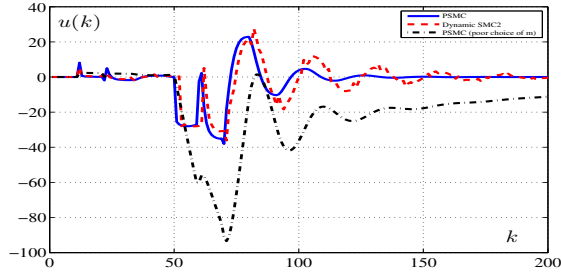


Fig. 4. Evolutions of the control input $u(k)$

Even though, the second dynamic Sliding Mode Controller is intended to control systems with time-delay, the Predictive Sliding Mode Controller, with a good choice of the discontinuous term magnitude m , presented better performances. In fact, the obtained results prove the capability of the proposed control law to eliminate chattering, to reject disturbances and to ensure fast convergence.

V. CONCLUSION

Delay time may cause more difficulties for process control. In this paper, the presented predictive sliding mode controller combines the design technique of SMC and MPC. A predictive sliding mode control strategy is proposed. It is shown that mixing both control techniques gives a new controller with a better robustness properties. An analysis of the robustness of this controller were presented. In presence of disturbances and parameters variation, the obtained simulation results show good performances in term of regulation, then disturbances rejection, and chattering reduction.

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REFERENCES

- [1] J.P. Richard, M. Ksouri, *Système à retard*, Edition CP, 2006.
- [2] J. P. Richard, F. Gouaisbaut and W. Perruquetti, *Sliding mode control in the presence of delay*, KYBERNETIKA, Vol. 37, pp. 277-294, 2001.
- [3] W. Perruquetti and J.P. Barbot, *Sliding Mode Control in Engineering*, Edition, Marcel Dekker, 2002.
- [4] F. Gouaisbaut, M. Dambrine, and J. P. Richard, *SRobust control of linear system with a time-varying delay : A sliding mode control design via LMI*, IMA Journal of Mathematical Control and Information, Vol. 19, pp. 83-94, 2002.

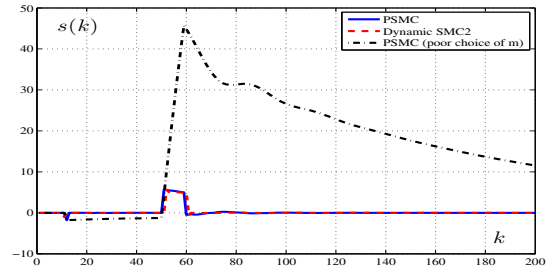


Fig. 5. Evolutions of the sliding function

- [5] W. C. Huimin and C.Gao, *Discrete Sliding-Mode Control of Uncertain Systems with Time Delays*, Proceedings of the International Conference on Modelling, Identification and Control, Okyama, Japan, pp. 882-885, 2010.
- [6] M. Yan and Y. Shi, *Robust sliding-mode control of uncertain Discrete time systems with time delay*, CIET Control Theory Appl, Vol. 2, pp. 662-674, 2008.
- [7] P. Lopez, A. S. Nouri, *Théorie élémentaire et pratique de la commande par les régimes glissants*, Springer-Verlag, 2006.
- [8] V. I. Utkin, H. C. Chang, *Sliding mode control on electro-mechanical systems*, Mathematical Problems in Engineering, Vol. 8, pp. 451-473, 2002.
- [9] M. Mihoub, A. S. Nouri, R. Ben Abdennour, *Real-time application of discrete second order sliding mode control to a chemical reactor*, Control Engineering Practice, pp. 1-7, 2009.
- [10] G. Bartolini, A. Ferrara, E. Usai, *Chattering Avoidance by Second-Order Sliding Mode Control*, IEEE Transaction Automatic Control, Vol. 43, No. 2, pp. 241-246, 1998.
- [11] A. Cavallo, C. Natale, *High-order sliding control of mechanical systems: theory and experiments*, Automatica, Vol. 12, pp. 1139-1149, 2004.
- [12] N. Abdennebi, A. S. Nouri, *A new sliding surface for discrete second order Sliding Mode Control of time delay systems*, Proceedings of the 9th International Multi-Conference on System, Signals and Devices, 2012.
- [13] H. Ben Mansour, A. S. Nouri, *Discrete Predictive Sliding Mode control of uncertain systems*, Proceedings of the 9th International Multi-Conference on System, Signals and Devices, 2012.
- [14] W. Gabin, D. Zambrano, E.F Camacho, *Sliding mode predictive control of a solar air conditioning plant*, Control Engineering Practice, Vol. 17, pp. 652-663, 2009.
- [15] L. Xiao, H. SU, X. Zhang, *Discrete Variable Structure Control Algorithm for Nonlinear Systems via Sliding Mode Prediction*, IEEE Transactions on Automatic Control, 2006.
- [16] M. De La Parte, O. Camacho, E. Camacho, *Developement of a GPC-based sliding mode controller*, ISA Transaction, Vol. 41, pp. 19-30, 2002.
- [17] H. Zaholan, W. Mao, L. Shuhuan, *Discrete sliding mode prediction control of uncertain switched systems*, Journal of systems Engineering and Electronics, Vol. 20, pp. 1065-1071, 2009.
- [18] W. Gao, Y. Wang, A. Homaifa, *Discrete-time variable structure control systems*, IEEE Transactions on Industrial Electronics, Vol. 42, No. 2, pp. 117-122, 1995.
- [19] J. Zhao, *SMPC for Discrete-time Singular Systems with Time-varying Delay*, Journal of Control Engineering and Technology (JCET), Vol. 1, pp. 59-64, 2011.
- [20] J. Zhao, J. Meng, L. Zhang, *Passivity-based Sliding Mode Predictive Control of discrete-time Singular Systems with time varying delay*, Proceeding of International conference on Consumer, Electronics, Communication and Networks (CECNET), 2011.
- [21] F. Da, *Sliding mode predictive control for long delay time systems*, Journal Physics Letter A, Elsevier, Vol. 348, pp.228-232, 2006.
- [22] M. Mihoub, A. S. Nouri, R. Ben Abdennour, *Multimodel discrete second order sliding mode control: Stability analysis and real time application on a chemical reactor*, In Tech, pp. 473-490, 2011.
- [23] A. Bartoszewicz *Discrete time quasi sliding mode control strategies*, IEEE Transaction on industrial Electronics, vol.45, no.4, pp. 633-637, 1998.