

ADAPTIVE BEAM FORMING IN CORRELATED INTERFERENCE ENVIRONMENT

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ABSTRACT

In the presence of signal correlated interference, a conventional adaptive beamformer fails to perform satisfactorily due to the signal cancellation effects. This paper suggests a new technique based signal interpolation followed by spatial correlation smoothing to de-correlate the signal from the interference for avoiding signal cancellation.

I. INTRODUCTION

Adaptive arrays have the desirable property of being able to respond to a weak signal in the presence of strong unknown interference. Conventional adaptive beam-forming algorithms (e.g., [1], [2]) perform satisfactorily only under the assumption that the signal and its interference(s) are mutually uncorrelated. There are however instances such as multipath propagation, narrow band jamming, etc. where such an assumption is not justified. In such situations, there occurs a signal cancellation phenomenon. Adaptive array in its efforts to cancel out the interference, ends up cancelling out the correlated portion of the signal as well. Recently, Widrow et. al [3] have suggested a method of de-correlating the signal and the interference by dithering the array. If the array is constantly moved in a direction orthogonal to the direction of arrival of the signal (henceforth called the look direction), the interference will get modulated by the array motion without in any way affecting the signal. In other words, the interference energy will get spread spatially and thereby decorrelating it from the signal.

It will be much more desirable to be able to introduce motion to the array through signal processing means rather than physically moving the array constantly and accurately. Shan and Kailath [4] have proposed the method of sub-array spatial correlation smoothing. However, this method can

appreciably reduce the effective size of the array. In this paper, we propose the use of signal interpolation followed by sub-array spatial correlation smoothing to overcome the effects of correlated interference.

II. PROBLEM OF ADAPTIVE BEAM-FORMING IN CORRELATED NOISE

Consider a linear adaptive array of N-elements as shown in Fig. 1. The optimum weight vector which minimizes the interference power without affecting the signal power is given by:

$$W_{opt} = \frac{R_{xx}^{-1} C_s}{(C_s^* R_{xx}^{-1} C_s)} \quad (1)$$

where

$$R_{xx} = E\{X(k)X^*(k)\} \quad (2)$$

(* denotes conjugate transpose)

is the autocorrelation matrix of the array measurement vector $X(k)$ and C_s is the so called look-direction vector and is given by

$$C_s^T = [1 e^{-j2\pi d/\lambda \sin\theta_s} e^{-j2\pi d/\lambda 2\sin\theta_s} \dots e^{-j2\pi d/\lambda (N-1)\sin\theta_s}] \quad (3)$$

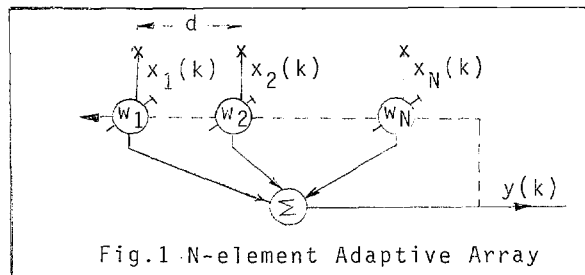
where, θ_s is the angle of arrival of the signal, d is the inter-element spacing and λ is the operating wavelength. Without loss of generality, we assume $\theta_s = 0^\circ$ (i.e., array broad-side). Then,

$$C_s^T = [1 \ 1 \ \dots \ 1] \quad (4)$$

The array measurement vector $X(k)$ is given by,

$$X^T(k) = s(k)C_s^T + \sum_{i=1}^m v_i(k)C_i^T + W^T(k) \quad (5)$$

where $s(k)$ is the amplitude of the look-direction signal, $v_i(k)$ is the amplitude of i^{th} interference arriving at the array from a direction θ_i and C_i in the corresponding direction vector. $W(k)$ represents the broad-band white Gaussian noise $N(0, \sigma^2)$.



Therefore,

$$R_{xx} = R_{ss} + CR_{vv}C^* + \sigma^2 I \quad (6)$$

Letting,

$$R_{nn} = CR_{vv}C^* + \sigma^2 I \quad (7)$$

where

$$C^* = [C_1^* : C_2^* : \dots : C_m^*]$$

If it is assumed that the signal and the interferences are mutually uncorrelated, then, the matrix R_{nn} has N non-zero the eigenvalues and N corresponding independent eigenvectors (e_1, e_2, \dots, e_N). Out of these, $(N - m)$ eigenvalues are equal to σ^2 . The remaining m eigenvalues ($> \sigma^2$) and the associated eigenvectors ($e_{m+1} \dots e_N$) correspond to the m independent interference sources. Under these conditions, it is easy to show that W_{opt} is such that,

$$W_{opt} \perp (e_{m+1} \dots e_N)$$

Consequently the output of the array,

$$y(k) = W_{opt}^* X(k) \quad (8)$$

does not contain any contribution from the interference sources.

The situation turns out to be quite different when the signal and the interferences are mutually correlated. It is shown in [4] that in such a situation, the eigenvectors ($e_{m+1} \dots e_N$) are mutually dependent and are described by a single eigenvector e_s which also corresponds to the signal as well. This implies that the matrix $CR_{vv}C^*$ has rank = 1. The resulting optimum weight vector W_{opt} which is orthogonal to ($e_{m+1} \dots e_N$) is also now orthogonal to e_s . Therefore, the array output y_k contains no contribution from the signal component $S_k C_s$. The array in its effort to cancel out the interference ends up cancelling out the signal as well. The technique suggested in [4] for combatting the effects of correlated interference consists of forming $p(> m)$ sub-arrays each consisting of $(m + 1)$ elements. Then,

$$X(k) = [x_1(k) \ x_2(k) \ \dots \ x_N(k)]^T$$

and, sub-array measurement vectors are formed as,

$$\begin{aligned} X^1(k) &= [x_1(k) \ x_2(k) \ \dots \ x_{m+1}(k)]^T \\ X^2(k) &= [x_2(k) \ x_3(k) \ \dots \ x_{m+2}(k)]^T \\ &\vdots \\ X^p(k) &= [x_p(k) \ x_{p+1}(k) \ \dots \ x_{p+m}(k)]^T. \end{aligned} \quad (9)$$

The sub-array correlation matrix r_{xx}^l is defined as,

$$r_{xx}^l = E\{X^l(k)X^{l*}(k)\} \quad (10)$$

and the smoothed correlation matrix is defined as,

$$\bar{R}_{xx} = \frac{1}{P} \sum_{l=1}^P r_{xx}^l = \frac{1}{PK} \sum_{k=1}^K \sum_{l=1}^P X^l(k)X^{l*}(k) \quad (11)$$

The matrix \bar{R}_{xx} can now be shown to be of full rank. Note that because of the process of correlation smoothing, the size of the array is reduced. In the next section we describe a new method for cancelling correlated interference which does not suffer from this drawback.

III. SIGNAL INTERPOLATION FOR COMBATTING CORRELATED INTERFERENCE

The measurement available at the j^{th} element of the array is given by,

$$x_j(k) = s(k) + \sum_{i=1}^m v_i(k) e^{j2\pi d/\lambda (j-1)U_i} + W_j(k)$$

$$j = 1, 2, \dots, N$$

where $U_i = \sin\theta_i$ and is called the wavenumber of the i^{th} interference source.

Assuming $d = \lambda/2$, gives,

$$x_j(k) = s(k) + \sum_{i=1}^m v_i(k) e^{j\pi (j-1)U_i} + W_j(k) \quad (12)$$

$$j = 1, \dots, N$$

The DFT of the signal $\{x_j(k)\}$, $j = 1, 2, \dots, N$ gives the direction or wavenumber spectra at time k . The inverse DFT of this spectra after appropriate scaling results in the original $\{x_j(k)\}$. Our problem of imparting motion to the array will be solved if we can interpolate the received signal between two array elements. The interpolated samples can then be thought to be the outputs obtained when the array is moved by the interpolation distance, i.e., when the elements occupy the interpolated positions. There are a variety of signal interpolation schemes available [5]. However linear interpolation using FFT and zero padding [6] is quite straightforward and computationally efficient. The FFT based interpolation can be implemented in the following three steps

- i) Compute the N -point FFT of the array received signal $\{x_j(k)\}$ $j = 1, \dots, N$.
- ii) Pad N -zeros in the output of the FFT to increase the number of points to $2N$ (in general to $3N, 4N, \dots$).
- iii) Compute the IFFT of the zero padded spectra sequence.

The interpolated samples can be thought of as the array measurement vector when it is moved a distance $d/2$ (in general $d/3, d/4$, etc.).

We can now form sub-arrays and carry out spatial correlation smoothing. Two different correlation smoothing schemes were tried out. These are,

- (A) Pad N zero to increase the number of samples to $2N$ and form sub-arrays as shown in Fig. 2.

(B) Pad ($N^2 - N$) zeros to increase the number of samples to N^2 and form sub-arrays as shown in Fig. 3.

IV. SIMULATION RESULTS

To verify the proposed technique an adaptive array consisting of $N = 8$ uniformly spaced elements was simulated on a digital computer. The simulated environment contained a weak look-direction signal of amplitude 0.1 and two strong correlated interference sources of amplitude = 10.0 at -20° and $+30^\circ$ to the array broad-side. In the first instance, both the signal and the interference were taken to be pure sinusoids of 0.20 Hz. In addition zero-mean white Gaussian noise with variance = 0.0075 was added to each array element independently.

The spatially smoothed autocorrelation matrix was computed based on the interpolation schemes (A) and (B) described earlier. The resulting \bar{R}_{xx} was used to compute W_{opt} as in (1). Fig. 4 and Fig. 5 show the normalized frequency spectra of the look direction signal and the array output $y(k)$ for schemes (A) and (B) respectively. The corresponding beam patterns when the array weights are set to W_{opt} are shown in Fig. 6 and Fig. 7. Note the recovery of the look direction signal and the correct placement of nulls in the direction of the interference sources. In difference to this, when a conventional adaptive beam-forming algorithm [2] was used, there was practically no useful output obtained and the placement of nulls was unrelated to the interference directions.

The same processing was repeated for the case of broad-band look direction signal. The look-direction signal was now taken to be a damped sinusoid giving a bandwidth of approximately 0.075 Hz. For the lack of space, only the array beam pattern obtained after computing W_{opt} according to scheme (A) is shown in Fig. 8. Once again, nulls are correctly steered in the direction of the interference sources.

V. CONCLUSION

A new method based on signal interpolation followed correlation smoothing for combatting the effects of correlated interference has been developed. In fact two different schemes for implementing the proposed technique have been looked into. Both these schemes have been found to give satisfactory results. In scheme (A) note that there is a reduction in the size of the array, but the effective number of elements is not reduced. In difference to this, in scheme (B) there is no reduction in the size of the number of elements in the array. However, this scheme requires more computation. Another important aspect of the proposed technique is that it should be possible to use the signal interpolation method for designing non-uniform and conformal adaptive arrays in correlated interference.

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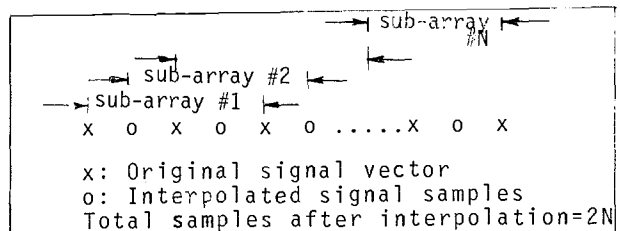


Fig.2 Interpolation Scheme A

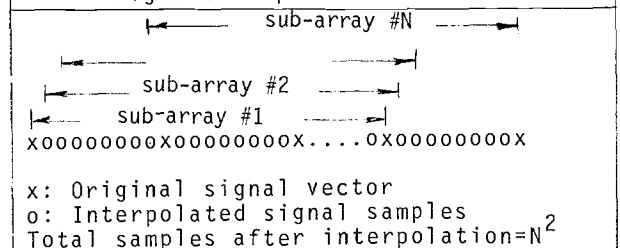


Fig.3 Interpolation Scheme B

