

Identification of nonlinear systems by the new representation ARX-Laguerre decoupled multimodel

Sameh Adaily, Tarek Garna*, Abdelkader Mbarek and Hassani Messaoud

Unité de Recherche Automatique Traitement de signal et Image (ATSI)
Ecole Nationale d'Ingénieurs de Monastir (ENIM), Université de Monastir
Rue Ibn El Jazzar, 5019 Monastir-Tunisa
*email : tarek.garna@enim.rnu.tn

Abstract— This paper proposes a new alternative in the multimodel approach by expanding each ARX sub-model on independent orthonormal Laguerre bases by filtering the process input and output using Laguerre orthonormal functions. The resulting multimodel, entitled ARX-Laguerre decoupled multimodel, ensures the parameter number reduction with a recursive and easy representation. However, such reduction is still constrained by an optimal choice of Laguerre pole characterizing each basis. To do so, we develop a pole optimization algorithm which constitutes an extension of that proposed by Tanguy *et al.* [17]. The ARX-Laguerre decoupled multimodel as well as the proposed pole optimization algorithm are illustrated and validated on a numerical simulation.

Keywords—Laguerre bases, ARX multimodel, ARX-Laguerre multimodel, optimization.

I. INTRODUCTION

In the literature, we can mention among the various structures of nonlinear models commonly used the Volterra series [1, 15, 18], the Volterra models with reduced complexity (Volterra-Laguerre, Volterra-BOG) [2, 9], the nonlinear block-structure models (Wiener and Hammerstein) [12, 14] and the multimodels [6, 13]. These latter are obtained by using the multimodel approach which is adapted for the modelisation of a large class of complex systems. The concept of this approach is based on the divide of the nonlinear system behavior into a set of operating regions. For each operating region a local model or a sub-model of simple linear structure can be used to describe the dynamic behavior. The overall dynamic behavior is obtained by considering the relative contribution of each sub-model by means of a weighting function associated with each operating region. As a result, the multimodels facilitate the extension of certain methods of identification, diagnosis and control developed in the context of linear systems to nonlinear systems. In addition, we note that there is a wide variety of choices to define the structure of the local linear models associated with the operating regions. Several authors have used local models in the form of input-output (ARX, ARMAX, ...) or of the state representation [4, 6, 7, 11, 13]. In recent years a new linear model was developed by exploiting the orthonormal filters such as filters of the Laguerre basis [10, 12]. This kind of modeling presents many advantages. In fact, it is not influenced either by the choice of sampling interval or by the process delay. Moreover these filters are characterized by the Laguerre pole, the optimal identification of which is achieved by

exploiting a priori information about the dominating pole of the system. This identification guarantees a significant parametric reduction [5, 10, 17]. These properties have encouraged some authors [15, 16] to exploit the Laguerre filters for each operating region in the multimodel approach instead the local models in the form of input-output. We note that this technique uses the filtering of the input by Laguerre orthonormal functions.

Thus, given the importance of reducing parametric and in order to collect the maximum of information about the system, it is interesting to extend this principle of the input filtering by Laguerre functions to filter both the input and the output in the multimodel case. To do this, we propose to use the representation ARX multimodel where each operating region is represented by the ARX model characterized by the output of the ARX multimodel and the input system. In this case, we use the orthonormal Laguerre functions for filtering the input and the output of the multimodel for each ARX sub-model. It consists in decomposing the parameters of the ARX sub-model associated with the input and the output on two independent Laguerre orthonormal bases. The resulting sub-model, called ARX-Laguerre sub-model, is a new linear representation with respect to the filtered input and the filtered multimodel output. Therefore, the resulting multimodel, entitled ARX-Laguerre decoupled multimodel, ensures the parameter number reduction with a recursive and easy representation compared to the classical representation ARX multimodel. This result depends essentially on the value of the Laguerre pole of each basis. In fact, for L sub-models we have $2L$ Laguerre poles and an optimal choice of each Laguerre pole is compulsory to lessen considerably the parameters number. We propose then an analytical solution for Laguerre poles optimization by extending the works of Tanguy *et al* [17]. The proposed optimization consists in minimizing the upper bound of the squared norm of the error resulting from the truncation of the series into a finite number of Laguerre functions. The Laguerre poles are obtained using the knowledge of the ARX-Laguerre decoupled multimodel parameters which are estimated by the standard parameter estimation methods like RLS method.

The paper is organized as follows: in section 1 we introduce the principle of the development of the new structure of ARX-Laguerre decoupled multimodel by developing the coefficients

of each ARX sub-model on two independent Laguerre bases. Also, we present the proposed recursive vectorial representation of the ARX-Laguerre decoupled multimodel. In Section 2 we treat the extension and generalization of the work of Tanguy *et al.* [17] for the analytical calculation and the iterative identification of the Laguerre poles characterizing the ARX-Laguerre decoupled multimodel. Finally, in section 3 some simulation results have been given to illustrate the efficiency of the identification procedure of the Laguerre poles and the proposed ARX-Laguerre multimodel where this proposed multimodel is compared to the classical ARX multimodel.

II. ARX-LAGUERRE DECOUPLED MULTIMODEL

The methodology of the multimodel approach is to replace the research of a single model representing the output $y(k)$ by searching for a family of sub-models. Each sub-model is defined by its output $y_s(k)$ associated with a weighting function $\mu_s(\gamma(k))$. The weighting functions are used to determine the relative contribution of each sub-model in the region where the system's output is changing. We note that the variable $\gamma(k)$ is indexing functions $\mu_s(\cdot)$ that are specific to the system. Generally, it is either the input $\gamma(k) = u(k)$ and/or either of the output of the system $\gamma(k) = y_m(k)$ [6, 13]. In our case, we consider $\gamma(k) = u(k)$ and a general representation of the multimodel structure can be written as:

$$y(k) = \sum_{s=1}^L \mu_s(u(k)) y_s(k) \quad (1)$$

where L is the number of the sub-models such as:

$$\sum_{s=1}^L \mu_s(u(k)) = 1 \text{ and } 0 \leq \mu_s(u(k)) \leq 1, \forall s=1, \dots, L \quad (2)$$

In the following, we consider that each sub-model is represented by a strictly causal ARX model and we propose to develop each ARX sub-model on two independent Laguerre bases.

A. Principle of expansion of ARX multimodel on Laguerre orthonormal bases

Each sub-model of the multimodel approach can be described by a strictly causal discrete time ARX model as:

$$y_s(k) = \sum_{j=1}^{\infty} h_a^s(j) y(k-j) + \sum_{j=1}^{\infty} h_b^s(j) u(k-j), s=1, \dots, L \quad (3)$$

where $u(k)$ and $y(k)$ are the system input and the output of the multimodel respectively. In this paragraph, we detail the development of each ARX sub-model on independent Laguerre orthonormal bases. Due to stability condition in the meaning of Bounded Input Bounded Output (BIBO) criterion, the coefficients h_a^s and h_b^s satisfy:

$$\sum_{j=1}^{\infty} |h_a^s(j)| < \infty, \sum_{j=1}^{\infty} |h_b^s(j)| < \infty, s=1, \dots, L \quad (4)$$

and the ARX sub-model (3) can be written as:

$$y_s(k) = \sum_{j=1}^{na} h_a^s(j) y(k-j) + \sum_{j=1}^{nb} h_b^s(j) u(k-j), s=1, \dots, L \quad (5)$$

such that:

$$\begin{cases} h_a^s(j) = 0 & \text{if } j > na \\ h_b^s(j) = 0 & \text{if } j > nb \end{cases} \quad (6)$$

where na and nb ($nb \leq na$) are the model order associated to the output and the input respectively. As the coefficients h_a^s and h_b^s are summable, they belong to Lebesgue space $\ell_2[0, +\infty[$ and since the Laguerre orthonormal functions form an orthonormal basis in such space, the coefficients h_a^s and h_b^s can be decomposed on two independent Laguerre bases $\mathfrak{S}_a^s = \{\ell_{n,a}^s\}_{n=0}^{\infty}$ and $\mathfrak{S}_b^s = \{\ell_{n,b}^s\}_{n=0}^{\infty}$ with respect to the output and the input respectively as:

$$h_a^s(j) = \sum_{n=0}^{\infty} g_{n,a}^s \ell_{n,a}^s(j, \xi_a^s) \text{ and } h_b^s(j) = \sum_{n=0}^{\infty} g_{n,b}^s \ell_{n,b}^s(j, \xi_b^s) \quad (7)$$

where ξ_a^s ($|\xi_a^s| < 1$) and ξ_b^s ($|\xi_b^s| < 1$) are the Laguerre poles, $i = a, b, s = 1, \dots, L$ and $g_{n,a}^s$ and $g_{n,b}^s$ are the Fourier coefficients. By substituting the linear combinations of relation (7) in the ARX sub-model defined by (5) and (6), the resulting sub-model, entitled the ARX-Laguerre is written as:

$$y(k) = \sum_{n=0}^{\infty} g_{n,a}^s x_{n,y}^s(k) + \sum_{n=0}^{\infty} g_{n,b}^s x_{n,u}^s(k) \quad (8)$$

where $x_{n,y}^s$ and $x_{n,u}^s$ are the filtered output and the filtered input respectively by Laguerre functions such as:

$$x_{n,y}^s(k) = \sum_{j=1}^{\infty} \ell_{n,a}^s(j, \xi_a^s) y(k-j) = \ell_{n,a}^s(k, \xi_a^s) * y(k) \quad (9)$$

$$x_{n,u}^s(k) = \sum_{j=1}^{\infty} \ell_{n,b}^s(j, \xi_b^s) u(k-j) = \ell_{n,b}^s(k, \xi_b^s) * u(k) \quad (10)$$

where $*$ is the convolution product. In practice, the infinite series in can be truncated to a finite orders Na and Nb as:

$$y_s(k) = \sum_{n=0}^{Na-1} g_{n,a}^s x_{n,y}^s(k) + \sum_{n=0}^{Nb-1} g_{n,b}^s x_{n,u}^s(k) \quad (11)$$

From (1) and (11), we represent in Figure 1 the parallel structure of the du ARX-Laguerre decoupled multimodel:

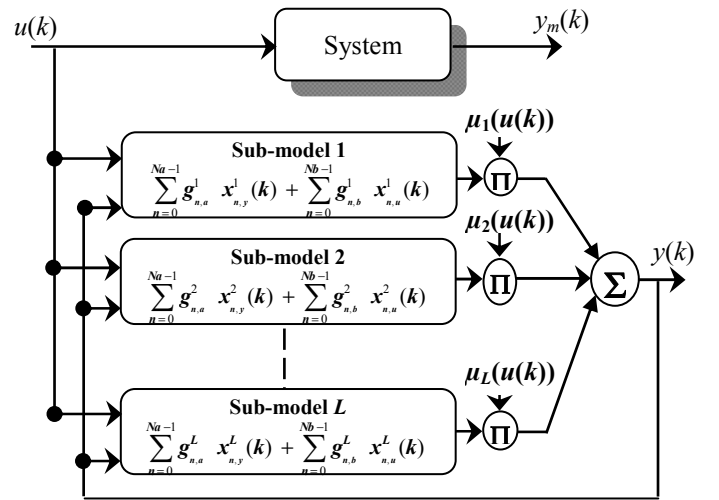


Figure 1. Parallel structure of the du ARX-Laguerre decoupled multimodel

B. Recursive vector representation

The Laguerre orthonormal functions $\ell_{n,i}^s, n=0, \dots, Ni-1, i = a, b, s=1, \dots, L$, are given by their Z-transform:

$$L_{n,i}^s(z) = \frac{\sqrt{1-(\xi_i^s)^2}}{z - \xi_i^s} \left(\frac{1 - \xi_i^s z}{z - \xi_i^s} \right)^n, \quad n = 0, 1, 2, \dots \quad (12)$$

From relation (12), the Z-transform of the Laguerre functions can be written in the following recurrent way:

$$\begin{cases} L_{0,i}^s(z) = \frac{\sqrt{1-(\xi_i^s)^2}}{z - \xi_i^s} \\ L_{n,i}^s(z) = \frac{1 - \xi_i^s z}{z - \xi_i^s} L_{n-1,i}^s(z), \quad n = 1, 2, \dots \end{cases} \quad i = a, b, s=1, \dots, L \quad (13)$$

By combining the recurrent form (13) with the Z-transform of the relations (9) and (10), we can formulate the recurrence relation for the filters $X_{n,y}^s(z) = Z[x_{n,y}^s(k)]$ and $X_{n,u}^s(z) = Z[x_{n,u}^s(k)]$, $n = 1, 2, \dots$ as:

$$\begin{cases} X_{0,y}^s(z) = \frac{\sqrt{1-(\xi_a^s)^2}}{z - \xi_a^s} Y(z) \\ X_{n,y}^s(z) = \frac{1 - \xi_a^s z}{z - \xi_a^s} X_{n-1,y}^s(z) \end{cases}, \quad \begin{cases} X_{0,u}^s(z) = \frac{\sqrt{1-(\xi_b^s)^2}}{z - \xi_b^s} U(z) \\ X_{n,u}^s(z) = \frac{1 - \xi_b^s z}{z - \xi_b^s} X_{n-1,u}^s(z) \end{cases} \quad (14)$$

From the two systems of relation (14) we obtain for each ARX-Laguerre sub-model (11) the following recursive vector representation for $s=1, \dots, L$:

$$\begin{cases} X^s(k) = A^s X^s(k-1) + b_y^s y(k-1) + b_u^s u(k-1) \\ y^s(k) = (c^s)^T X^s(k) \end{cases} \quad (15)$$

with :

$$-X^s(k) = [x_{0,y}^s(k), \dots, x_{Na-1,y}^s(k), \dots, x_{0,u}^s(k), \dots, x_{Nb-1,u}^s(k)]^T \in \mathbb{R}^{Na+Nb} \quad (16)$$

$$-c^s = [g_{0,a}^s, \dots, g_{Na-1,a}^s, \dots, g_{0,b}^s, \dots, g_{Nb-1,b}^s]^T \in \mathbb{R}^{Na+Nb} \quad (17)$$

- A^s is an $Na+Nb$ - dimensional square matrix:

$$A^s = \begin{pmatrix} A_y^s & 0_{Na, Nb} \\ 0_{Nb, Na} & A_u^s \end{pmatrix} \quad (18)$$

with $0_{m,q}$ is an $(m \times q)$ - dimensional null matrix. A_y^s and A_u^s are two square matrices with dimensions Na and Nb respectively, defined as:

$$A_y^s = \begin{bmatrix} \xi_a^s & 0 & \dots & 0 \\ 1 - (\xi_a^s)^2 & \xi_a^s & \dots & 0 \\ -\xi_a^s (1 - (\xi_a^s)^2) & 1 - (\xi_a^s)^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ (-\xi_a^s)^{Na-2} (1 - (\xi_a^s)^2) & (-\xi_a^s)^{Na-3} (1 - (\xi_a^s)^2) & \dots & \xi_a^s \end{bmatrix} \quad (19)$$

$$A_u^s = \begin{bmatrix} \xi_b^s & 0 & \dots & 0 \\ 1 - (\xi_b^s)^2 & \xi_b^s & \dots & 0 \\ -\xi_b^s (1 - (\xi_b^s)^2) & 1 - (\xi_b^s)^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ (-\xi_b^s)^{Nb-2} (1 - (\xi_b^s)^2) & (-\xi_b^s)^{Nb-3} (1 - (\xi_b^s)^2) & \dots & \xi_b^s \end{bmatrix} \quad (20)$$

- b_y^s and b_u^s are two $Na+Nb$ - dimensional vectors:

$$b_y^s = \sqrt{1-(\xi_a^s)^2} \begin{bmatrix} 1 \\ -\xi_a^s \\ (-\xi_a^s)^2 \\ \vdots \\ (-\xi_a^s)^{Na-1} \\ 0_{Nb,1} \end{bmatrix}, \quad b_u^s = \sqrt{1-(\xi_b^s)^2} \begin{bmatrix} 0_{Na,1} \\ 1 \\ -\xi_b^s \\ (-\xi_b^s)^2 \\ \vdots \\ (-\xi_b^s)^{Nb-1} \end{bmatrix} \quad (21)$$

where $0_{Ni,1}$ is a null vector of dimension Ni , $i=a, b$.

Therefore, from the relations (1) and (15) we can propose the following recursive vector representation of the ARX-Laguerre decoupled multimodel:

$$\begin{cases} \underline{X}(k) = \underline{A} \underline{X}(k-1) + \underline{b}_y y(k-1) + \underline{b}_u u(k-1) \\ y(k) = \underline{C}^T \underline{M}_\mu \underline{X}(k) \end{cases} \quad (22)$$

with:

$$-\underline{X}(k) = \left[(X^1(k))^T, \dots, (X^L(k))^T \right]^T \in \mathbb{R}^{L(Na+Nb)} \quad (23)$$

- \underline{A} is an $L(Na+Nb)$ - dimensional square matrix:

$$\underline{A} = \begin{pmatrix} A^1 & 0_{Na+Nb} & \dots & 0_{Na+Nb} \\ 0_{Na+Nb} & A^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0_{Na+Nb} \\ 0_{Na+Nb} & \dots & 0_{Na+Nb} & A^L \end{pmatrix} \quad (24)$$

where 0_{Na+Nb} is an $Na+Nb$ - dimensional null square matrix.

- \underline{b}_y et \underline{b}_u are two $L(Na+Nb)$ - dimensional vectors:

$$\underline{b}_y = \left[(b_y^1)^T, \dots, (b_y^L)^T \right]^T, \quad \underline{b}_u = \left[(b_u^1)^T, \dots, (b_u^L)^T \right]^T \quad (25)$$

$$-\underline{C} = [c_1^T, \dots, c_L^T]^T \in \mathbb{R}^{L(Na+Nb)} \quad (26)$$

- \underline{M}_μ is an $L(Na+Nb)$ - dimensional square matrix:

$$\underline{M}_\mu = \begin{pmatrix} \mu_1 I_{Na+Nb} & 0_{Na+Nb} & \dots & 0_{Na+Nb} \\ 0_{Na+Nb} & \mu_2 I_{Na+Nb} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0_{Na+Nb} \\ 0_{Na+Nb} & \dots & 0_{Na+Nb} & \mu_L I_{Na+Nb} \end{pmatrix} \quad (27)$$

with I_{Na+Nb} is an $Na+Nb$ - dimensional identity matrix.

III. OPTIMISATION OF LAGUERRE POLES

The parameter number of the new ARX-Laguerre decoupled multimodel (22) is $L(Na+Nb)$. Then, comparing the relations (5) and (11), a significant reduction of the parameter number is satisfied if $Na+Nb \ll na+nb$. This latter depends mainly on the poles ξ_a^s and ξ_b^s characterizing each basis so that the provided multimodel better describes the nonlinear system dynamics with small truncating orders. We propose then a separate optimization with an analytical solution for Laguerre poles optimization based on the works of Tanguy *et al.* [17]. To do so, we consider the following cost functions for $s=1, \dots, L$:

$$J_a^s = \frac{1}{\|h_a^s\|^2} \sum_{n=0}^{\infty} n (g_{n,a}^s)^2 \quad \text{and} \quad J_b^s = \frac{1}{\|h_b^s\|^2} \sum_{n=0}^{\infty} n (g_{n,b}^s)^2 \quad (28)$$

where:

$$\|h_a^s\|^2 = \sum_{j=0}^{\infty} (h_a^s(j))^2, \quad \|h_b^s\|^2 = \sum_{j=0}^{\infty} (h_b^s(j))^2 \quad (29)$$

We define the following quantities:

$$M_{1,a}^s = \frac{1}{\|h_a^s\|^2} \sum_{j=0}^{\infty} j (h_a^s(j))^2, \quad M_{1,b}^s = \frac{1}{\|h_b^s\|^2} \sum_{j=0}^{\infty} j (h_b^s(j))^2 \quad (30)$$

$$M_{2,a}^s = \frac{1}{\|h_a^s\|^2} \sum_{j=0}^{\infty} j h_a^s(j) h_a^s(j-1), \quad M_{2,b}^s = \frac{1}{\|h_b^s\|^2} \sum_{j=0}^{\infty} j h_b^s(j) h_b^s(j-1) \quad (31)$$

Then we obtain:

$$J_i^s = \frac{(1+M_{1,i}^s) (\xi_i^s)^2 - 2 M_{2,i}^s \xi_i^s + M_{1,i}^s}{(1 - (\xi_i^s)^2)}, \quad i = a, b, s = 1, \dots, L \quad (32)$$

and the optimal poles $\xi_{opt,a}^s$ and $\xi_{opt,b}^s$ solutions of $\partial J_a^s / \partial \xi_a^s$ and $\partial J_b^s / \partial \xi_b^s$ respectively, $s=1, \dots, L$, can be computed as:

$$\xi_{opt,i}^s = \begin{cases} \rho_i^s - \sqrt{(\rho_i^s)^2 - 1} & \text{if } \rho_i^s > 1 \\ \rho_i^s + \sqrt{(\rho_i^s)^2 - 1} & \text{if } \rho_i^s < -1 \end{cases}, \quad i = a, b \quad (33)$$

with:

$$\rho_i^s = \frac{2 M_{1,i}^s + 1}{2 M_{2,i}^s} \quad (34)$$

In order to write the quantities $M_{1,i}^s$ and $M_{2,i}^s$, $i = a, b$, $s=1, \dots, L$, and to compute the optimal Laguerre poles as functions of Fourier coefficients, we define:

$$T_{1,a}^s = \sum_{n=0}^{\infty} (2n+1) (g_{n,a}^s)^2 \quad \text{and} \quad T_{1,b}^s = \sum_{n=0}^{\infty} (2n+1) (g_{n,b}^s)^2 \quad (35)$$

$$T_{2,a}^s = 2 \sum_{n=0}^{\infty} n g_{n,a}^s g_{n-1,a}^s \quad \text{and} \quad T_{2,b}^s = 2 \sum_{n=0}^{\infty} n g_{n,b}^s g_{n-1,b}^s \quad (36)$$

and then $M_{1,i}^s$ and $M_{2,i}^s$ are given by:

$$\begin{cases} M_{1,i}^s = \frac{(1+(\xi_i^s)^2) T_{1,i}^s + 2 \xi_i^s T_{2,i}^s}{2 (1 - (\xi_i^s)^2) \|h_i^s\|^2} - \frac{1}{2} \\ M_{2,i}^s = \frac{(1+(\xi_i^s)^2) T_{2,i}^s + 2 \xi_i^s T_{1,i}^s}{2 (1 - (\xi_i^s)^2) \|h_i^s\|^2} \end{cases}, \quad i = a, b, \quad s=1, \dots, L \quad (37)$$

Due to (37) the parameter ρ_i^s given by relation (34) can be written as functions of Fourier coefficients:

$$\rho_i^s = \frac{(1+(\xi_i^s)^2) T_{1,i}^s + 2 \xi_i^s T_{2,i}^s}{(1+(\xi_i^s)^2) T_{2,i}^s + 2 \xi_i^s T_{1,i}^s}, \quad i = a, b, \quad s = 1, \dots, L \quad (38)$$

We conclude that the relations (33) and (38) enables to determine the $2L$ optimal poles from the coefficients of ARX-Laguerre multimodel without computing the coefficients of the ARX multimodel. Moreover, we note that the optimal values of Laguerre poles do not depend on the truncating orders Na and Nb . From Tanguy *et al.* [17] we have:

$$2 \|h_i^s\|^2 \frac{\partial J_i^s}{\partial \xi_i^s} = - \frac{2}{(1 - (\xi_i^s)^2)} T_{2,i}^s, \quad i = a, b, \quad s = 1, \dots, L \quad (39)$$

Consequently, from (39) $T_{2,i}^s$ is zero when J_i^s is minimal and the reciprocal is true; that is, when $T_{2,i}^s$ is nearly zero, then J_i^s is close to its minimal value and therefore the pole ξ_i^s is close to its optimal value $\xi_{opt,i}^s$. This property will be used to develop an offline identification iterative algorithm of the ARX-Laguerre multimodel. The convergence of this algorithm is controlled by the weak values of $T_{2,i}^s$, $i = a, b$.

Algorithm 1: Iterative identification of the optimal poles

1. Assume that we have H input/output pairs (u, y_m) .
2. Fix the truncating orders $Na > 1$ and $Nb > 1$ and choose arbitrary two initial stable poles $\xi_{a,int}^s$ and $\xi_{b,int}^s$, $s = 1, \dots, L$.
3. Estimate the Fourier coefficients $g_{n,a}^s$ et $g_{n,b}^s$ with the RLS method and compute $T_{1,i}^s$ and $T_{2,i}^s$, $i = a, b$, $s = 1, \dots, L$.
4. If $T_{2,a}^s$ and $T_{2,b}^s$ are close to zero \Rightarrow End.
Else compute ρ_a^s and ρ_b^s using (38) and determine the new poles $\xi_{opt,a}^s$ and $\xi_{opt,b}^s$ using (33) then go to step 3.

IV. NUMIRECAL SIMULATION

For evaluating the performances of the proposed ARX-Laguerre decoupled multimodel and the identification approach of Laguerre poles let's consider the following nonlinear model:

$$y_m(k) = \frac{y_m(k-1) y_m(k-2) y_m(k-3) u(k-2) [y_m(k-3)-1] + u(k-1)}{(1+y_m^2(k-2)+y_m^3(k-3))}$$

1500 input/output observations were collected from the process where the process input $u(k)$ was chosen to be a pseudo random sequence with variable amplitude between -0.4 and 1.2. The evolution of $u(k)$ and the corresponding output $y_m(k)$ are drawn in Figures 2 and 3 respectively. The output is corrupted by an additive white and gaussian noise which is independent of the input signal and with a signal to noise ratio equals to 30dB. We evaluate the performances of the resulting ARX-Laguerre decoupled multimodel by computing the Normalized Mean square Error (NMSE) on a measurement window F :

$$NMSE = \frac{\sum_{k=1}^F [y_m(k) - y(k)]^2}{\sum_{k=1}^F [y_m(k)]^2}$$

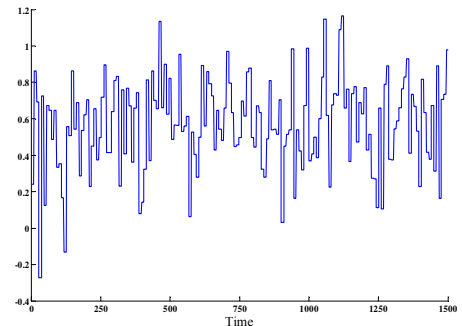


Figure 2. Process input $u(k)$

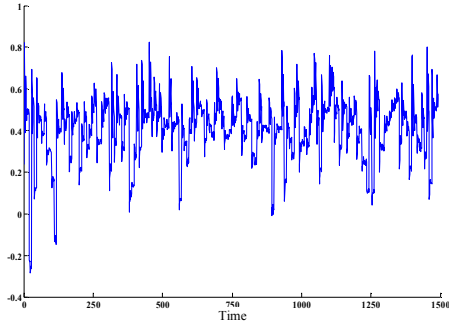


Figure 3. Precess output $y_m(k)$

We consider $L = 2$ the number of sub-models where the two weighting functions $\mu_1(u(k))$ and $\mu_2(u(k))$ (Figures 4) are constructed from the normalization of the gaussian functions

$$w_1(u(k)) = \exp\left(-\frac{(u(k)-0.18)^2}{(0.9)^2}\right) \text{ and } w_2(u(k)) = \exp\left(-\frac{(u(k)-0.52)^2}{(0.9)^2}\right)$$

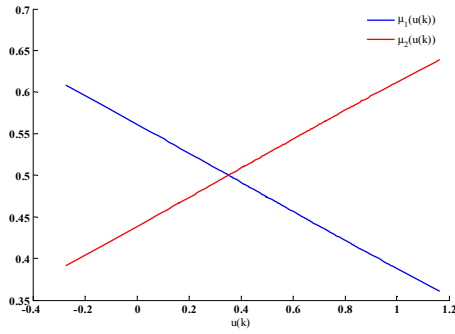


Figure 4. Weighting functions $\mu_1(u(k))$ and $\mu_2(u(k))$

In Table 1, we summarize the performances of the algorithm 1 for two combinations of truncating orders ($Na = Nb = 2$) and ($Na = Nb = 3$). Moreover, Figures 5 and 6 illustrate for the fixed values of Na and Nb the behavior of Laguerre poles estimated iteratively through the proposed optimization algorithm. Figures 7 and 8 plot the evolution of the criteria $R_{2,a}^s$ and $R_{2,b}^s$, $s = 1, 2$.

Na	Nb	ξ_a^1	ξ_b^1	ξ_a^2	ξ_b^2	$NMSE$ (%)
2	2	-0.388	-0.424	-0.338	-0.245	0.319
3	3	-0.379	-0.433	-0.329	-0.248	0.272

Table 1. Optimal values of Laguerre poles for different truncating orders

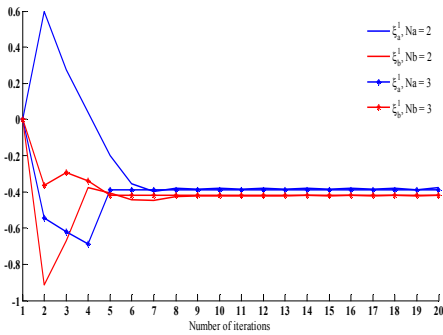


Figure 5. Iterative Identification of Laguerre poles ξ_a^1 and ξ_b^1 for $Na = Nb = 2$ and $Na = Nb = 3$

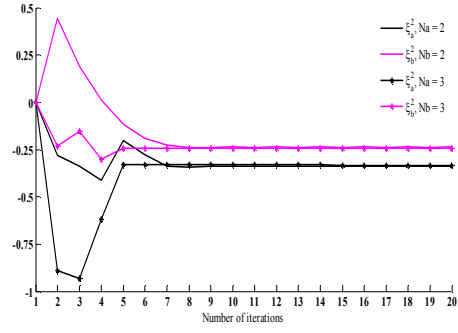


Figure 6. Iterative Identification of Laguerre poles ξ_a^2 and ξ_b^2 for $Na = Nb = 2$ and $Na = Nb = 3$

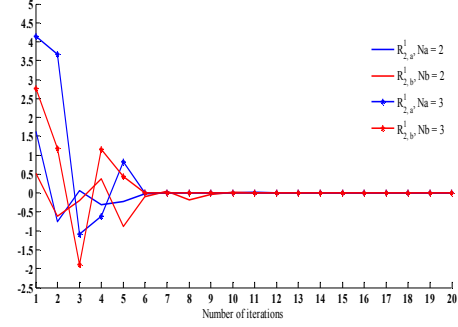


Figure 7. Evolution of the criteria $R_{2,a}^1$ and $R_{2,b}^1$

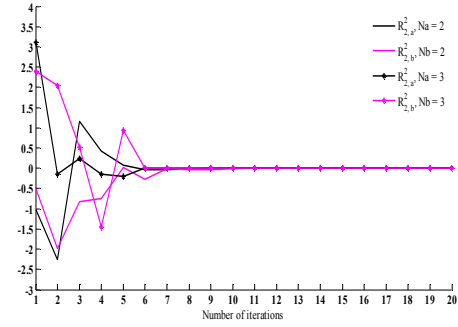


Figure 8. Evolution of the criteria $R_{2,a}^2$ and $R_{2,b}^2$

From Table 1, we note the slow variation of poles with truncating orders. However, from Figures 5 and 6 an increase of the truncating order implies a fast convergence of estimated poles but with computation expense and higher accuracy expressed by small values of the $NMSE$ (Table 1). From Figures 7 and 8 the convergence criteria $R_{2,a}^s$ and $R_{2,b}^s$, $s = 1, 2$, characterizes and shows the convergence of algorithm 1 of the poles identification. We note too that the iterative identification of Laguerre poles ensures the parameter number reduction. In fact, the modeling performance throughout the validation phase on 800 observations is shown in Figures 9 and 10 where we illustrate the evolution of the outputs of ARX-Laguerre and ARX multimodels for the same number of parameters equals to 4. For the ARX-Laguerre decoupled multimodel with $Na = Nb = 1$, the $NMSE$ equals 1.38 %. For $na = nb = 1$ it resorts that the $NMSE$ relative to ARX multimodel is 3.59 %. So we note the good correspondence between outputs in Figure 7 and the discordance in Figure 10, that illustrates the advantage of the ARX-Laguerre decoupled multimodel.

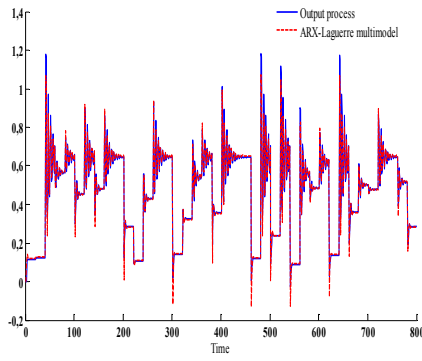


Figure 9. Validation of the ARX-Laguerre multimodel, $N_a = N_b = 1$

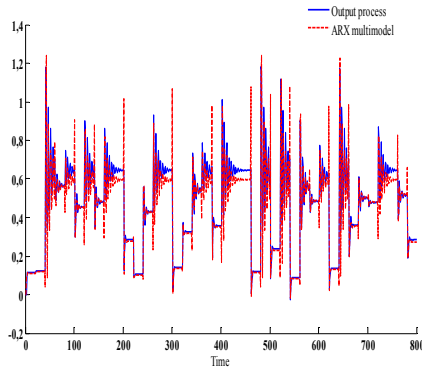


Figure 10. Validation of the ARX multimodel, $n_a = n_b = 1$

V. CONCLUSION

In this paper, a new approach for the input-output modelling of nonlinear systems has been introduced which overcomes the parameter complexity of the ARX multimodel where each sub-model is represented by an ARX model. This reduction has been insured by developing each ARX sub-model parameters on two independent Laguerre bases associated to the input and the output. The proposed multimodel is entitled the ARX-Laguerre decoupled multimodel presented as a simple recurrent representation. Each of the Laguerre bases is characterized by one Laguerre pole so that its optimal value ensures a significant parameter reduction of the proposed multimodel. Therefore, we have extended the works of Tanguy *et al.* by proposing an optimization algorithm to provide in an analytical way the optimal values for Laguerre poles. This optimization depends on Fourier coefficients of the ARX-Laguerre decoupled multimodel which are identified by the RLS method. The resulting multimodel as well as the poles optimization procedure are tested in numerical simulation.

REFERENCES

- [1] S. Boyd and L.O. Chua, "Fading memory and the problem of approximating nonlinear operators with Volterra series" IEEE Tr. Circuits and Systems vol. CAS-32, No. 11, 1985, pp. 1150-1171.
- [2] R.J.G.B. Campello, G. Favier and W.C. Amaral, "Optimal expansions of discrete-time Volterra models using Laguerre functions" Proc. of IFAC SYSID, pp. 1845-1849, Rotterdam, The Netherlands, Aug. 2003.
- [3] H. Demuth, M. Beale and M. Hagan, "Neural Network Toolbox 5, User's Guide" The MathWorks, 2007.
- [4] D. Filev, "Fuzzy modeling of complex systems". International Journal of Approximate Reasoning, Vol. 5, Issue 3, 1991, pp. 281-290.
- [5] Y. Fu and G.A. Dumont, "An Optimum Time-scale for Discrete Time Laguerre Network" IEEE Transaction on Automatic control. Vol. 38, Issue 6, 1993, pp. 934 - 938.
- [6] K. Gasso, "Identification des systèmes dynamiques non-linéaires : approche multi-modèle" Thèse de doctorat à l'INPL : Institut National Polytechnique de Lorraine, France, 2000.
- [7] T. A. Johansen and A. B. Foss, "Constructing NARMAX using ARMAX models" International Journal of Control, Vol. 58, Issue 5, 1993, pp. 1125-1153.
- [8] T. A. Johansen and B. A. Foss, "Identification of non-linear system structure and parameters using regime decomposition" Automatica, Vol. 31, Issue 2, 1995, pp. 321-326.
- [9] A.Y. Kibangou, G. Favier and M.M. Hassani, "Laguerre-Volterra Filters Optimization Based on Laguerre Spectra" EURASIP Journal on Advances in Signal Processing, Vol. 17, Issue 2005, 2005, pp. 2874-2887.
- [10] R. Malti, "Représentation de systèmes discrets sur la base des filtres orthogonaux - Application à la modélisation de systèmes dynamiques multivariables" Thèse de doctorat à l'INPL : Institut National Polytechnique de Lorraine, France, 1999.
- [11] R. Murray-Smith and T. Johansen, "Multiple model Approaches to Modelling and Control" Taylor & Francis, London". 1997.
- [12] S. J. Norquay, A. Palazoglu and J. A. Romagnoli, "Application of Wiener model predictive control (WMPC) to a pH neutralization experiment" IEEE transactions on control systems technology, Vol. 7, Issue 4, 1999, pp. 437-445.
- [13] R. Orjuela, "Contribution à l'estimation d'état et au diagnostic des systèmes représentés par des multimodèles" Thèse de doctorat à l'INPL : Institut National Polytechnique de Lorraine, France, 2008.
- [14] P. Saha, S.H. Krishnan, V.S.R. Rao and S.C. Patwardhan, "Modeling and Predictive Control of MIMO Nonlinear Systems Using Wiener-Laguerre Models". Chemical Engineering Communications, Vol. 191, Issue 8, 2004, pp. 1083-1119.
- [15] D. Sbarbaro, "Context sensitive networks for modelling nonlinear dynamic systems" Proceedings of 3rd European Control Conference, ECC 95, Rome (Italy), 1995, pp. 2420-2424.
- [16] D. Sbarbaro and Johansen T.A., "Multiple local Laguerre models for modelling nonlinear dynamic systems of the Wiener class" IEE Proceedings, Control Theory Applications, Vol. 144, Issue 5, 1997, pp. 375-380.
- [17] N. Tanguy, R. Morvan, P. Vilbé and L. C. Calvez, "Online optimization of the time scale in adaptive Laguerre-based filters" IEEE Trans. on Signal Processing, Vol. 48, Issue 4, 2000, pp. 1184-1187.
- [18] G. Yongsheng, T. Liwei, W. Ming and W. Sha, "Nonlinear system identification of gear crack failure based on second order Volterra kernel" International Journal of Modelling, Identification and Control, Vol. 11, Issue 3/4, 2010, pp. 173 - 179.