Three- dimensional industrial process tomography using electrical and electromagnetic tomography: recent developments

HY Wei, C Yang, L Ma, Z Ye, A Yao,M Soleimani
Department of Electronic and Electrical Engineering
University of Bath, UK
Email: m.soleimani@bath.ac.uk

Abstract —**Electrical and Electromagnetic imaging are widely used in many application areas. Most works are done in 2D imaging mode. Recently, volumetric (3D) imaging of passive electromagnetic properties using low frequency electrical and electromagnetic imaging is being developed. This paper presents the latest results of 3D electrical and electromagnetic tomography imaging. The results of experimental data will be presented based on most commonly used finite element modelling of the forward model and the standard Tikhonov regularized inverse solution. Challenges and opportunities will be discussed in presentation.**

Keywords-Electrical and electromagnetic tomography; ECT; EIT; MIT; three-dimensional process tomography

I. Introduction

Process tomography is an emerging field of sensing and monitoring with several major industrial applications [1]. Low frequency electromagnetic tomography techniques (less than 20 MHz) are used to non-invasively create cross sectional images of the objects with contrasts in one or more of the passive electromagnetic properties (PEP) including conductivity, permittivity and permeability [2]. Magnetic induction tomography (MIT) is a relatively new member of the electromagnetic imaging family, which works based the eddy current in conductive objects. Electrical impedance tomography (EIT) and electrical capacitance tomography (ECT) have been studied in this paper.

Three-dimensional imaging techniques have been developed for these imaging modalities. This paper presents the 3D imaging results. The forward modeling and sensitivity calculation has been presented and the standard Tikhonov regularization method has been used in all these techniques. Some useful references for more details can be found in [1-8].

II. Forward Modelling

The forward model in ECT, EIT and MIT are all based on some approximation to the Maxwell's equations. Here we briefly present the forward modeling in these imaging modalities. Gradient based image reconstruction methods such as Tikhonov regularization technique require sensitivity maps also known in form of Jacobian matrix, which has been discussed in [2].

1) ECT

Forward problem in ECT is a process of simulating the current measurements on electrode displacements when the excitation electrodes are set to a fixed voltage given the geometry of the electrical sensor array and permittivity distribution inside the electric field. Figure 1 is a sketch of a 2D ECT system which includes shields and the imaging area.
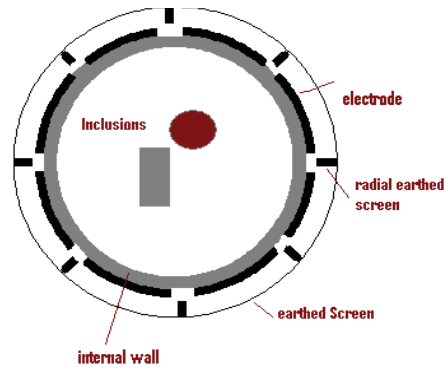

Figure 1 ECT System

As shown in figure 1, there are variety of combination of electrodes can be used so that there could be many different ECT protocols. In typical ECT protocol, each of the electrodes are set to some fixed voltage value in turn while the others are set to zero(grounded), the total charge is measured on each of the remaining electrodes.   Assuming the electrostatic approximation and no free charge we will have $\nabla \times E = 0$ for an electric potential u, and $E = -\nabla u$. The mathematical model for an ECT forward problem can be written as

$$\nabla \cdot (\varepsilon \nabla u) = 0 \quad \text{in } \Omega \tag{1}$$

Where ε is the dielectric permittivity, Ω indicates the region inside the cylindrical screen excluding the electrodes and radial screens (can possibly be an infinite region). The electric potential is fixed on each electrode and can be viewed as

$$u = v_k \qquad \text{on } E_k \tag{2}$$

This is the boundary conditions. E_k means the k_{th} electrode, v_k is a fixed value indicated the voltage on an excitation electrode and is zero on sensing electrodes. The electric charge on the k_{th} electrode is given by

$$Q_k = \int_{E_k} \varepsilon \frac{\partial u}{\partial n} dS$$

$$(3)$$

Where n is the inward normal on the k_{th} electrode. The finite element method should be satisfactorily solved this problem. By using Galerkin's approximation, the boundary value problem reduces to a linear system of equations

$$K(\varepsilon)U = Q$$

$$(4)$$

In this equation, K represents the discrete representation of the operator $\nabla \cdot \varepsilon \nabla$, Q is the boundary condition term and U is the vector of electric potential solution. In experimental ECT systems, the capacitance data are normalized using calibration. The normalized capacitance is (for each capacitance measurement)

$$\lambda = \frac{C_{meas} - C_{air}}{C_{high} - C_{air}}$$

$$(5)$$

The capacitances between all the combination pairs of the electrodes should be measured in order to perform a 'body-scan 'of the imaging volume. Each of these measurements has a unique sensitivity weighting over the cross-section of the imaging volume, and hence is independent of the others. Generally, for a system contains N-electrodes, the number of independent measurements can be calculated simply by
The N(N-1)/2 measurements are obtained via appropriate sensor electronics and sent to the control computer. Then the MATLAB can generate a tomography image of the dielectric distribution from the measurements and their corresponding sensitivity distribution maps. Figure 2 shows one of our meshed models and the sensitivity map of two electrodes in free space in first electrode layers.
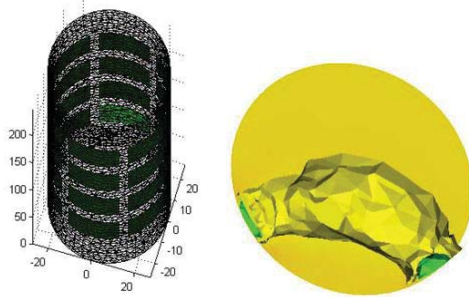

Figure 2 3D ECT Meshed Model (Left) Sensitivity Map (Right)

2) EIT
Figure 3 shows the sketch of a typical EIT system. Defer to the ECT, electrodes are now attached inside the system pipe

which would be directly in contact with the medium region. Here we present a method for the forward model and a general approach for the inverse problem that will be used EIT as well.
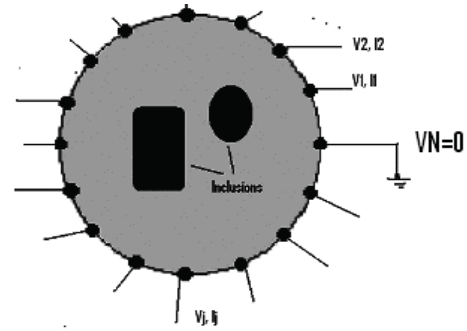

Figure 3 EIT Measurement Model

In order to solve the EIT inverse problem we need to solve the forward problem, which is the model of the measurement process, which can also be solved using a finite element method. As defined previously under low-frequency assumptions, the full Maxwell's equations can be simplified to the complex-valued Laplace equation:

$$\nabla \cdot (\sigma^* \nabla u) = 0 \qquad (6)$$

Where u is the complex-valued electric potential and σ^* is the complex conductivity of the medium.

$\sigma^* = \sigma + i\omega\varepsilon_o\varepsilon_r$, for ω is the angular frequency, ε_o and ε_r are the absolute and relative permittivity). Appropriate boundary conditions need to be enforced to enable an accurate model for EIT. In this work we use the complete electrode model, which takes into account both the shunting effect of the electrodes and the contact impedance between the electrodes and medium. Using this boundary condition the EIT model includes

$$U + z_l\sigma\frac{\partial u}{dn} = V_l, \ x \in e_l, l = 1,2,\ldots,L \qquad (7)$$

$$\int_{e_l} \sigma\frac{\partial u}{dn} dS = I_l, \ x \in e_l, l = 1,2,\ldots,L \qquad (8)$$

$$\sigma\frac{\partial u}{dn} = 0, \ x \in \partial\Omega/\cup_l^L e_l \qquad (9)$$

where z_l is the effective contact impedance between the lth electrode and the tissue, n is the outward normal, V is the complex-valued voltage, I is the complex-valued current and e_l denotes the electrode l. $x \in \partial\Omega/\cup_l^L e_l$ indicates a point on the boundary not under the electrodes. A finite element method (FEM) based forward model has been used here.

3) *MIT*

Figure 4 shows the sensor arrangement for a 2D MIT system. In MIT mutual inductance of pairs of the coils are measured shown in figure 5.
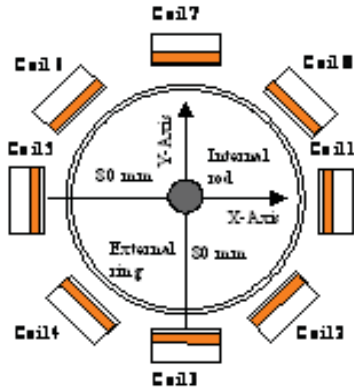

Figure 4 MIT Model

The forward problem in MIT is a classical eddy current problem. This problem can be formulated in terms of the magnetic vector potential A for the sinusoidal waveform excitation case using complex phasor notation

$$\nabla \times (\frac{1}{\mu} \nabla \times \mathbf{A}) + i\omega\sigma\mathbf{A} = \mathbf{J}_s \quad (11)$$

where σ is electrical conductivity (the imaginary part $-i\omega\varepsilon$ can be neglected due to high conductivity samples), μ is magnetic permeability, ω is frequency in rad/sec and J_s is the applied current density in an excitation coil. In this study we used a so called A, A formulation for the eddy current problem and for that we had to mesh the coils as part of discretisation of the domains.
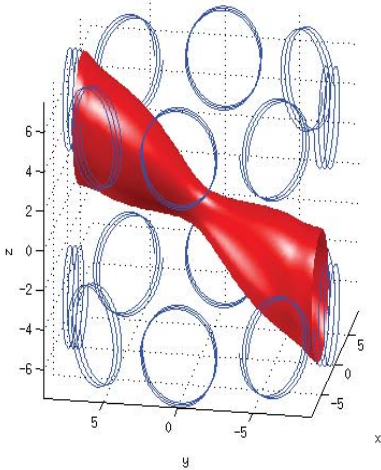
III. INVERSION METHOD

In image reconstruction, only linear algorithms are considered. The linear method is based on most commonly used Tikhonov regularization, which uses a universal regularization technique for solving the ill-posed inverse problem in the following manner $x = (J^T J + \alpha I)^{-1} J^T z$, where x is the image pixel vector (which represent electrical conductivity of the pixels), z is the measurement vector, J is the sensitivity matrix (Jacobian matrix), I is an identity matrix, and α is the regularization parameter.

IV. RESULTS

Figure 6 shows experimental data for 3D MIT system with a two array of sensors 8 in each plane. Figure 7 shows reconstruction of 3D EIT using a 12 electrodes sensor two arrays of 6 in each plane. Figure 8 shows reconstruction of 3D ECT using a 32 electrodes system.
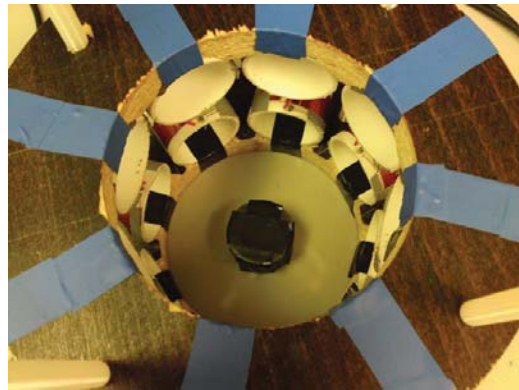

Figure 5 Sensitivity Map of MIT System

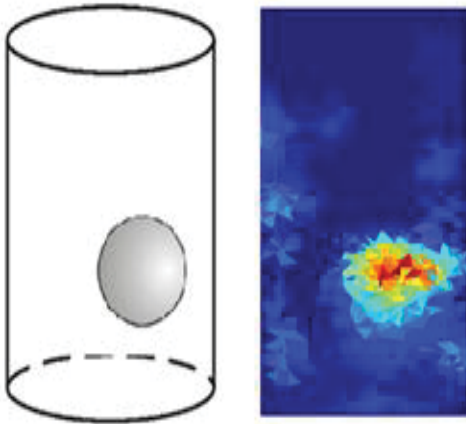**Figure 6 MIT Result Object (top) Resulting Image (bottom)**

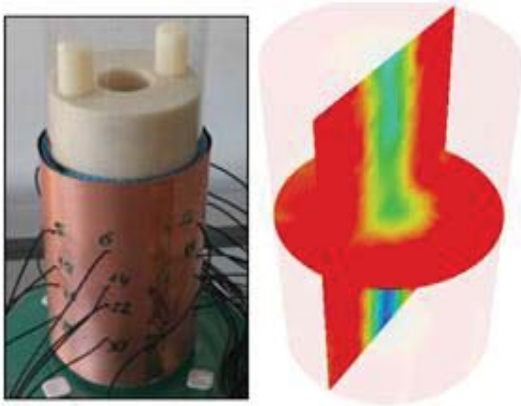**Figure 7 EIT Result Object (left) Resulting Image (right)**

Figure 8 ECT Result Object (left) Resulting Image (right) [9]

V.   CONCLUSIONS

Three-dimensional tomography provides new opportunities in industrial process applications. Although the 3D imaging is being developed in academic environments, they have been less used in industrial settings. The main challenges currently facing are the computational aspects including memory issues and computational time. The linear system of equations arising from forward and the inverse problems in 3D bring great deal of challenges for numerical analysts. Selection of suitable linear solver, regularization type, regularization parameter and computational structure (CPU or GPU or HPC) are the main challenges in 3D imaging in coming years. Furthermore, more results will be shown during the presentation.

VI.   REFERENCES

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