

# ORING LOSS MODEL FOR IMPLEMENTATION IN SIGNAL PROCESSING SYSTEMS FOR DATA DISPLAY

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## ABSTRACT

A generalized mathematical model is developed to compute the ORing loss for a system composed of square law detectors followed by electrical integrators, an ORing device, and a display. The ORing loss is computed as a function of the detection probability, the false alarm probability, the number of channels ORed, and the number of lines on the display.

It is concluded that the ORing loss decreases with an increase in the detection probability and a decrease in the false alarm probability.

## INTRODUCTION

Many acoustic signal processing systems produce a large quantity of data that can not be displayed concurrently to an operator in real time for analysis. Therefore, a data reduction technique is necessary. One technique that provides a reduction of data is ORing, a process where only the maximum of a set ( $X_1, X_2, \dots, X_N$ ) is selected and displayed.

The subsequent analysis is devoted to a signal processing system composed of square law detectors followed by electrical integrators, an ORing device, and a display. A generalized mathematical model is developed to characterize the performance of this system in terms of detection probabilities and false alarm probabilities.

Some relevant past work on this topic is given in [1] through [3].

## MATHEMATICAL ANALYSIS

The system of interest is shown in Fig. 1. The input to the ORing device,  $X_1, X_2, \dots, X_N$ , contains either N channels of noise or N-1 channels of noise and one channel of signal plus noise. The random variables,  $X_1, X_2, \dots, X_N$ , are statistically independent, and they are identically, independently distributed (i.i.d) for the noise-only case. The channel inputs to the ORing device will be assumed to be Gaussian distributed.

The input signal-to-noise ratio to the ORing device ( $SNR_I$ ) in Fig. 1 is given by

$$SNR_I = 20 \log \left[ (m_1 - m_0) / \sigma \right] \\ = 20 \log d_I \quad (\text{dB}) \quad , \quad (1)$$

where  $d_I = (m_1 - m_0) / \sigma =$  input deflection coefficient

$m_1 =$  mean of the signal plus noise at the input to the ORing device

$m_0 =$  mean of the noise at the input to the ORing device

$\sigma =$  standard deviation of the noise at the input to the ORing device.

There will be a performance loss when ORing N channels of data. Therefore, if the same detection probability and false alarm probability is desired for N channels ( $N > 1$ ) as for one channel, additional SNR is needed at the input to the ORing device. The additional SNR needed at the input to the ORing device for N channels is the ORing loss ( $SNR_{LOSS_{OR}}$ ):

$$SNR_{LOSS_{OR}} = 20 \log \left( d_{I_N} / d_{I_1} \right) \quad (\text{dB}) \quad , \quad (2)$$

where  $d_{I_N} =$  input deflection coefficient for N channels ( $N > 1$ )

$d_{I_1} =$  input deflection coefficient for one channel.

The ORing loss relative to the detector input ( $SNR_{LOSS_D}$ ) is given by

$$SNR_{LOSS_D} = 10 \log \left( d_{I_N} / d_{I_1} \right) \quad (\text{dB}) \quad . \quad (3)$$

The output of the ORing device is

$$Y = \text{Max}(X_1, X_2, \dots, X_N) \quad . \quad (4)$$

For the noise-only case, the cumulative distribution of Y,  $F_0(Y)$ , is

$$\begin{aligned}
F_0(y) &= \text{Prob}(Y \leq y \mid \text{all noise}) \\
&= \text{Prob}(X_1, X_2, \dots, X_N \leq y \mid \text{all noise}) \\
&= P_0^N(y) \quad .
\end{aligned} \tag{5}$$

The probability density function for the noise-only case,  $f_0(y)$ , is defined as

$$\begin{aligned}
f_0(y) &= dF_0(y)/dy \\
&= N P_0^{N-1}(y) p_0(y) \quad .
\end{aligned} \tag{6}$$

For the case in which one of the  $X_i$ 's is signal plus noise, the cumulative distribution of  $Y$ ,  $F_1(y)$ , is

$$\begin{aligned}
F_1 &= \text{Prob}(X_1, X_2, \dots, X_N \leq y \mid \text{signal}) \\
&= P_0^{N-1}(y) P_1(y) \quad .
\end{aligned} \tag{7}$$

The probability density function for the signal plus noise case,  $f_1(y)$ , is defined as

$$\begin{aligned}
f_1(y) &= dF_1(y)/dy \\
&= (N-1) P_0^{N-2}(y) p_0(y) P_1(y) \\
&\quad + P_0^{N-1}(y) p_1(y) \quad .
\end{aligned} \tag{8}$$

The output,  $Y$ , of the ORing device may be expressed in terms of a deflection coefficient,  $d_{0Y}$ :

$$d_{0Y} = (\mu_{Y1} - \mu_{Y0}) / \sigma_{Y0} \quad , \tag{9}$$

where

$$\begin{aligned}
\mu_{Y1} &= \text{mean of the signal plus noise after ORing} \\
\mu_{Y0} &= \text{mean of the noise after ORing} \\
\sigma_{Y0} &= \text{standard deviation of the noise after ORing}
\end{aligned}$$

In order to evaluate Eq. (9), it is necessary to compute  $\mu_{Y0}$ ,  $\mu_{Y1}$ , and  $\sigma_{Y0}$ .

The mean values,  $\mu_{Y0}$  and  $\mu_{Y1}$ , are

$$\mu_{Y0} = \int y f_0(y) dy = N \int y P_0^{N-1}(y) p_0(y) dy \tag{10}$$

$$\begin{aligned}
\mu_{Y1} &= \int y f_1(y) dy = \int y \left[ (N-1) P_0^{N-2}(y) p_0(y) P_1(y) \right. \\
&\quad \left. + P_0^{N-1}(y) p_1(y) \right] dy \quad .
\end{aligned} \tag{11}$$

The standard deviations,  $\sigma_{Y0}$  and  $\sigma_{Y1}$ , for the signal-absent and signal-present cases are

$$\sigma_{Y0} = \left( \overline{Y_0^2} - \mu_{Y0}^2 \right)^{1/2} \tag{12}$$

$$\sigma_{Y1} = \left( \overline{Y_1^2} - \mu_{Y1}^2 \right)^{1/2} \quad , \tag{13}$$

where

$$\overline{Y_0^2} = \int y^2 f_0(y) dy = N \int y^2 P_0^{N-1}(y) p_0(y) dy$$

$$\begin{aligned}
\overline{Y_1^2} &= \int y^2 f_1(y) dy = \int y^2 \left[ (N-1) P_0^{N-2}(y) p_0(y) P_1(y) \right. \\
&\quad \left. + P_0^{N-1}(y) p_1(y) \right] dy \quad .
\end{aligned}$$

The output,  $Z$ , of the eyeball integrator on the display can be expressed in terms of an output deflection coefficient,  $d_{0Z}$ , in the following way:

$$\begin{aligned}
d_{0Z} &= (\mu_{Z1} - \mu_{Z0}) / \sigma_{Z0} \\
&= \sqrt{L} d_{0Y} \quad ,
\end{aligned} \tag{14}$$

where  $L$  is the number of lines on the display.

It is necessary to determine the cumulative distribution functions,  $P_0(y)$  and  $P_1(y)$ , and the probability density functions,  $p_0(y)$  and  $p_1(y)$ , in Eqs. (10) through (13). It is assumed that sufficient post-detection integration has been performed in Fig. 1 to invoke the Central Limit Theorem.

Employing Gaussian input statistics, we obtain the mean values and the standard deviations for the signal-absent and the signal-present cases:

$$\mu_{Y0} = m_0 + \sigma A_N \tag{15}$$

$$\mu_{Y1} = m_0 + \sigma C_N(d_I) \tag{16}$$

$$\sigma_{Y0} = \sigma (B_N - A_N^2)^{1/2} \tag{17}$$

$$\sigma_{Y1} = \sigma (D_N(d_I) - C_N^2(d_I))^{1/2} \quad , \tag{18}$$

where

$$A_N = N \int x \phi(x) \Phi^{N-1}(x) dx$$

$$B_N = N \int x^2 \phi(x) \Phi^{N-1}(x) dx$$

$$C_N(d_I) = (N-1) \int x \phi(x) \Phi^{N-2}(x) \Phi(x-d_I) dx \\ + \int x \Phi^{N-1}(x) \phi(x-d_I) dx$$

$$D_N(d_I) = (N-1) \int x^2 \phi(x) \Phi^{N-2}(x) \Phi(x-d_I) dx \\ + \int x^2 \Phi^{N-1}(x) \phi(x-d_I) dx$$

$$\Phi(x) = (1/2\pi)^{1/2} \int_{-\infty}^x e^{-t^2/2} dt = \int_{-\infty}^x \phi(t) dt$$

$d_I = (m_1 - m_0)/\sigma$  = input deflection coefficient defined in Eq. (1).

The deflection coefficient at the output of the eyeball integrator,  $d_{OZ}$ , is now obtained by substituting Eqs. (15) through (17) into Eq. (14):

$$d_{OZ} = \sqrt{L} \left( (C_N(d_I) - A_N) / (B_N - A_N^2)^{1/2} \right) \quad (19)$$

The statistics of the eyeball integrator output,  $Z$ , must be evaluated. Since consideration will be given to large values of  $L$ ,  $Z$  will be assumed to be Gaussian distributed. Therefore, the detection probability,  $P_D$ , and the false alarm probability,  $P_F$  of the system in Fig. 1 is as follows:

$$P_D = \int_T^\infty \phi_1(z) dz = \Phi \left( \frac{u_{Z1} - T}{\sigma_{Z1}} \right) \quad (20)$$

$$P_F = \int_T^\infty \phi_0(z) dz = \Phi \left( \frac{u_{Z0} - T}{\sigma_{Z0}} \right) \quad (21)$$

where  $T$  is the threshold, and  $\phi(z) = \Phi(z)$ . Solving for  $P_D$  by eliminating the threshold,  $T$ , in Eqs. (20) and (21) gives

$$P_D = \Phi \left( \frac{d_{OZ} + \Phi^{-1}(P_F)}{K} \right) \quad (22)$$

where  $K = \sigma_{Y1}/\sigma_{Y0}$ . For the case of no ORing ( $N=1$ )

$$P_D = \Phi \left( \sqrt{L} d_I + \Phi^{-1}(P_F) \right) \text{ for } N=1 \quad (23)$$

Equations (1), (15) through (19), (22), and (23) were programmed on a digital computer. The input SNR to the ORing device was varied from -10 to +6 dB. The detection probability was computed for a specified  $P_F$ ,  $N$ , and  $L$ . Finally, Eq. (3) was used to compute the ORing loss.

## CONCLUSIONS

The analytical results are presented in Figs. 2 through 7. An examination of these figures reveals that the ORing loss decreases with an increase in the detection probability and with a decrease in the false alarm probability for a fixed number of channels ORed and a fixed number of lines on the display.

## REFERENCES

- [1] W. A. Struzinski, "ORing Loss Data for Square Law Detectors Followed by an ORing Device and an Accumulator," J. Acoust. Soc. Am., 72(1), 191-195 (1982).
- [2] W. A. Struzinski, "ORing Loss for Quantizers Followed by an ORing Device and an Accumulator," IEEE Trans. Acoust., Speech, Signal Processing, ASSP-30, No. 4 668-671 (1982).
- [3] A. H. Nuttall, "Signal-to-Noise Ratio Requirements for Greatest-of Device Followed by Integrator," NUSC Technical Memorandum No. TC-13-75, Naval Underwater Systems Center, New London, CT, 24 July 1975.

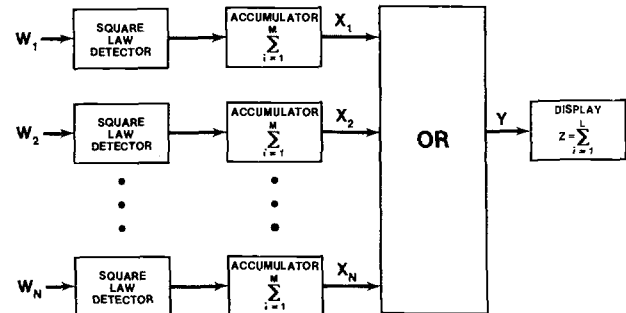


Fig. 1 General system diagram

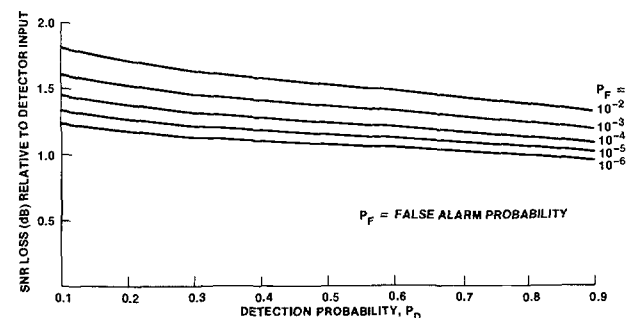


Fig. 2 ORing loss for 2 channels ORed and 32 lines on the display.

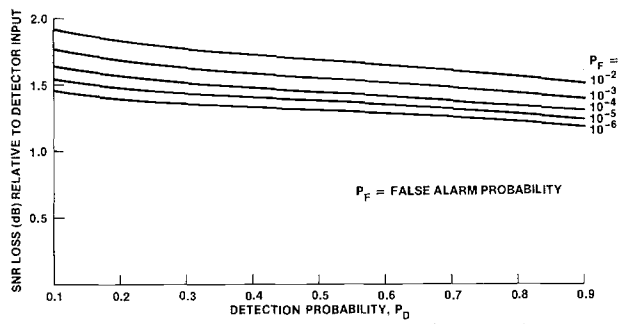


Fig. 3 ORing loss for 2 channels ORed and 64 lines on the display.

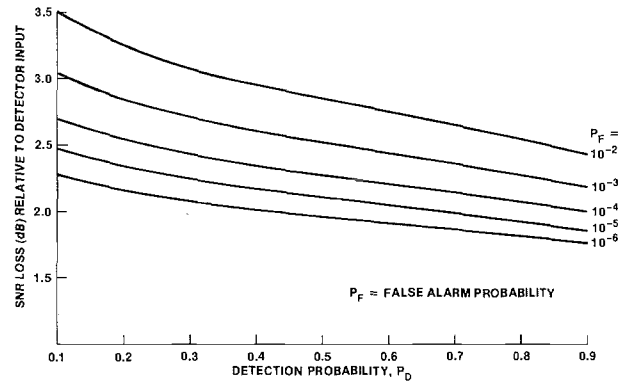


Fig. 4 ORing loss for 4 channels ORed and 32 lines on the display.

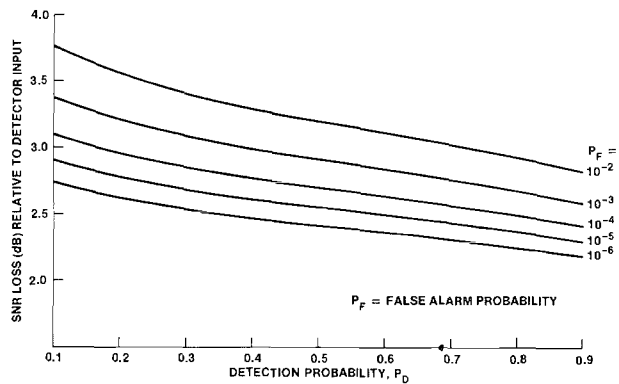


Fig. 5 ORing loss for 4 channels ORed and 64 lines on the display.

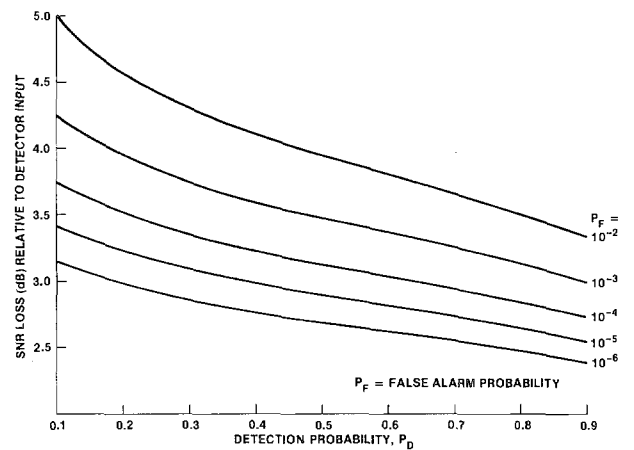


Fig. 6 ORing loss for 8 channels ORed and 32 lines on the display.

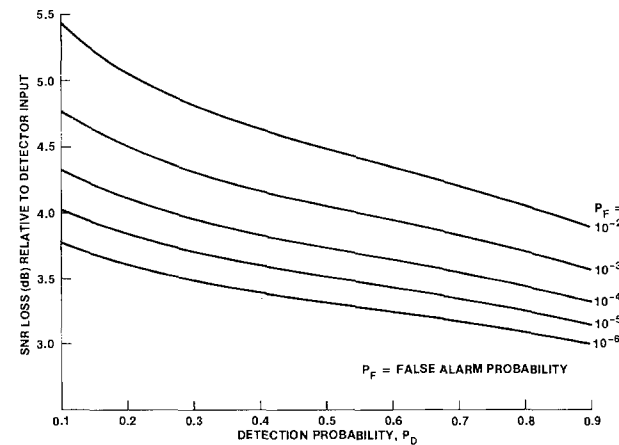


Fig. 7 ORing loss for 8 channels ORed and 64 lines on the display.