

Power Allocation for Cognitive Radio Networks Under Channel Uncertainty

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Abstract—Cognitive radio (CR) networks can re-use the RF spectrum licensed to the primary user (PU) network by carefully controlling the interference to the PUs. However, due to lack of explicit support from the PU system, CR sensing algorithms often face difficulty in acquiring CR-to-PU channels accurately. Moreover, the sensing algorithms cannot detect silent PU receivers, which nevertheless have to be protected. In order to achieve aggressive spectrum re-use even in such challenging scenarios, a CR power control problem with probabilistic interference constraints is formulated. Both log-normal shadowing and small-scale fading uncertainties are taken into account through suitable approximations. In particular, a weighted sum-rate maximization problem is considered, whose Karush-Kuhn-Tucker points are obtained via sequential geometric programming. Numerical tests verify the performance of our novel approach.

Index Terms—Cognitive radio, power control, channel uncertainty, geometric programming.

I. INTRODUCTION

Cognitive radio (CR) systems aim to make opportunistic use of the RF spectrum that has been licensed to (but is not fully utilized by) primary user (PU) systems. Provided that the interference channels from the CR transmitters to the PU receivers can be accurately acquired through CR spectrum sensing algorithms, judicious control of transmission powers of the CR systems can protect the PU transmissions while maximizing the performance metric of the CR network. Such a configuration is often termed as spectrum underlay [1].

However, typically due to lack of collaboration mechanisms between PU and CR systems, reliably detecting the presence of PU transmissions, and estimating the CR-to-PU channels require considerable effort. More challenging is acquiring passive PU receivers, which do not transmit RF energy but just listen. Nonetheless, those receivers still need to be protected under the PU-CR hierarchy.

One way to estimate the potential PU receiver locations, and eventually their channel gains, is to rely on the idea of channel gain cartography [2]. Once the locations of the PU transmitters are acquired through CR sensing [3], the channel gain maps allow estimation of the corresponding PU coverage regions, in which the PU receivers must reside [4].

CR power control in underlay scenarios has been studied in the literature often under the assumption of perfect CR-to-PU channel knowledge [5], [6], [7]. Limited-rate feedback from

the PU system on instantaneous channel gains was assumed available, and exploited for CR resource allocation in [8]. Focusing on a single CR link and a PU link, [9] analyzed the ergodic capacity under Rayleigh fading using various channel uncertainty models. Uncertain CR-to-PU channels were considered in [10], where the CR network sum rate was maximized under a constraint on the outage probability of a single PU; however, the instantaneous interference caused to the PU was approximated by using a deterministic coefficient for the average interference gain multiplied by the sum of the CR powers, which holds only under a dense CR network assumption. Also, the outage constraint was reported for i.i.d. Rayleigh-distributed channels.

The present work deals with multiple CR and PU links under both spatially correlated log-normal shadow fading and small-scale Nakagami fading. Our approach is to exploit statistical channel knowledge of the CR-to-PU channels, which may be obtained through location information of the PU receivers, or based on the channel gain cartography. Since the wireless channel involves shadow fading as well as small-scale fading, both sources of uncertainty must be taken into account. As PU protection constraints must be enforced with high reliability, probabilistic constraints will be imposed in order to guarantee that the interference power experienced by PU receivers falls below a tolerable level with a given high probability.

Since exact computation of the interference probabilities is not tractable, approximation techniques are employed. Specifically, the sum of interfering signal powers is approximated as log-normal through the use of Fenton-Wilkinson method [11]. The resulting chance-constrained weighted sum-rate maximization problem is non-convex, but can be re-formulated so that its Karush-Kuhn-Tucker (KKT) solutions are obtained via sequential geometric programming (GP). This is remarkable considering the fact that the power allocation problem with *perfect* channel knowledge is also typically non-convex and requires a sequential GP approach [12]. The sequential GP formulation can also benefit from the availability of efficient interior-point solvers optimized for geometric programs.

The rest of the paper is organized as follows. Sec. II describes the system model and formulates the optimization problem. Sec. III approximates the probabilistic interference constraints. Problem re-formulation and solution based on sequential GP are described in Sec. IV. Sec. V provides the results from numerical tests, and Sec. VI concludes the paper.

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II. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a CR network comprising K transmitter-receiver pairs, sharing spectrum licensed to a PU system. It is assumed that the locations of the CR transmitters and receivers are known to the CR system. Let \mathbf{x}_k and \mathbf{u}_k denote the locations of the k -th CR transmitter and receiver, respectively, where $k \in \{1, 2, \dots, K\}$.

In order to make opportunistic use of the spectrum under PU-CR hierarchy, the CR system first employs spectrum sensing algorithms to detect the PU system activity in space and time domains [3], [2]. Based on the sensing results, power control is performed to prevent excessive interference to the PU system, and to maximize the CR network performance. The focus of the present work is on this resource allocation task.

Let $g_{\mathbf{x}_k \rightarrow \mathbf{u}_k}$ denote the channel gain of link $\mathbf{x}_k \rightarrow \mathbf{u}_k$ at a given time instant, which can be expressed as

$$g_{\mathbf{x}_k \rightarrow \mathbf{u}_k} = g_k \|\mathbf{x}_k - \mathbf{u}_k\|^{-\eta} s_{\mathbf{x}_k \rightarrow \mathbf{u}_k} |h_{\mathbf{x}_k \rightarrow \mathbf{u}_k}|^2 \quad (1)$$

where g_k collects the antenna and other propagation gains, η is the path loss exponent, $s_{\mathbf{x}_k \rightarrow \mathbf{u}_k}$ denotes shadow fading, and $h_{\mathbf{x}_k \rightarrow \mathbf{u}_k}$ small-scale fading of the link. Shadowing can be accurately modeled as log-normal, and the small-scale fading as Nakagami- m distributed [13, Ch. 2]. It is also assumed that the small-scale fading is independent across different links, and of the shadow fading. On the other hand, shadowing may be correlated across links with correlation given in [14], [15].

Let $p_k \in (0, p_k^{\max}]$ denote the transmission power of CR k , capped by p_k^{\max} , and $\mathbf{p} := [p_1, \dots, p_K]^T$ collect the transmit-powers of all K CRs. Let also π_k denote the received PU signal power as well as other interference measured at the k -th CR receiver. Then, the instantaneous signal-to-interference-plus-noise power ratio (SINR) at CR receiver k can be expressed as

$$\gamma_k := \frac{p_k g_{\mathbf{x}_k \rightarrow \mathbf{u}_k}}{\sum_{i=1, i \neq k}^K p_i g_{\mathbf{x}_i \rightarrow \mathbf{u}_k} + \pi_k + \sigma_k^2}, \quad k = 1, 2, \dots, K \quad (2)$$

where σ_k^2 is the receiver noise power at CR k . Define $\boldsymbol{\gamma} := [\gamma_1, \dots, \gamma_K]^T$.

Suppose that the positions of R PU receivers $\{\mathbf{r}_r\}_{r=1}^R$ have been acquired via spectrum sensing [2]. Alternatively, $\{\mathbf{r}_r\}$ may denote the candidate positions of the PU receivers that serve as reference positions for constraining interference to the PU system. Supposing incoherent superposition of CR waveforms [16], the interference power experienced at PU receiver r due to CR transmissions is given by

$$i_r := \sum_{k=1}^K p_k g_{\mathbf{x}_k \rightarrow \mathbf{r}_r}, \quad r = 1, 2, \dots, R. \quad (3)$$

The power control problem of interest is to maximize the weighted sum-rate of the CR system while constraining interference to the PU system. Other types of objective functions such as proportional fairness, harmonic-mean fairness, and max-min fairness utilities can be alternatively accommodated

by the proposed framework. These extensions will be reported in the journal version of this work.

If the CR-to-CR channel gains $\{g_{\mathbf{x}_k \rightarrow \mathbf{u}_k}\}$ as well as the CR-to-PU channel gains $\{g_{\mathbf{x}_k \rightarrow \mathbf{r}_r}\}$ were perfectly known to the CR network, the optimal transmission power vector \mathbf{p} that maximizes a weighted sum-rate without causing harmful interference to the PU system would be obtained by solving

$$(P1) \quad \max_{\mathbf{p} > \mathbf{0}} \sum_{k=1}^K w_k \log_2(1 + \gamma_k(\mathbf{p})) \quad (4)$$

$$\text{subject to } p_k \leq p_k^{\max}, \quad k = 1, \dots, K \quad (5)$$

$$i_r(\mathbf{p}) \leq i_r^{\max}, \quad r = 1, \dots, R \quad (6)$$

where $\{w_k\}$ are non-negative weights, and i_r^{\max} denotes the maximum interference power that can be tolerated by PU receiver r . Problem (P1) is in general non-convex, but a successive GP technique can be employed to obtain a locally optimal solution efficiently [12].

In practice, precise knowledge of the CR-to-PU channels requires explicit coordination between the PU and the CR systems. In typical CR scenarios where such coordination is infeasible, statistical knowledge of $\{g_{\mathbf{x}_k \rightarrow \mathbf{r}_r}\}$ may be collected and used. In this case, to protect the PU transmissions under channel uncertainty, probabilistic interference constraints are well motivated. Then, the problem of interest becomes

$$(P2) \quad \max_{\mathbf{p} > \mathbf{0}} \sum_{k=1}^K w_k \log_2(1 + \gamma_k(\mathbf{p})) \quad (7)$$

$$\text{subject to } p_k \leq p_k^{\max}, \quad k = 1, \dots, K \quad (8)$$

$$\text{Prob}\{i_r > i_r^{\max}\} \leq \epsilon_r, \quad r = 1, \dots, R \quad (9)$$

where $\epsilon_r > 0$ is a prescribed parameter representing the upper-bound on the probability that the interference due to CR transmissions exceeds a given threshold i_r^{\max} at PU receiver r . On top of the non-convexity issue, an additional challenge in solving (P2) is the chance constraint (9). In the next section, a suitable approximation method will be employed to first address the challenge associated with (9).

III. APPROXIMATION OF INTERFERENCE CONSTRAINTS

In order to solve (P2), the chance constraints (9) must be written explicitly in terms of the optimization variable \mathbf{p} . As the random variable (r.v.) i_r involves summation of powers affected by possibly correlated shadow fading and small-scale fading, direct characterization of its distribution will not lead to a tractable optimization formulation. To sidestep this hurdle, the distribution of i_r is approximated in the sequel.

First, let us focus on the composite fading $a_{\mathbf{x}_k \rightarrow \mathbf{r}_r} := s_{\mathbf{x}_k \rightarrow \mathbf{r}_r} \cdot |h_{\mathbf{x}_k \rightarrow \mathbf{r}_r}|^2$, which is denoted in dB as $A_{\mathbf{x}_k \rightarrow \mathbf{r}_r} := 10 \log_{10} a_{\mathbf{x}_k \rightarrow \mathbf{r}_r}$. Upon denoting the mean and the variance of the shadowing component $S_{\mathbf{x}_k \rightarrow \mathbf{r}_r} := 10 \log_{10} s_{\mathbf{x}_k \rightarrow \mathbf{r}_r}$ in dB as $\mu_{S_{\mathbf{x}_k \rightarrow \mathbf{r}_r}}$ and $\sigma_{S_{\mathbf{x}_k \rightarrow \mathbf{r}_r}}^2$, respectively, the probability density function (*p.d.f.*) of r.v. $a_{\mathbf{x}_k \rightarrow \mathbf{r}_r}$ is given by the Gamma-log-

normal density

$$f_{a_{\mathbf{x}_k \rightarrow \mathbf{r}_r}}(a) = \int_0^\infty \left(\frac{m}{u}\right)^m \frac{a^{m-1}}{\Gamma(m)} e^{-\frac{ma}{u}} \cdot \frac{1}{\sqrt{2\pi\kappa\sigma_{S_{\mathbf{x}_k \rightarrow \mathbf{r}_r}}} u} e^{-\frac{(10 \log_{10}(u) - \mu_{S_{\mathbf{x}_k \rightarrow \mathbf{r}_r}})^2}{2\sigma_{S_{\mathbf{x}_k \rightarrow \mathbf{r}_r}}^2}} du \quad (10)$$

where $\kappa := \frac{1}{10} \ln 10$, and $\Gamma(\cdot)$ is the Gamma function. It is known that this can be well-approximated by the log-normal density as [13, Ch. 2]

$$f_{a_{\mathbf{x}_k \rightarrow \mathbf{r}_r}}(a) \approx \frac{1}{\sqrt{2\pi\kappa\sigma_{A_{\mathbf{x}_k \rightarrow \mathbf{r}_r}}} a} e^{-\frac{(10 \log_{10}(a) - \mu_{A_{\mathbf{x}_k \rightarrow \mathbf{r}_r}})^2}{2\sigma_{A_{\mathbf{x}_k \rightarrow \mathbf{r}_r}}^2}} \quad (11)$$

where $\mu_{A_{\mathbf{x}_k \rightarrow \mathbf{r}_r}}$ and $\sigma_{A_{\mathbf{x}_k \rightarrow \mathbf{r}_r}}^2$ are given by

$$\mu_{A_{\mathbf{x}_k \rightarrow \mathbf{r}_r}} = \kappa^{-1} \left(-\ln m - C + \sum_{m'=1}^{m-1} \frac{1}{m'} \right) + \mu_{S_{\mathbf{x}_k \rightarrow \mathbf{r}_r}} \quad (12)$$

$$\sigma_{A_{\mathbf{x}_k \rightarrow \mathbf{r}_r}}^2 = \kappa^{-2} \zeta(2, m) + \sigma_{S_{\mathbf{x}_k \rightarrow \mathbf{r}_r}}^2 \quad (13)$$

respectively. Here, $C \approx 0.5772$ is Euler's constant and $\zeta(\cdot, \cdot)$ the Hurwitz zeta function. The approximation in (11) is quite accurate in the propagation scenarios of practical interest, but starts to deteriorate when $m = 1$ (Rayleigh fading) and $\sigma_{S_{\mathbf{x}_k \rightarrow \mathbf{r}_r}} < 6$ dB [17]. The approximation was used in [16] in a cellular network context, and more recently for interference modeling in CR networks [18].

Under (11), the overall channel gain $g_{\mathbf{x}_k \rightarrow \mathbf{r}_r}$ is also approximately log-normal [cf. (1)]. Let $G_{\mathbf{x}_k \rightarrow \mathbf{r}_r} := 10 \log_{10} g_{\mathbf{x}_k \rightarrow \mathbf{r}_r}$ denote the Gaussian-approximated channel gain in dB of link $\mathbf{x}_k \rightarrow \mathbf{r}_r$, and let $\mu_{G_{\mathbf{x}_k \rightarrow \mathbf{r}_r}} := 10 \log_{10}(g_0 \|\mathbf{x}_k - \mathbf{r}_r\|^{-\eta}) + \mu_{A_{\mathbf{x}_k \rightarrow \mathbf{r}_r}}$ and $\sigma_{G_{\mathbf{x}_k \rightarrow \mathbf{r}_r}} := \sigma_{A_{\mathbf{x}_k \rightarrow \mathbf{r}_r}}$ be its mean and standard deviation. Furthermore, let $C_{G_{\mathbf{x}_k \rightarrow \mathbf{r}_r}, G_{\mathbf{x}_j \rightarrow \mathbf{r}_r}} := \mathbb{E}\{(G_{\mathbf{x}_k \rightarrow \mathbf{r}_r} - \mu_{G_{\mathbf{x}_k \rightarrow \mathbf{r}_r}})(G_{\mathbf{x}_j \rightarrow \mathbf{r}_r} - \mu_{G_{\mathbf{x}_j \rightarrow \mathbf{r}_r}})\}$ denote the cross-covariance of $G_{\mathbf{x}_k \rightarrow \mathbf{r}_r}$ and $G_{\mathbf{x}_j \rightarrow \mathbf{r}_r}$ for $(k, r) \neq (j, n)$.

Thus, the r.v. i_r can be viewed as a sum of (possibly correlated) log-normal r.v.'s. Since exact expression for the distribution of a sum of log-normally distributed r.v.'s is still too cumbersome for our purpose, further approximation is pursued. The Fenton-Wilkinson method [11] and the Schwartz-Yeh method [19] were originally developed to approximate the distribution of a sum of *independent* log-normal r.v.'s to another log-normal distribution via a cumulant-matching technique [20]. The methods have been extended to the case of *correlated* log-normal r.v.'s in [21]. The Fenton-Wilkinson method-based technique is suitable for our purpose due to its accuracy over a wide range of parameters, and the simple expressions it yields. More recent work exists along this direction [22], but may be again too complex for our optimization framework.

Let $I_r := 10 \log_{10} i_r$, and consider a Gaussian r.v. \tilde{I}_r with mean $\mu_{\tilde{I}_r}$ and variance $\sigma_{\tilde{I}_r}^2$. By matching the first two moments

of i_r and $\tilde{i}_r := e^{\kappa \tilde{I}_r}$, one can determine [21]

$$\mu_{\tilde{I}_r} = \kappa^{-1} \ln \left(\frac{\xi_{I_r,1}}{\xi_{I_r,2}^{1/2}} \right) \quad (14)$$

$$\sigma_{\tilde{I}_r}^2 = \kappa^{-2} \ln \left(\frac{\xi_{I_r,2}}{\xi_{I_r,1}} \right) \quad (15)$$

where

$$\xi_{I_r,1} := \left(\sum_{k=1}^K p_k a_{r,k} \right)^2 \quad (16)$$

$$\xi_{I_r,2} := \sum_{k=1}^K p_k^2 b_{r,k} + 2 \sum_{k=1}^{K-1} \sum_{j=k+1}^K p_k p_j b'_{r,k,j} \quad (17)$$

$$a_{r,k} := e^{\kappa \mu_{G_{\mathbf{x}_k \rightarrow \mathbf{r}_r}} + \frac{\kappa^2}{2} \sigma_{G_{\mathbf{x}_k \rightarrow \mathbf{r}_r}}^2} \quad (18)$$

$$b_{r,k} := e^{2\kappa \mu_{G_{\mathbf{x}_k \rightarrow \mathbf{r}_r}} + 2\kappa^2 \sigma_{G_{\mathbf{x}_k \rightarrow \mathbf{r}_r}}^2} \quad (19)$$

$$b'_{r,k,j} := e^{\kappa \mu_{G_{\mathbf{x}_k \rightarrow \mathbf{r}_r}} + \kappa \mu_{G_{\mathbf{x}_j \rightarrow \mathbf{r}_r}}} \cdot e^{\frac{\kappa^2}{2} (\sigma_{G_{\mathbf{x}_k \rightarrow \mathbf{r}_r}}^2 + \sigma_{G_{\mathbf{x}_j \rightarrow \mathbf{r}_r}}^2 + 2C_{G_{\mathbf{x}_k \rightarrow \mathbf{r}_r}, G_{\mathbf{x}_j \rightarrow \mathbf{r}_r}})} \quad (20)$$

Thus, given the first- and the second-order statistics of the shadow fading processes in dB scale, and the m parameter of the Nakagami- m -distributed small-scale fading, the distribution of the dB-scale interference powers $\{I_r\}$ at PU locations $\{\mathbf{r}_r\}$ can be approximated as Gaussian with mean and variance given in terms of the optimization variable \mathbf{p} .

Based on the foregoing discussion, and with $I_r^{\max} := 10 \log_{10} i_r^{\max}$, the interference constraints (9) can be approximated by

$$\text{Prob}\{\tilde{I}_r(\mathbf{p}) > I_r^{\max}\} = Q \left(\frac{I_r^{\max} - \mu_{\tilde{I}_r}(\mathbf{p})}{\sigma_{\tilde{I}_r}(\mathbf{p})} \right) \leq \epsilon_r, \quad r = 1, 2, \dots, R \quad (21)$$

where $Q(x) := \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$ is the standard Gaussian tail function. Constraints (21) can be equivalently written as

$$\mu_{\tilde{I}_r}(\mathbf{p}) + Q^{-1}(\epsilon_r) \sigma_{\tilde{I}_r}(\mathbf{p}) \leq I_r^{\max}, \quad r = 1, 2, \dots, R. \quad (22)$$

IV. SEQUENTIAL GP SOLUTION

Plugging (14)-(15) directly into (22) does not lead to a tractable formulation. Instead, upon introducing a set of positive auxiliary variables $\mathbf{z}_r := [z_{r,1}, z_{r,2}]^T$, $r = 1, 2, \dots, R$, the following set of constraints equivalent to (22) is considered.

$$\mu_{\tilde{I}_r}(\mathbf{p}) \leq \ln(z_{r,1}), \quad r = 1, \dots, R \quad (23)$$

$$\sigma_{\tilde{I}_r}^2(\mathbf{p}) \leq \ln(z_{r,2}), \quad r = 1, \dots, R \quad (24)$$

$$\phi_r(\mathbf{z}_r) := \ln(z_{r,1}) + Q^{-1}(\epsilon_r) \sqrt{\ln(z_{r,2})} - I_r^{\max} \leq 0, \quad r = 1, \dots, R. \quad (25)$$

Using (14)-(15), it is possible to express after some manipulations (23) and (24) as the following ratios of posynomials

$$\frac{\xi_{I_r,1}^2(\mathbf{p})}{\xi_{I_r,2}(\mathbf{p})z_{r,1}^{2\kappa}} \leq 1, \quad r = 1, \dots, R \quad (26)$$

$$\frac{\xi_{I_r,2}(\mathbf{p})}{\xi_{I_r,1}(\mathbf{p})z_{r,2}^{\kappa^2}} \leq 1, \quad r = 1, \dots, R. \quad (27)$$

Based on the preceding discussion, and introducing additional non-negative auxiliary variables $\mathbf{q} := [q_1, q_2, \dots, q_K]^T$, the problem to solve becomes

$$(P3) \quad \min_{\mathbf{p} \succ \mathbf{0}, \{\mathbf{z}_r \succ \mathbf{0}\}, \mathbf{q} \succ \mathbf{0}} \prod_{k=1}^K q_k^{w_k} \quad (28)$$

$$\text{subject to } \frac{\sum_{i=1, i \neq k}^K p_k g_{\mathbf{x}_i \rightarrow \mathbf{u}_k} + \pi_k + \sigma_k^2}{\sum_{i=1}^K p_k g_{\mathbf{x}_i \rightarrow \mathbf{u}_k} + \pi_k + \sigma_k^2} q_k^{-1} \leq 1, \quad k = 1, \dots, K \quad (29)$$

$$p_k \leq p_k^{\max}, \quad k = 1, \dots, K \quad (30)$$

$$\frac{\xi_{I_r,1}^2(\mathbf{p})}{\xi_{I_r,2}(\mathbf{p})z_{r,1}^{2\kappa}} \leq 1, \quad r = 1, \dots, R \quad (31)$$

$$\frac{\xi_{I_r,2}(\mathbf{p})}{\xi_{I_r,1}(\mathbf{p})z_{r,2}^{\kappa^2}} \leq 1, \quad r = 1, \dots, R \quad (32)$$

$$\phi_r(\mathbf{z}_r) \leq 0, \quad r = 1, \dots, R. \quad (33)$$

Problem (P3) involves constraints expressed as ratio-of-posynomials in (29) and (31)–(32), which are non-convex. Another source of non-convexity is (33). Thus, it is in general difficult to obtain a globally optimal solution. Recall that this is true even for the case of perfect channel knowledge in (P1). Therefore, we resort to a successive convex approximation method to obtain (at least locally) optimal solutions [23]. Remarkably, it can be shown that this approach boils down to sequential GP as in the perfect channel knowledge case [12].

A. Successive Convex Approximation

Here, the general successive convex approximation method is briefly described [23]. Consider an optimization problem

$$\min_{\mathbf{p} \in \mathcal{P}} f_0(\mathbf{p}) \quad (34)$$

$$\text{subject to } f_k(\mathbf{p}) \leq 0, \quad k = 1, 2, \dots, K \quad (35)$$

where $f_0(\mathbf{p})$ is convex and differentiable, $f_k(\mathbf{p})$, $k = 1, \dots, K$, are differentiable functions, and the feasible region $\mathcal{F} := \{\mathbf{p} \in \mathcal{P} | f_k(\mathbf{p}) \leq 0, k = 1, \dots, K\}$ is compact. Then, starting from a feasible point $\mathbf{p}^{(0)} \in \mathcal{F}$, a series of approximate problems is solved to locate a KKT point of the original (non-convex) problem. For each $k = 1, \dots, K$, let $\tilde{f}_k(\mathbf{p}; \mathbf{p}^{(j)})$ denote the surrogate function for $f_k(\mathbf{p})$, which may depend on the solution $\mathbf{p}^{(j)}$ to the problem of the (previous) $(j-1)$ -st iteration. The approximate problem to solve per iteration j is

$$\min_{\mathbf{p} \in \mathcal{P}} f_0(\mathbf{p}) \quad (36)$$

$$\text{subject to } \tilde{f}_k(\mathbf{p}; \mathbf{p}^{(j)}) \leq 0, \quad k = 1, 2, \dots, K \quad (37)$$

and its feasible region is denoted by $\mathcal{F}^{(j)}$. Provided that $\tilde{f}_k(\mathbf{p}; \mathbf{p}^{(j)})$ satisfies the following conditions **c1**–**c3** for each $k = 1, \dots, K$, the series of solutions to the approximate problems converge to the KKT point of the original problem.

$$\mathbf{c1)} \quad f_k(\mathbf{p}) \leq \tilde{f}_k(\mathbf{p}; \mathbf{p}^{(j)}), \quad \forall \mathbf{p} \in \mathcal{F}^{(j)}$$

$$\mathbf{c2)} \quad f_k(\mathbf{p}^{(j)}) = \tilde{f}_k(\mathbf{p}^{(j)}; \mathbf{p}^{(j)})$$

$$\mathbf{c3)} \quad \nabla f_k(\mathbf{p}^{(j)}) = \nabla \tilde{f}_k(\mathbf{p}^{(j)}; \mathbf{p}^{(j)})$$

B. Sequential GP

In order to apply the successive convex approximation method to (P3), appropriate surrogate constraints for the non-convex constraints need to be determined. For the ratio of posynomials in (29) and (31)–(32), the single condensation method is adopted for simplicity.

In the single condensation method, each ratio of posynomials is approximated by a constraint on a posynomial. Specifically, a constraint given by

$$\frac{\sum_i n_i(\mathbf{p})}{\sum_i d_i(\mathbf{p})} \leq 1 \quad (38)$$

with monomials $\{n_i(\mathbf{p})\}$ and $\{d_i(\mathbf{p})\}$ is approximated as

$$\frac{\sum_i n_i(\mathbf{p})}{\prod_i \left(\frac{d_i(\mathbf{p})}{\alpha_i} \right)^{\alpha_i}} \leq 1 \quad (39)$$

where $\alpha_i := d_i(\mathbf{p}^{(j)}) / \sum_n d_n(\mathbf{p}^{(j)})$ for a given point $\mathbf{p}^{(j)}$. Then, by viewing the l.h.s.'s of (38) and (39) as $f_k(\mathbf{p})$ and $\tilde{f}_k(\mathbf{p}; \mathbf{p}^{(j)})$, respectively, it can be readily shown that they satisfy conditions **c1**–**c3** in Sec. IV-A [12].

To handle the non-convexity in (33), it is first noted that $\phi_r(\mathbf{z}_r)$ is a concave function for $\mathbf{z}_r \succ \mathbf{0}$, which can be easily verified by examining the second-order derivatives of each term [cf. (25)], and recalling that the sum of concave functions is concave. Thus, an upper-bound of $\phi_r(\mathbf{z}_r)$ that satisfies **c1**–**c3** can be obtained via the supporting hyperplane as

$$\tilde{\phi}_r(\mathbf{z}_r; \mathbf{z}_r^{(j)}) := \frac{z_{r,1}}{z_{r,1}^{(j)}} + \frac{Q^{-1}(\epsilon_r)z_{r,2}}{2z_{r,2}^{(j)}\sqrt{\ln z_{r,2}^{(j)}}} - c_r(\mathbf{z}_r^{(j)}) \quad (40)$$

$$c_r(\mathbf{z}_r^{(j)}) := 1 + \frac{Q^{-1}(\epsilon_r)}{2\sqrt{\ln z_{r,2}^{(j)}}} - \phi_r(\mathbf{z}_r^{(j)}). \quad (41)$$

It is apparent from (41) and (33) that $c_r(\mathbf{z}_r^{(j)}) > 0$ provided that $\mathbf{z}_r^{(j)}$ is a feasible point, and $\epsilon_r < 0.5$. Therefore, a surrogate constraint for (33) is

$$\frac{1}{c_r(\mathbf{z}_r^{(j)})} \left(\tilde{\phi}_r(\mathbf{z}_r; \mathbf{z}_r^{(j)}) + c_r(\mathbf{z}_r^{(j)}) \right) \leq 1, \quad r = 1, \dots, R. \quad (42)$$

The l.h.s. is affine, and thus it is a posynomial.

Overall, the problem to solve in the j -th successive iteration is given by (P3) with constraints (29) and (31)–(32) replaced by their surrogates per (39), as well as (33) substituted with (42). It is immediate that this problem is a GP problem, which involves minimizing a posynomial (in our case, a monomial) subject to posynomial inequality constraints. GP problems can be solved efficiently through optimized interior-point methods.

Although GP problems are not convex in their original form, their globally optimal solution can be obtained by convex reformulation through a log change of variables [24].

V. NUMERICAL TESTS

Consider the scenario depicted in Fig. 1, where $K = 3$ CR links (shown in solid lines) are present, and a single ($R = 1$) PU link (dashed line) is also active with transmit-power 0.1 W. The path loss parameters are set as $g_k = 1$ for all k and $\eta = 3.5$. $m = 1$ is used for Nakagami- m fading. Spatially correlated log-normal shadowing was generated with mean 0, standard deviation 10 dB, and coherence distance 30 m. The maximum transmit-power for the CR system is $p_k^{\max} = 5$ W, and the weights are $w_k = 1$, for all $k = 1, 2$, and 3. The interference threshold was set to $I_r^{\max} = -80$ dBW, and the interference probability bound to $\epsilon_r = 0.01$ for $r = 1$.

To verify the effect of interference constraints, the complementary cumulative distribution function (*c.c.d.f.*) of the received interference power at the PU receiver is plotted in Fig. 2. The curve with square markers corresponds to the case where only the location information was utilized to obtain the channel gain statistics; in this case, the mean of shadow fading is set to 0 dB, and the correlations were computed based on the method described in [15]. The curve with ‘X’ markers represents the case where shadowing measurements were used to estimate the CR-PU channel gain via a spatial interpolation technique (channel gain cartography) [2]. Thus, the channel gain was estimated reliably with error standard deviation of about 4 dB. In these two cases, (P3) was solved to obtain the CR transmission powers. The curves with the circle markers and the ‘+’ markers depict, respectively, the cases of perfect channel knowledge and path loss-only model with shadowing and small-scale fading completely neglected. In the last two cases, (P1) was solved to obtain optimal powers; 5,000 independent realizations were used to generate the plot.

From Fig. 2, it is clearly seen that solving (P3) enforces the desired interference constraint, which also verifies the accuracy of the associated approximations. In fact, with crude channel gain statistics, the interference constraint is seen to be over-satisfied, while channel gain cartography leads to a tightly met constraint. On the contrary, neglecting uncertainty in the channel gains grossly violates the constraints.

Fig. 3 shows the *c.c.d.f.*’s of the SINRs $\{\gamma_k\}$ of the CR links. The solid, dashed, and dotted curves represent different CR links, and the markers are used in the same manner as in Fig. 2. It can be seen that especially when the channel gains are estimated from measurements, the performance degradation from the perfect channel gain case is not too large. The path loss-only case exhibits better SINRs than the proposed schemes, but only at severe interference to the PU system.

To see how close the obtained KKT solutions are to the globally optimal solution, an exhaustive grid-based search was performed. Fig. 4 depicts the weighted sum-rate objectives obtained by solving (P1) and (P3) for 50 different channel realizations. It is seen that the sequential GP-based objectives

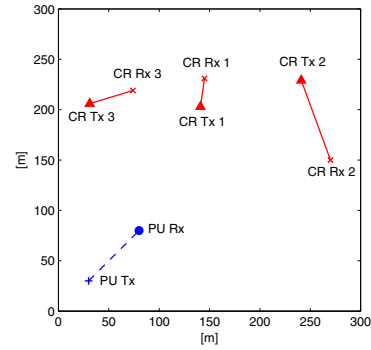


Fig. 1. Test scenario.

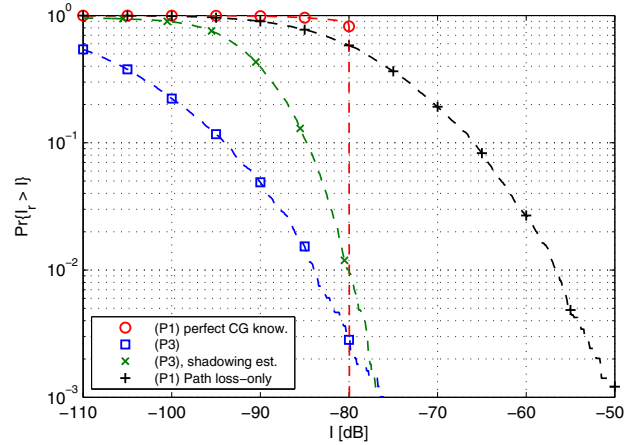


Fig. 2. Interference power *c.c.d.f.*

very often coincide with the globally optimal objectives even in the case with channel uncertainty.

VI. CONCLUSIONS

A CR power allocation algorithm was developed under CR-to-PU channel gain uncertainty arising from shadowing and small-scale fading effects. Probabilistic interference constraints were imposed to protect the PU transmissions with prescribed reliability. Fenton-Wilkinson-type approximations were employed to model the received interference power at the PUs as log-normal, which leads to a tractable optimization formulation. A weighted sum-rate maximization problem for the CR network was considered. Due to non-convexity of the problem, a successive convex approximation technique was adopted to obtain a KKT solution. This approach boils down to a sequential GP algorithm, which can be implemented using efficient interior-point methods. Numerical tests verified the validity and the performance of the proposed design.

REFERENCES

[1] J. M. Peha, “Approaches to spectrum sharing,” *IEEE Commun. Mag.*, vol. 43, no. 2, pp. 10–12, Feb. 2005.

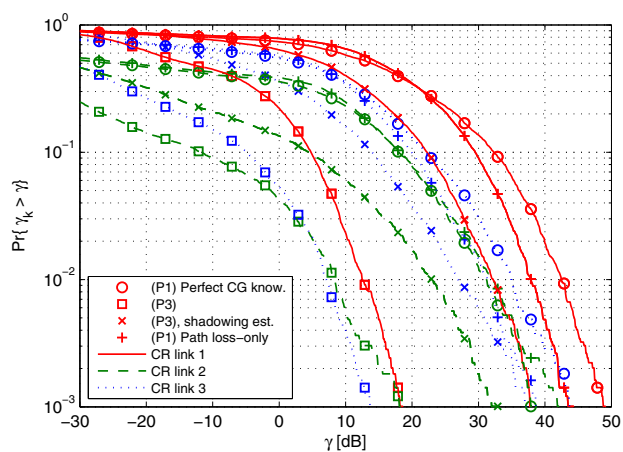


Fig. 3. CR SINR c.c.d.f.

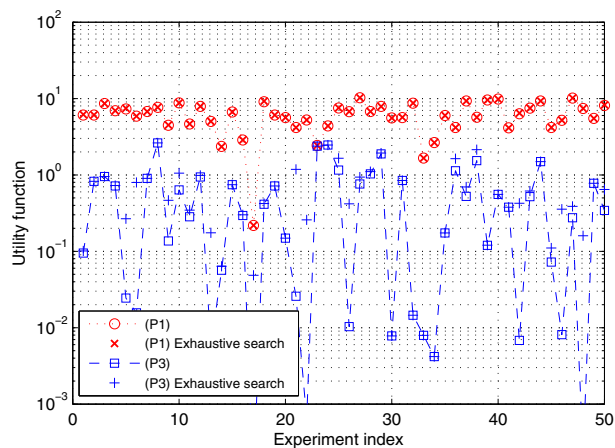


Fig. 4. Achieved sum-rates.

[8] S.-J. Kim, E. Dall'Anese, and G. Giannakis, "Cooperative spectrum sensing for cognitive radios using Krigeed Kalman filtering," *IEEE J. Sel. Topics Sig. Proc.*, Feb. 2011, to appear.

[9] J.-A. Bazerque and G. B. Giannakis, "Distributed spectrum sensing for cognitive radio networks by exploiting sparsity," *IEEE Trans. Sig. Proc.*, vol. 58, no. 3, pp. 1847–1862, Mar. 2010.

[10] E. Dall'Anese, S.-J. Kim, and G. Giannakis, "Global channel gain tracking using distributed Krigeed Kalman filtering," *IEEE Trans. Veh. Technol.*, Jun. 2010, submitted.

[11] A. Ghasemi and E. S. Sousa, "Fundamental limits of spectrum-sharing in fading environments," *IEEE Trans. Wireless Commun.*, vol. 6, no. 2, pp. 649–658, Feb. 2007.

[12] I. Mitliagkas, N. Sidiropoulos, and A. Swami, "Convex approximation-based joint power and admission control for cognitive underlay networks," in *Proc. of the IEEE IWCMC Conf.*, Crete, Greece, 2008, pp. 28–32.

[13] R. Zhang and Y. C. Liang, "Exploiting multi-antennas for opportunistic spectrum sharing in cognitive radio networks," *IEEE J. Sel. Topics Sig. Proc.*, vol. 2, no. 1, pp. 88–102, Feb. 2008.

[14] A. G. Marques, X. Wang, and G. B. Giannakis, "Dynamic resource management for cognitive radios using limited-rate feedback," *IEEE Trans. Sig. Proc.*, vol. 57, no. 9, pp. 3651–3666, Sep. 2009.

[15] H. A. Suraweera, P. J. Smith, and M. Shafi, "Capacity limits and performance analysis of cognitive radio with imperfect channel knowledge," *IEEE Trans. Veh. Technol.*, vol. 59, no. 4, pp. 1811–1822, May 2010.

[16] B. Zayen, M. Haddad, A. Hayar, and G. E. Øien, "Binary power allocation for cognitive radio networks with centralized and distributed user selection strategies," *Physical Communication*, vol. 1, no. 3, pp. 183 – 193, 2008.

[17] L. F. Fenton, "The sum of lognormal probability distributions in scatter transmission systems," *IRE Trans. Commun. Syst.*, vol. 8, no. 1, pp. 57–67, Mar. 1960.

[18] M. Chiang, C. W. Tan, D. P. Palomar, D. O'Neill, and D. Julian, "Power control by geometric programming," *IEEE Trans. Wireless Commun.*, vol. 6, no. 7, pp. 2640–2651, Jul. 2007.

[19] G. L. Stüber, *Principles of Mobile Communication*, 2nd ed. Boston, MA: Kluwer Academic Publishers, 2001.

[20] C. Oestges, N. Czink, B. Bandemer, P. Castiglione, F. Kaltenberger, and A. J. Paulraj, "Experimental characterization and modeling of outdoor-to-indoor and indoor-to-indoor distributed channels," *IEEE Trans. Veh. Technol.*, 2010, to appear; downloadable at <http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=5415672>.

[21] P. Agrawal and N. Patwari, "Correlated link shadow fading in multi-hop wireless networks," *IEEE Trans. Wireless Commun.*, vol. 8, no. 9, pp. 4024–4036, Aug. 2009.

[22] R. Prasad and A. Kegel, "Improved assessment of interference limits in cellular radio performance," *IEEE Trans. Veh. Technol.*, vol. 40, no. 2, pp. 412–419, May 1991.

[23] M.-J. Ho and G. L. Stüber, "Co-channel interference of microcellular systems on shadowed Nakagami fading channels," in *Proc. of the IEEE Veh. Tech. Conf.*, Secaucus, NJ, May 1993, pp. 568–571.

[24] X. Hong, C.-X. Wang, and J. Thompson, "Interference modeling of cognitive radio networks," in *Proc. of the IEEE Veh. Tech. Conf.*, Singapore, May 2008, pp. 1851–1855.

[25] S. Schwartz and Y. Yeh, "On the distribution function and moments of power sums with lognormal components," *Bell Syst. Tech. J.*, vol. 61, no. 7, pp. 1441–1462, Sep. 1982.

[26] D. C. Schleher, "Generalized Gram-Charlier series with application to the sum of log-normal variates," *IEEE Trans. Info. Theory*, vol. 23, no. 2, pp. 275–280, Mar. 1977.

[27] A. A. Abu-Dayya and N. C. Beaulieu, "Outage probabilities in the presence of correlated lognormal interferers," *IEEE Trans. Veh. Technol.*, vol. 43, no. 1, pp. 164–173, Feb. 1994.

[28] N. B. Mehta, J. Wu, A. F. Molisch, and J. Zhang, "Approximating a sum of random variables with a lognormal," *IEEE Trans. Wireless Commun.*, vol. 6, no. 7, pp. 2690–2699, Jul. 2007.

[29] B. R. Marks and G. P. Wright, "A general inner approximation algorithm for nonconvex mathematical programs," *Oper. Res.*, vol. 26, no. 4, pp. 681–683, Jul.-Aug. 1978.

[30] S. Boyd, S.-J. Kim, L. Vandenberghe, and A. Hassibi, "A tutorial on geometric programming," *Optim. Eng.*, vol. 8, pp. 67–127, 2007.