

# PARCS MAGNETIC FIELD MEASUREMENT: LOW FREQUENCY MAJORANA TRANSITIONS AND MAGNETIC FIELD INHOMOGENEITY<sup>1</sup>

J. H. Shirley and S. R. Jefferts  
Time and Frequency Division  
National Institute of Standards and Technology  
325 Broadway  
Boulder, Colorado 80305

*Abstract* - Knowledge of the magnetic field is required to determine the quadratic Zeeman bias of a cesium frequency standard. The small magnetic fields (C-field) used in PARCS (PARCS is the primary atomic reference clock in space project), may have sufficient inhomogeneity that it is not practical to measure the field by observing Ramsey fringes on a field-sensitive hyperfine line at low launch velocities. An alternative method is to observe low-frequency (Majorana) transitions among the Zeeman sublevels. However, this method does not average the C-field in the same way as Ramsey excitation. We have computed the error incurred in using this method for some simple models of inhomogeneity. We have then estimated the field homogeneity requirements needed for this method to produce adequate results. Experiments with the NIST cesium fountain are in qualitative agreement.

*Keywords* - Magnetic field inhomogeneity, Majorana transitions, PARCS

## INTRODUCTION

In a cesium frequency standard the magnetic field is normally measured by observing a field-dependent hyperfine line near the clock transition. But the narrow velocity distribution used in PARCS [1] gives a lineshape with many Ramsey fringes making it difficult to pick out the central one. Also, the magnetic fields used in PARCS may be so small (10nT) that the relative field inhomogeneity may be large enough to wash out the Ramsey fringes or move them off the Rabi pedestal. These considerations have led us to consider alternative methods for measuring the magnetic field.

We propose here a method using low-frequency Majorana transitions ( $\Delta F = 0$ ,  $\Delta m = \pm 1$ ) among the Zeeman sublevels of the selected hyperfine level. These can be observed by PARCS using the normal state-selection technique to put atoms into the  $F = 3$ ,  $m = 0$  state. No excitation is done in the first Ramsey cavity. The Majorana transitions are driven with a low-frequency transverse magnetic field covering the drift region. Any remaining  $F = 3$ ,

$m = 0$  atoms are removed to the  $F = 4$ ,  $m = 0$  state by microwave excitation in the second Ramsey cavity. Both  $F = 3$  and  $F = 4$  atoms are then detected in the normal manner. The detected  $F = 3$  atoms are those which have made a transition to a nonzero  $m$  value. Because the excitation time can be as long as the drift time for Ramsey excitation, the resulting resonance can be almost as narrow as the Ramsey fringes.

However, this method does not average magnetic field inhomogeneity in the same way as microwave Ramsey excitation does. Hence its use will give a different value for the magnetic field bias. In the following we examine how the field is averaged during Majorana excitation. We compare the resulting bias with the inhomogeneity bias associated with Ramsey excitation. We then invert the problem to specify the field homogeneity required to keep these biases below a given relative uncertainty on the clock transition. Finally, we give the results of some experiments with the cesium fountain NIST-F1.

## THEORY - - A Little Bit

We shall adopt the theoretical methods used by Shirley et al. [2]. We describe an inhomogeneous magnetic field by

$$B(t) = B_0[1 - \epsilon f(t)], \quad (1)$$

where  $B_0$  is a fixed nominal field value chosen for convenience,  $\epsilon$  is a measure of the relative amplitude of inhomogeneity, and  $f(t)$  contains all the variation of the field. For definiteness we specify  $|f(t)| \leq 1$ . In the absence of inhomogeneity let  $\nu_Z$ , proportional to  $B_0$ , be the separation of the  $\Delta m = 0$  lines in the hyperfine spectrum,  $\delta\nu_{QZ} = 8\nu_Z^2/\nu_{\text{hfs}}$  be the quadratic Zeeman bias of the clock transition, and  $\nu_{\text{hfs}}$  be the unperturbed clock transition frequency defining the second. A Ramsey measurement of a field-sensitive line yields  $\nu_Z[1 - \epsilon\langle f \rangle]$  where

<sup>1</sup> Contribution of NIST, an agency of the US government; not subject to copyright in the United States

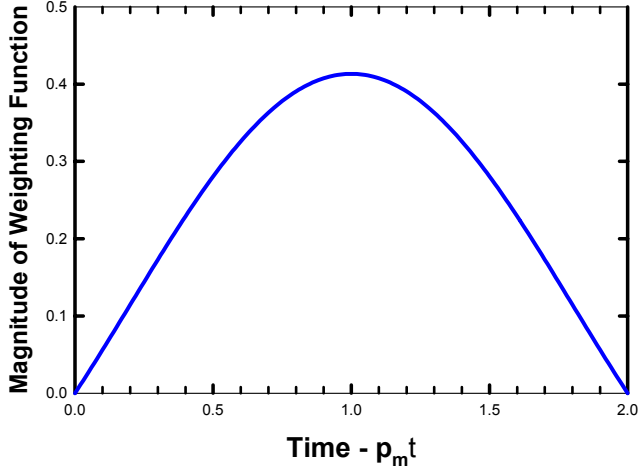


Fig 1. Weighing function for inhomogeneity averaging.

$$\langle f \rangle = \frac{1}{T} \int_0^T f(t) dt \quad (2)$$

is the time average of  $f(t)$  across the drift region.

A measurement of the Majorana transition across the same region yields the resonance frequency  $\nu_M[1-\epsilon f_M]$  where  $\nu_M$  is the resonance frequency associated with  $B_0$  and  $f_M$  is a weighted time average dependent on the amplitude and detuning of the excitation. The ratio of  $\nu_M$  to  $\nu_Z$  is  $(g_J - 9g_I)/2(g_J - g_I)$  or 0.5008 for the  $F = 3$  state of cesium. Here  $g_J$  and  $g_I$  are the electron and nuclear g-factors. The difference from 0.5000 can be significant in using the Majorana measurement to predict the quadratic Zeeman bias.

For constant excitation amplitude  $f_M$  can be written as

$$f_M = \frac{\int_0^T f(t) [\cos p_m(2t - T) - \cos p_m T] p_m dt}{\sin p_m T - p_m T \cos p_m T} \quad (3)$$

Here  $p_m$  is half the generalized Rabi frequency  $p_m^2 = b^2 + \omega_m^2/4$ ,  $b$  is half the Rabi frequency (proportional to the amplitude of the excitation field), and  $\omega_m$  is the frequency detuning at which the line center is measured. We have assumed that the center frequency of the resonance is found by slow square-wave frequency modulation of amplitude  $\omega_m$  (see Fig. 2). Equation (3) was derived by inserting (1) into the time-dependent Schrödinger equation to create a time-dependent detuning and then solving to first order in  $\epsilon$  [3]. The weighting function in the integrand is a truncated cosine symmetric about  $T/2$ . It is shown in Fig. 1 for  $p_m T = 1.96$ , a value corresponding to a modulation frequency equal to the

half width, and the lowest  $b$  value giving a maximum transition probability on resonance. Its effect is to de-emphasize the inhomogeneity near the beginning and end of the excitation. The denominator normalizes the integral and is related to the slope of the lineshape.

Our first result from (3) is that any inhomogeneity anti-symmetric about  $T/2$  does not contribute to  $f_M$ . In the following table we list some sample symmetric inhomogeneity functions  $f(t)$ , their mean  $\langle f \rangle$ , mean square  $\langle f^2 \rangle$ , and their excitation mean  $f_M$ .

Table I. Inhomogeneity Functions and their Means

Name	$f(t)$	$\langle f \rangle$	$\langle f^2 \rangle$	$f_M$
Quadratic	$(1-2t/T)^2$	1/3	1/5	$(2S\sin a + D)/3D - 2/a^2$
Cosine	$\cos(2\pi t/T)$	0	1/2	$-a^2 \sin a / (\pi^2 - a^2) D$

End steps:

$$\begin{aligned} 1, & \quad 0 \leq t \leq \tau & r & \quad r & [\sin a - \sin(a - r) - \arccos a]/D \\ 0, & \quad \tau \leq t \leq T - \tau \\ 1, & \quad T - \tau \leq t \leq T \end{aligned}$$

In the above table  $a = p_m T$ ,  $r = 2\tau/T$ , and  $D = \sin a - a \cos a$ . For the lowest optimum excitation and modulation at the half height of the resonance we have  $bT = 0.342$ ,  $\omega_m T = 3.86$ , and  $p_m T = 1.96$ . For this value of  $a$  the values of  $f_M$  for the quadratic and cosine functions are 0.182 and -0.353 respectively. For the end step function we find -0.508  $r^2$  for small  $r$ . These values are not greatly different from those at small  $a$ .

## INHOMOGENEITY REQUIREMENTS

Since  $f_M$  differs from both the mean and root-mean-square values of  $f$ , its use in evaluating the quadratic Zeeman effect will lead to an incorrect result. Below we compare this error with a familiar inhomogeneity bias. We then estimate how small the inhomogeneity should be to keep either bias below a specified level in the accuracy budget of a standard.

The quadratic Zeeman effect of the clock transition as seen by the atoms is given by [2, Eq. 99]

$$\delta\nu_{QZ}(\text{atom}) = \delta\nu_{QZ} [1 - 2\epsilon \langle f \rangle + \epsilon^2 \langle f^2 \rangle].$$

If we measure the field by observing a field-dependent hyperfine transition and calculate the quadratic Zeeman effect, we obtain [2, Eq. 98]

$$\delta\nu_{QZ}(\text{calc}) = \delta\nu_{QZ} [1 - 2\epsilon \langle f \rangle + \epsilon^2 \langle f \rangle^2].$$

The difference between these expressions is the mean-square vs square-of-the-mean bias

$$\delta\nu_{QZ}(\text{square}) = \varepsilon^2 \delta\nu_{QZ}[\langle f \rangle^2 - \langle f^2 \rangle].$$

For this bias to be less than some specified relative uncertainty  $\delta y$  for the clock transition we must have  $|\delta\nu_{QZ}(\text{square})| \leq \delta y \nu_{hfs}$ . This requirement can be re-expressed as a requirement on the degree of inhomogeneity:

$$\varepsilon \leq (\nu_{hfs} / \nu_Z) \sqrt{\delta y / 8[\langle f^2 \rangle - \langle f \rangle^2]}. \quad (4)$$

Similarly, if we measure the field by low-frequency Majorana transitions, we compute the quadratic Zeeman bias as

$$\delta\nu_{QZ}(\text{Maj}) = \delta\nu_{QZ}[1 - 2\varepsilon f_M + \varepsilon^2 f_M^2],$$

yielding a bias that for small  $\varepsilon$  is dominated by the terms linear in  $\varepsilon$ . The corresponding inhomogeneity requirement is

$$\varepsilon \leq (\nu_{hfs} / \nu_Z)^2 \delta y / 16 |f_M - \langle f \rangle|. \quad (5)$$

Except for a factor of order unity this requirement is just the square of the preceding one. Both involve the quantity  $(\nu_{hfs} / \nu_Z) \sqrt{\delta y}$ .

Table II. Homogeneity Requirements for Three Standards

Clock Type Name	Uncertainty Goal $\delta y$	C-field and $\nu_Z$	Ramsey Method	Majorana Method
Thermal Beam NIST-7	$10^{-15}$	$5.6 \mu\text{T}$ $40 \text{ kHz}$	$\varepsilon < 0.007$	$\varepsilon < 10^{-5}$
Cesium Fountain NIST-F1	$10^{-16}$	$0.085 \mu\text{T}$ $600 \text{ Hz}$	$\varepsilon < 0.13$	$\varepsilon < 0.004$
Space Clock PARCS	$10^{-17}$	$0.01 \mu\text{T}$ $70 \text{ Hz}$	$\varepsilon < 0.4$	$\varepsilon < 0.07$

In table II (above) we present numerical evaluations of (4) and (5) for three primary standards of different types, their associated operating fields, and relative uncertainty goals for inhomogeneity bias. Neither NIST-7 nor NIST-F1 meet

the homogeneity requirement for using the Majorana method. But PARCS can meet it. Note that the Ramsey method condition for PARCS assumes that Ramsey fringes can be observed on the field sensitive line, which will not be the case for large inhomogeneity. For example, a linear field gradient along the longitudinal axis of several per cent will offset the Rabi pedestal so far that fringes cannot be seen.

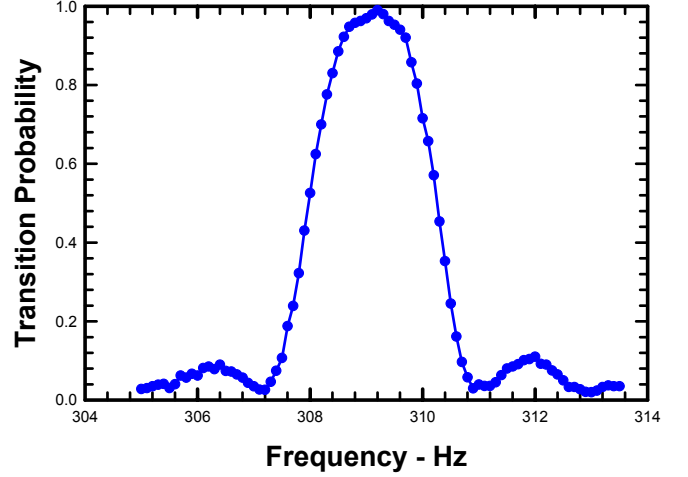


Figure 2 – Lineshape of the Majorana resonance in NIST-F1

## EXPERIMENTS

The cesium fountain primary frequency standard NIST-F1 [4] was used for preliminary experiments since it has a somewhat suitable configuration and detailed magnetic field measurements had already been made. Figure 3 shows the magnetic field in NIST-F1 as a function of distance above the cavity center. The measurements were made by 0.1 s pulses of low frequency excitation timed to coincide with the atomic cloud being at apogee. The short pulses ensure that the field is averaged over only about 1 cm height. In normal operation the atoms traverse the field from left to right, slowing as they approach apogee. They then reverse traversing the field from right to left. Hence they always see a symmetric inhomogeneity.

The Majorana measurements were made by using the state-selection cavity to prepare atoms in the  $F = 3, m = 0$  state. The Ramsey cavity was not excited. Instead, coils on the sides of the drift tube provided a transverse low-frequency field covering most of the region above the Ramsey cavity. This field was left on continuously. The atoms thus experienced the field throughout their flight time. Unfortunately, the cutoff waveguide tube at the exit of the Ramsey cavity is thick enough to partially shield the low-frequency field. Hence the exact region being averaged

during the low-frequency excitation is somewhat ambiguous. Figure 2 shows a sample lineshape for a Majorana transition.

The measurements were made at four different toss heights. For each toss height Table III gives the difference between the field measured by Majorana excitation and that computed by numerically averaging the field as the atoms see it down to the end of the cutoff tube (11 cm). It also gives the bias computed from (3) by modeling the inhomogeneity in Fig. 3 by a combination of functions in Table I. The agreement is qualitatively correct. The disagreement at the lowest toss height may indicate that the stronger excitation field required

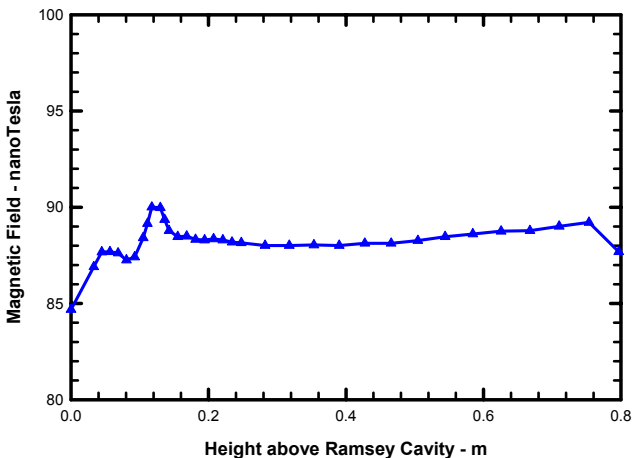


Figure 3 – C-Field inhomogeneity in NIST-F1 as a function of height above the center of the Ramsey cavity.

for the shorter excitation time penetrates deeper into the cutoff tube.

Table III. Measured and Predicted Inhomogeneity Biases

Toss height (cm)	11.4	24.4	38.2	66.4
Measured Bias (nT)	-0.46	-0.17	-0.12	+0.05
Predicted Bias (nT)	-0.32	-0.18	-0.11	+0.08

These biases are to be compared with a total field of about 89 nT. The uncertainty in both the measured and predicted biases is about 0.03 nT.

### SUMMARY

We have shown how an inhomogeneous field is averaged during excitation. We have compared the resulting bias with other inhomogeneity biases. We then specified the field homogeneity required to keep these biases below a given

relative uncertainty on the clock transition. Experiments with the cesium fountain NIST-F1 were in qualitative agreement.

### ACKNOWLEDGEMENTS

We are pleased to acknowledge Bob Drullinger, Mike Lombardi, David Smith and Liz Donley for helpful comments on the manuscript. Andrea DeMarchi originally suggested the use of Majorana transitions on NIST-F1. Bill Klipstein and Eric Burt have both provided useful feedback on using the method for the PARCS project.

### REFERENCES

- [1] T. P. Heavner et al., “PARCS - A primary reference clock in space,” *Proc. First International Symposium on Microgravity Research & Applications in Physical Science and Biotechnology*, pp. 739-745, January 2001.
- [2] J. H. Shirley, W. D. Lee, and R. E. Drullinger, “Accuracy evaluation of the primary frequency standard NIST-7,” *Metrologia* vol. 38, pp. 427-458, 2001.
- [3] Details of the derivation will be presented in a manuscript presently under preparation.
- [4] S. R. Jefferts et. al., “Accuracy evaluation of NIST-F1,” *Metrologia* vol. 39, pp. 321-335, 2002.