

Ergodic Capacity of MIMO Triply Selective Rayleigh Fading Channels

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Abstract—New results are presented for the ergodic capacity of spatially-correlated, time-varying and frequency-selective (*i.e.*, triply selective) MIMO Rayleigh fading channels. Simplified capacity formulas are also derived for special cases such as SIMO and MISO triply selective fading channels. A closed form formula is proposed that quantifies the effect of the frequency-selective fading on the ergodic capacity into an intersymbol interference (ISI) degradation factor. It is discovered that, in general frequency-selective MIMO channels, the ISI inter-tap correlations will reduce the ergodic capacity comparing to the frequency flat fading channel. Only in the special case when the fading does not have ISI inter-tap correlations will the ergodic capacity be the same as that of the frequency flat channel. The new capacity results are experimentally verified via Monte-Carlo simulations.

I. INTRODUCTION

Multiple-Input Multiple-Output (MIMO) wireless communication has recently received significant attention due to its enormous channel capacity potential in rich scattering environment [1]-[3]. The ergodic capacity results have been well established for MIMO Rayleigh fading channels which are spatially correlated (including spatially uncorrelated), time quasi-static, and frequency nonselective, see [4]-[22] and the references therein. These capacity results are based on the assumption that the MIMO channels have neither Doppler spread nor delay spread, which is not the case in many moderate and high mobility and high data rate mobile communication applications.

The capacity studies for MIMO frequency-selective Rayleigh fading channels has also received some attention [23]-[29]. Specifically, in [25], it was reported that OFDM-based MIMO frequency-selective (delay spread) channels will in general provide advantages over frequency flat fading channels not only in terms of outage capacity but also in terms of ergodic capacity. However, in [29], it was reported that frequency-selectivity does not affect the ergodic capacity of wide-band MIMO channels, which is agreeable with the single-input single-output (SISO) ergodic capacity results in [32]. Both [29] and [32] are based on the assumption that the discrete-time sampled channel impulse response has no inter-tap correlation. Recently, it was reported by Xiao *et al* [35] and Paulraj *et al* [19] that the sampled fading channel taps are in general inter-tap correlated due to the transmit pulse-shaping and receive matched filters.

In this paper, we consider the ergodic capacity of a MIMO system that undergoes inter-tap correlated (including inter-

tap uncorrelated as a special case) frequency-selective, time-varying and spatially correlated fading, which is referred to as *triply selective* fading in this paper. Due to the time variation, we assume that the channel state information is unknown to the transmitter but perfectly known to the receiver. Therefore, the equal power allocation scheme is used at the transmitter. New results for the ergodic capacity are derived for MIMO triply selective Rayleigh fading channels. Simplified capacity formulas are also derived for special cases such as single-input multiple-output (SIMO) and multiple-input single-output (MISO) systems. We find that the inter-tap correlations of frequency-selective fading channels can have significant impact on the ergodic capacity. This impact is quantified into an ISI degradation factor in a closed form formula. In a general frequency-selective fading channel, the ergodic capacity is reduced by the ISI degradation factor. In the special case when the ISI has no inter-tap correlations, the ISI degradation factor is one, and the ergodic capacity is the same as that of the frequency flat channel. The theoretical results have been verified by extensive Monte-Carlo simulations using improved Jakes' Rayleigh fading simulator [35], [36].

II. CHANNEL MODELS AND PRELIMINARIES

Consider a wideband MIMO wireless channel. Assume that the transmit pulse shaping filter $p_T(t)$ and the receive matched filter $p_R(t)$ are normalized with unit energy. Assume also that each physical fading subchannel $g_{m,n}(t, \tau)$ is wide-sense stationary uncorrelated scattering (WSSUS) [33] Rayleigh fading with normalized unit energy. The continuous-time MIMO channel can be accurately converted to the following discrete-time MIMO fading channel model with proper delay [35]

$$\mathbf{y}(k) = \sum_{l=0}^{L-1} \mathbf{H}(l, k) \cdot \mathbf{x}(k-l) + \mathbf{v}(k), \quad k = 0, 1, \dots, \infty, \quad (1)$$

where the input $\mathbf{x}(k) = [x_1(k), x_2(k), \dots, x_N(k)]^t$, the noise $\mathbf{v}(k) = [v_1(k), v_2(k), \dots, v_M(k)]^t$, and the output $\mathbf{y}(k) = [y_1(k), y_2(k), \dots, y_M(k)]^t$, with the superscript $(\cdot)^t$ being the transpose; L is the channel length which is depending on the transmit filter, delay spread power profiles and receive filter; and the matrix $\mathbf{H}(l, k)$ is the lT_s delayed channel matrix at

time instant k , defined by

$$\mathbf{H}(l, k) = \begin{bmatrix} h_{1,1}(l, k) & h_{1,2}(l, k) & \cdots & h_{1,N}(l, k) \\ h_{2,1}(l, k) & h_{2,2}(l, k) & \cdots & h_{2,N}(l, k) \\ \vdots & \ddots & \ddots & \vdots \\ h_{M,1}(l, k) & h_{M,2}(l, k) & \cdots & h_{M,N}(l, k) \end{bmatrix}, \quad (2)$$

where $h_{m,n}(l, k)$ is the (m, n) th subchannel's l th tap coefficient with time-varying index k .

Based on the physical fading channel assumptions described in [35], the composite discrete-time fading channel coefficients $h_{m,n}(l, k)$ are zero-mean complex-valued Gaussian random variables. The correlation function between the channel coefficients $h_{m,n}(l, k)$ and $h_{p,q}(l, k)$ is given by [35]

$$\mathcal{E} [h_{m,n}(l_1, k_1) \cdot h_{p,q}^*(l_2, k_2)] = \Psi_{RX}(m, p) \cdot \Psi_{TX}(n, q) \cdot \Psi_{ISI}(l_1, l_2) \cdot \Psi_{DPR}(k_1, k_2), \quad (3)$$

where the superscript $*$ denotes the conjugate, $\mathcal{E}[\cdot]$ denotes the expectation. The matrices Ψ_{RX} , Ψ_{TX} , Ψ_{ISI} and Ψ_{DPR} are the receive correlation coefficient matrix, the transmit correlation coefficient matrix, the intersymbol interference (ISI) inter-tap correlation coefficient matrix, and the temporal correlation coefficient matrix, respectively.

We give three specific remarks on the elements of these four matrices. First, $\Psi_{RX}(m, p)$ is the receive correlation coefficient between receive antennas m and p related to angle spread at the receiver with $0 \leq |\Psi_{RX}(m, p)| \leq \Psi_{RX}(m, m) = 1$, and $\Psi_{TX}(n, q)$ is the transmit correlation coefficient between transmit antennas n and q related to angle spread at the transmitter with $0 \leq |\Psi_{TX}(n, q)| \leq \Psi_{TX}(n, n) = 1$. Second, the coefficient $\Psi_{ISI}(l_1, l_2)$ is related to the channel fading power delay profile, the transmit filter, and the receive filter. Its calculation is given by (17) of [35]. Even if the physical channel $g_{m,n}(t, \tau)$ is WSSUS channel which means no inter-path correlation, the discrete-time sampled channel $h_{m,n}(l, k)$ will generally have inter-tap correlations [35], [19] because of the convolution between $p_T(t)$, $g_{m,n}(t, \tau)$ and $p_T(t)$. Our third remark goes to Ψ_{DPR} . Different fading model will have different Ψ_{DPR} . For the commonly used Clarke's 2-D isotropic scattering model-based Rayleigh fading, $\Psi_{DPR}(k_1, k_2) = J_0(2\pi F_d(k_1 - k_2)T_s)$, with $J_0(\cdot)$ being the zero-order Bessel function of the first kind, F_d the maximum Doppler frequency, and T_s the symbol period. The first three matrices satisfy $\text{tr}(\Psi_{RX}) = M$, $\text{tr}(\Psi_{TX}) = N$, and $\text{tr}(\Psi_{ISI}) = 1$ [35] due to normalizations.

This discrete-time MIMO channel model (3) is a generalized model describing triply selective MIMO channels. It contains many existing channel models as special cases. For example, 1), if $L = 1$ and $F_d = 0$, then the channel model becomes the spatially correlated, time quasi-static, and frequency flat model [5]. 2) If $L = 1$, $F_d = 0$, $\Psi_{TX} = \mathbf{I}_N$, and $\Psi_{RX} = \mathbf{I}_M$, then the model becomes the spatially uncorrelated, time quasi-static, and frequency flat model [1]. 3) If $M = 1$ and $N = 1$, then our model becomes the doubly selective fading model for SISO systems [34]. 4) If $L = 1$ and Ψ_{DPR} is an identity

matrix, then this model becomes a symbol-wise temporally independent fading model [1].

When the channel has intersymbol interference (frequency-selective), the channel capacity has to be analyzed based on a block of K output symbols $\{\mathbf{y}(k+1), \mathbf{y}(k+2), \dots, \mathbf{y}(k+K)\}$ at the receiver. The MIMO channel with ISI is then represented by

$$\mathbf{Y}_K = \mathcal{H} \mathbf{X}_{K+L-1} + \mathbf{V}_K, \quad (4)$$

where $\mathbf{Y}_K = [\mathbf{y}^t(k+1), \mathbf{y}^t(k+2), \dots, \mathbf{y}^t(k+K)]^t$, the input vector \mathbf{X}_{K+L-1} is circularly symmetric complex Gaussian (with padded zeros to clear out the ISI memory), and the noise vector \mathbf{V}_K is the additive white complex Gaussian random vector whose entries are i.i.d. and circularly symmetric, and

$$\mathcal{H} = \begin{bmatrix} \mathbf{H}(L-1, k+1) & \mathbf{H}(L-2, k+1) & \cdots & \mathbf{H}(0, k+1) & 0 & 0 \\ 0 & \mathbf{H}(L-1, k+2) & \ddots & \mathbf{H}(1, k+2) & \ddots & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & \mathbf{H}(L-1, k+K) & \cdots & \mathbf{H}(0, k+K) \end{bmatrix}.$$

When the channel matrix \mathcal{H} is perfectly known to the receiver but unknown to the transmitter, the equal power allocation scheme is employed. Then the instantaneous mutual information (per input symbol) is defined as

$$\mathcal{I}_K(k) = \frac{1}{K+L-1} \cdot \left[\log_2 \det \left(\mathbf{I}_{KM} + \frac{\gamma}{N} \mathcal{H} \mathcal{H}^\dagger \right) \right], \quad \text{b/s/Hz}, \quad (5)$$

where $\gamma = \frac{\mathcal{P}}{\sigma^2}$ is the normalized SNR with σ^2 being the receive noise power at each receive antenna and \mathcal{P} being the average total transmission power over the N antennas, and the superscript $(\cdot)^\dagger$ denotes the conjugate transpose. For a large $K \gg L$, the factor $1/(K+L-1)$ in (5) can be approximated by $1/K$. The ergodic capacity \mathcal{C}_{MIMO}^{av} is given by

$$\mathcal{C}_{MIMO}^{av} = \lim_{K \rightarrow \infty} \frac{1}{K} \left\{ \mathcal{E}_{\mathcal{H}} \left[\log_2 \det \left(\mathbf{I}_{KM} + \frac{\gamma}{N} \mathcal{H} \mathcal{H}^\dagger \right) \right] \right\}. \quad (6)$$

Before leaving this section, we make a remark on equation (6). It is well known that for the special MIMO channel with time quasi-static and frequency flat fading, which corresponds to the number of ISI taps $L = 1$ in the channel model (4), the channel matrix \mathcal{H} can be simplified and decomposed [4], [5] directly into $\mathcal{H} = \mathbf{H}(0, k) = \Psi_{RX}^{1/2} \mathbf{H}_W \Psi_{TX}^{1/2}$, where \mathbf{H}_W is a random matrix with $M \times N$ independent and identically distributed (i.i.d.) complex Gaussian random variables. Unfortunately, a similar form of decomposition does not exist for the triply selective MIMO fading channel with a general channel matrix \mathcal{H} ($K \neq 1$ and $L \neq 1$). Therefore, the Wishart (random) matrix theory [39], [40], [41] can not be directly employed to study the triply selective fading channel capacity (6).

III. NEW RESULTS FOR ERGODIC CAPACITY

In this section, we first present the ergodic capacity results for MIMO triply selective Rayleigh fading channels. Then we simplify the results for SIMO and MISO triply selective Rayleigh fading channels.

Theorem 1: For the triply selective fading MIMO channel characterized by equations (1)-(3), the ergodic capacity defined by (6) is equivalent to the following expression

$$\mathcal{C}_{MIMO}^{av} = \frac{1}{2\pi} \int_0^{2\pi} \mathcal{E}_{\mathbf{H}_W} \left\{ \log_2 \det \left[\mathbf{I}_M + \frac{\gamma}{N} f(\omega) \Psi_{RX} \mathbf{H}_W \Psi_{TX} \mathbf{H}_W^\dagger \right] \right\} d\omega, \quad (7)$$

where \mathbf{H}_W is an $(M \times N)$ matrix whose elements are normalized i.i.d. complex Gaussian random variables, and $f(\omega)$ is the channel power spectrum function determined solely by Ψ_{ISI} as follows

$$f(\omega) = 1 + 2 \sum_{i=1}^{L-1} a_i \cos(i\omega), \quad a_i = \sum_{l=0}^{L-1-i} \Psi_{ISI}(l, l+i).$$

Proof: Omitted for brevity.

Proposition 1: The triply selective fading MIMO channel ergodic capacity given by (7) can be accurately computed by

$$\mathcal{C}_{MIMO}^{av} = \mathcal{E}_{\mathbf{H}_W} \left\{ \log_2 \det \left[\mathbf{I}_M + \frac{\gamma}{N} \cdot \gamma_{ISI} \cdot \Psi_{RX} \mathbf{H}_W \Psi_{TX} \mathbf{H}_W^\dagger \right] \right\}, \quad (8)$$

where γ_{ISI} is the ISI degradation factor due to the channel ISI inter-tap correlations, determined by

$$\gamma_{ISI} = (2^{C_\gamma} - 1) / \gamma \quad (9)$$

with

$$C_\gamma = \frac{1}{2\pi} \int_0^{2\pi} \log_2 [1 + \gamma \cdot f(\omega)] d\omega. \quad (10)$$

Remark 1: The advantage of (7) over the capacity definition (6) is that the infinite sized channel matrix \mathcal{H} in (6) is reduced into a finite and small sized (*i.e.*, $M \times N$) random matrix \mathbf{H}_W in (7). Furthermore, the $L \times L$ ISI inter-tap correlation matrix Ψ_{ISI} is also converted to a scalar function $f(\omega)$ under the condition that the multiple subchannels share the same ISI fading characteristics [35]. This condition is met if the base station antenna separations are much smaller than the distance between the base station and the mobile station which is usually the case in practice. This salient feature of (7) is obtained through the decomposition property (3). It makes the capacity analysis of triply selective fading channels mathematically manageable. Proposition 1 makes one step forward to simplify the computation of the underline ergodic capacity.

Remark 2: It is noted that for spatially uncorrelated cases, *i.e.*, $\Psi_{TX} = \mathbf{I}_N$ and $\Psi_{RX} = \mathbf{I}_M$, the MIMO triply selective fading channel becomes the MIMO doubly selective fading channel, and the capacity result (8) has semi-analytic solution as shown in [42]. For spatially semicorrelated cases, *i.e.*, $\Psi_{TX} = \mathbf{I}_N$ or $\Psi_{RX} = \mathbf{I}_M$, the capacity formula (8) can be derived to have deterministic expression by utilizing the techniques proposed in [13], and [16] for frequency flat fading channels. For the case that both Ψ_{TX} and Ψ_{RX} are non-identity matrices, an upper bound can be derived for (8) by employing the procedure presented in [15] for frequency flat fading channels. Since the deterministic expression and the upper bound are somewhat complicated, details are omitted for brevity.

The ergodic capacity formula given by (8) can be significantly simplified for SIMO and MISO systems as shown below.

Proposition 2: For SIMO triply selective Rayleigh fading channels, the individual subchannels are spatially correlated, *i.e.*, Ψ_{RX} is not an identity matrix. Since $N = 1$ and $\Psi_{TX} = \mathbf{I}$, the ergodic capacity (8) can be simplified to be

$$\mathcal{C}_{SIMO}^{av} = \int_0^\infty \log_2 (1 + \gamma \cdot \gamma_{ISI} \cdot \lambda) \cdot p_\lambda(\lambda) \cdot d\lambda, \quad (11)$$

where $p_\lambda(\lambda) = \sum_{k=1}^M \frac{\beta_k}{\sigma_k} \exp\left(-\frac{\lambda}{\sigma_k}\right)$ with

$\beta_k = \prod_{i=1, i \neq k}^M \frac{\sigma_k}{\sigma_k - \sigma_i}$, and σ_i being the i th eigenvalue of Ψ_{RX} .

Proposition 3: For MISO triply selective Rayleigh fading channels, the individual subchannels are also spatially correlated, *i.e.*, Ψ_{TX} is not an identity matrix. Since $M = 1$ and $\Psi_{RX} = \mathbf{I}$, the ergodic capacity (8) can be simplified to be

$$\mathcal{C}_{MISO}^{av} = \int_0^\infty \log_2 \left(1 + \frac{\gamma}{N} \cdot \gamma_{ISI} \cdot \lambda \right) \cdot p_\lambda(\lambda) \cdot d\lambda, \quad (12)$$

where $p_\lambda(\lambda) = \sum_{k=1}^N \frac{\beta_k}{\sigma_k} \exp\left(-\frac{\lambda}{\sigma_k}\right)$ with

$\beta_k = \prod_{i=1, i \neq k}^N \frac{\sigma_k}{\sigma_k - \sigma_i}$, and σ_i being the i th eigenvalue of Ψ_{TX} .

The proof of Propositions 2 and 3 are omitted for brevity.

IV. SIMULATION RESULTS

To verify the theoretical ergodic capacity results presented in Section III, we have conducted extensive simulations which employs the discrete-time time-varying frequency-selective Rayleigh fading MIMO channel model described in Section II with different channel conditions such as Doppler spread F_d , channel length L , block length K , and antenna numbers M and N . To keep the paper within the length limit, we only present some of the simulation results.

Figure 1 depicts the ergodic capacity for the SISO, 2×2 , and 4×4 systems over three fading channel conditions. It is shown that every simulated curve is in excellent agreement with the corresponding theoretical curve. Comparing the ergodic capacities of the three different MIMO systems, we can see that the ergodic capacity increases as the number of antennas. The inter-tap uncorrelated frequency-selective channel has the same ergodic capacity as that of the frequency flat fading channel. However, when the frequency-selective Rayleigh fading channel has inter-tap correlations, its ergodic capacity is smaller than that of the channel with no inter-tap correlations.

Figure 2 shows the ergodic capacity for a 2×1 system over three fading channels. All simulation results are in excellent agreement with the theoretically calculated results based on Proposition 2. It is clearly shown that both the spatial

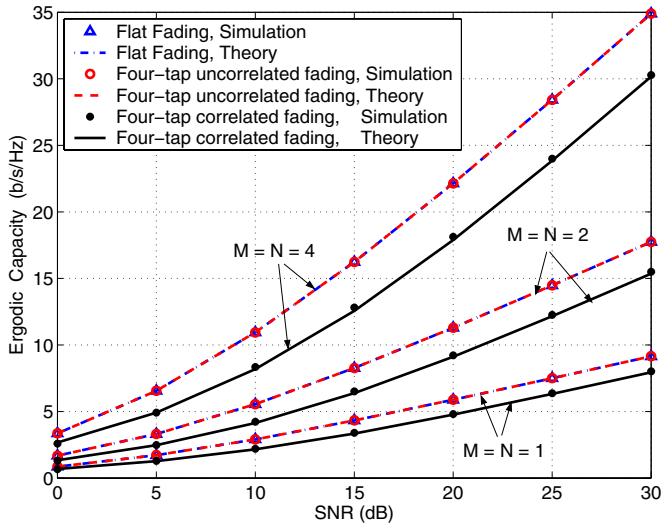


Fig. 1. Ergodic Capacity vs SNR for the SISO, 2×2 and 4×4 systems over: 1) spatially-uncorrelated frequency flat fading channel; 2) spatially-uncorrelated frequency-selective four-tap-uncorrelated fading channel; 3) spatially-uncorrelated frequency-selective four-tap-correlated fading channel with $\Psi_{ISI}(i, j) = \frac{0.95^{|i-j|}}{4}$.

correlation and the ISI inter-tap correlation will reduce the ergodic capacity if the channel state information is only known to the receiver.

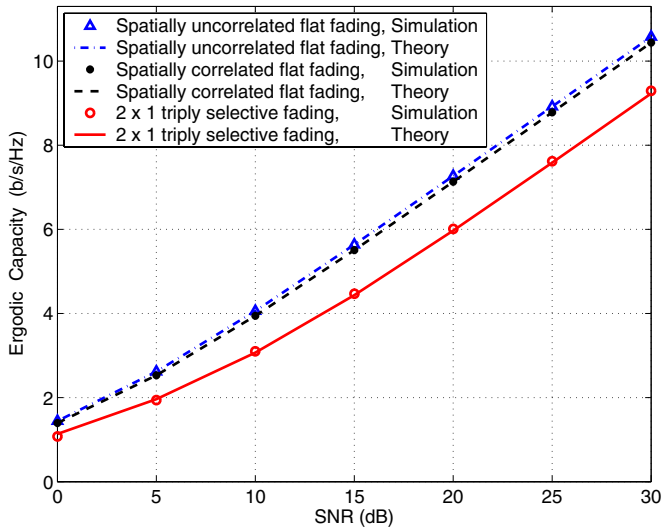


Fig. 2. Ergodic Capacity vs SNR for a 2×1 system over: 1) spatially-uncorrelated frequency flat fading; 2) spatially-correlated frequency flat fading with $\Psi_{RX} = [1 \ 0.7; \ 0.7 \ 1]$; 3) triply selective fading with $\Psi_{RX} = [1 \ 0.7; \ 0.7 \ 1]$ and $\Psi_{ISI}(i, j) = \frac{0.95^{|i-j|}}{4}$.

Figure 3 plots the ergodic capacity for a 2×2 system over three fading channels. Again, all the simulation results are in excellent agreement with the theoretical results obtained from Theorem 1 and Proposition 1. It is also indicated that the spatial correlation and the ISI inter-tap correlation reduce the ergodic capacity for equal power allocation at the transmitter.

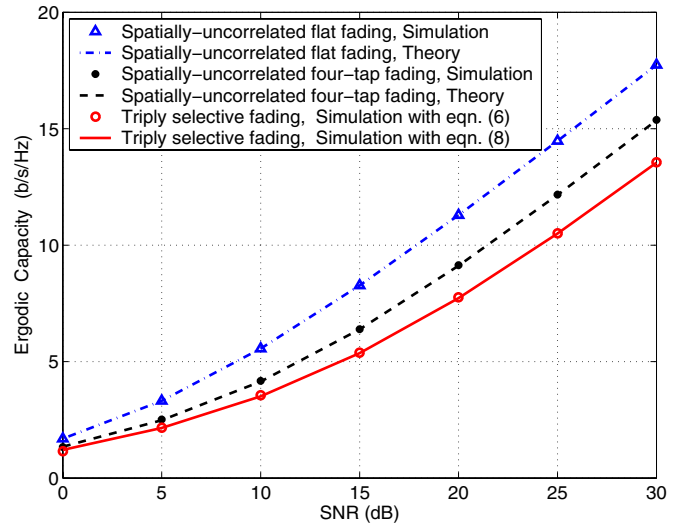


Fig. 3. Ergodic Capacity vs SNR for a 2×2 system over: 1) spatially-uncorrelated frequency flat fading; 2) spatially-uncorrelated frequency-selective fading with $\Psi_{ISI}(i, j) = \frac{0.95^{|i-j|}}{4}$; 3) triply selective fading with $\Psi_{TX} = [1 \ 0.7; \ 0.7 \ 1]$, $\Psi_{RX} = [1 \ 0.7; \ 0.7 \ 1]$ and $\Psi_{ISI}(i, j) = \frac{0.95^{|i-j|}}{4}$.

V. CONCLUSION

The ergodic capacity is investigated for triply selective (spatially-correlated, time-varying and frequency-selective) MIMO Rayleigh fading channels. A closed form formula has been derived that quantifies the effect of the ISI fading on the ergodic capacity into an ISI degradation factor γ_{ISI} . In the special case when the ISI fading does not have inter-tap correlations, $\gamma_{ISI} = 1$, and the ergodic capacity is the same as that of the frequency flat channel. In the more general cases of frequency-selective MIMO channels, $\gamma_{ISI} < 1$, and the inter-tap correlations of the ISI fading will reduce the ergodic capacity. A set of simplified results has been derived for SIMO and MISO systems. The new formulae have been experimentally verified via Monte-Carlo simulations.

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