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A NOVEL METHOD FOR AGING ESTIMATION OF CRYSTAL OSCILLATORS

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Abstract

The frequency aging of a quartz crystal oscillator is an important characteristic of frequency stability. Aging estimation for frequency correction of a precision crystal oscillator is a valid approach in improving oscillator performance. This paper presents an estimation and prediction model in which the frequency and frequency aging are described by a family of shifted logarithmic functions. Weights of logarithmic function are estimated recursively based upon the present data inputs and the last data outputs to avoid massive data storage. Outlier points in frequency measurements are filtered using a robust estimation computation. Finally, the frequency of an oscillator is predicted based upon the best estimated weights of logarithmic functions.

Introduction

With the continuing interest of military application in low cost - high performance time-keeping devices, precision quartz crystal oscillators are playing an important role in today's military electronic equipments. However, in a battlefield environment, a frequency reference may not be available, due to communication security or jamming, for correcting the frequency of oscillators. To solve this problem, a virtual oscillator can be used which acts as a temporary frequency reference to correct frequency error based on statistical estimation, as shown in Figure 1. A virtual oscillator is a mathematical model which optimally estimates the frequency and frequency aging of an oscillator, so that the frequency error of the oscillator can be predicted and corrected without the frequency reference. When the frequency reference is available, the virtual oscillator will update its parameters based on the frequency reference. When the frequency reference is not available, the virtual oscillator will operate based on the past behavior of the oscillator and predict the frequency error produced by frequency aging effect. At the same time, the oscillator will be calibrated by the frequency prediction of the virtual oscillator in correcting the frequency error. Therefore, estimation and prediction of frequency and frequency aging are important in frequency correction and time-keeping.

Stein and Filler [1] estimated frequency and frequency-aging of oscillators based on n -samples: $z(1)$,

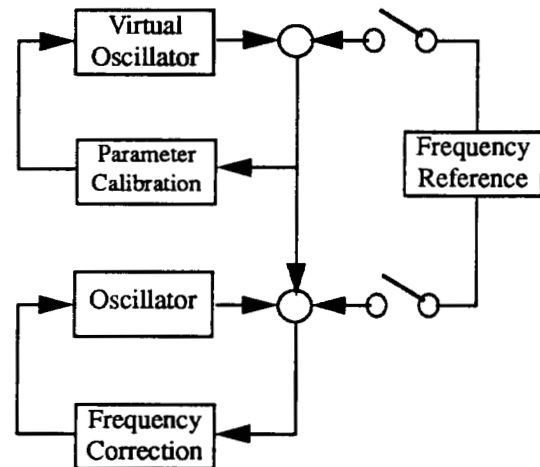


Figure 1 Frequency Correction by using a Virtual Oscillator

$z(2), \dots, z(n)$, where

$$z(k) = \frac{f(k) - f(1)}{f(1)}$$

and where $f(k)$ is a frequency output of an oscillator measured at instant $t(k)$, $k=1, 2, \dots, n$. Then, the frequency beyond the instant $t(n)$ is predicted by a frequency prediction model

$$y(n+l) = \alpha \cdot [t(n+l) - t(n)] + y(n) \quad (1)$$

$$l = 1, 2, \dots$$

Notice that this is a linear function. Where, $y(k)$, $k=1, 2, \dots, n$ is an estimation of $z(k)$ at the instant $t(k)$, $y(n+l)$, $l=1, 2, \dots, n$ is a prediction of $z(n+l)$ at the instant $t(n+l)$, and

$$\alpha = \frac{d}{dt}y(t)$$

is the frequency aging. For convenience, we call $y(k)$ as a frequency estimation, $y(n+l)$ as a frequency prediction, and $z(k)$ as a frequency measurement, throughout this paper.

Later, Su and Filler [2,3] improved Kalman filter frequency and frequency-aging prediction by introducing a nonlinear frequency model in which the frequency is predicted by a logarithmic function.

$$y(n+l) = a_1 \log \left(\frac{t(n+l) + d_0}{t(n) + d_0} \right) + y(n) \quad (2)$$

$$l = 1, 2, \dots$$

Then, Kalman filter is applied to determine the unknown parameters a_1 and the frequency estimation $y(k)$, $k=1, 2, \dots, n$. Since the long-term frequency aging of a quartz crystal oscillator is most likely described by a logarithmic function [4,5], this approach gives the more realistic prediction. However, the goodness-of-fit of Equation 2 is affected by the choice of shift-parameter d_0 . As shown in Figure 2, a bad choice of d_0 will lead to an incorrect prediction result. Furthermore, Kalman filter estimation requires knowledge of the noise covariances, which is usually not available for a quartz crystal oscillator in practice.

Mathematical Model of Frequency Aging

To overcome the above problems, a new frequency model is considered as shown below

$$y(k) = a_0(n) + \sum_{j=1}^M a_j(n) \log(t(k) + d_0 + d \cdot (j-1)) \quad (3)$$

$$1 < k \leq n$$

Where, d_0 and d are positive numbers. Parameter $a_j(n)$, $j=0, 1, \dots, M$, are estimated recursively based on the last step data outputs and the present data inputs. The objective of optimal estimation is to minimize the merit function

$$J = \sum_{k=1}^n w(k) \cdot [z(k) - y(k)]^2 \quad (4)$$

with best-fit parameters a_j , $j=0, 1, \dots, M$. After estimating parameters a_0, a_1, \dots, a_M at instant $t(n)$, the frequency at instant $t(n+1)$ can be predicted by

$$y(n+l) = a_0(n) + \sum_{j=1}^M a_j(n) \log(t(n+l) + d_0 + d \cdot (j-1)) \quad (5)$$

$$l = 1, 2, \dots$$

Equation 5 is made of a basic function $\log(t+d_0)$ with its x-dimensional shifts of $d, 2d, \dots, (M-1)d$, and a y-dimensional shift of a_0 . The previous model in Equation 2 is a special case of Equation 5 by choosing $M=1$. Comparing Equation 2 with Equation 5, the later has a family of logarithmic functions with additional terms: $\log(t+d_0+d)$, $\log(t+d_0+2d), \dots, \log(t+d_0+(M-1)d)$. Each term has a unique zero-crossing point on x-coordinate. Those terms

are candidates for fitting data $z(k)$. Since the new model has more terms, and an additional of M free parameters a_2, \dots, a_M , it is a better approach in describing frequency aging. Although the best value of parameters d_0, d , and M are not given in this algorithm, we will show that the model in Equations 3 or 5 is insensitive to those parameters.

Parameter Estimation

Giving a measurement $z(k)$ and an instant $t(k)$, the value of $a_j(k)$ will be computed recursively as follows:

$$a_j(k) = \sum_{i=0}^M [X(k)]_{ij}^{-1} \cdot Z_i(k) \quad (6)$$

$$j = 0, 1, 2, \dots, M$$

Where,

$$Z_i(k) = Z_i(k-1) + w^2(k) \cdot z(k) \cdot x_i(k) \quad (7)$$

$$Z_i(0) = 0$$

$$i = 1, 2, \dots, M$$

is calculated by using the data in step $k-1$, $[X(k)]^{-1}$ is the inverse matrix of $X(k)$, $[X(k)]_{ij}^{-1}$ is the ij -th entry of matrix $[X(k)]^{-1}$, and $X(k)$ is obtained by computing

$$X_{ij}(k) = X_{ij}(k-1) + w(k) \cdot x_i(k) \cdot x_j(k) \quad (8)$$

$$X_{ij}(0) = 0$$

$$i, j = 1, 2, \dots, M$$

Where, $x_j(k)$ is defined as

$$x_j(k) = \begin{cases} \log(t(k) + d_0 + d \cdot (j-1)) & 1 < j < M \\ 1 & j = 0 \end{cases} \quad (9)$$

and $w(k)$ is a weight at the step k which will be discussed later. Notice that the estimation of $a_j(k)$ requires only an M by M matrix $X(k)$, an M -dimensional vector $Z(k)$, and scalars $z(k)$ and $t(k)$. Therefore the size of data storage for optimal estimation is a function of M rather than a function of n . Usually, n is much larger than M .

Sensitive of Choosing Parameter d_0

To show the improvement in reducing the sensitivity of choosing shift-parameter d_0 , we rewrite Equation 2 to the following form

$$y_1(k) = a_1 \log(t(k) + d_0) + a_0 \quad (10)$$

$$k = 1, 2, \dots$$

Then, we compare it with the multiple logarithmic functions case (new model)

$$y_7(k) = \sum_{j=1}^7 a_j \log(t(k) + d_0 + (j-1) \cdot 0.2) + a_0 \quad (11)$$

by fitting them to the frequency measurement $z(k)$ taken from Oscillator #1. Where, the parameter a_j is estimated by equation 4 with $w(k)=1$.

Experiment 1:

First, choose $d_0=0.4$ for Equation 10 and plot $y_1|_{(d_0=0.4)}$ in Figure 2, where the solid-line stands for frequency measurements, and the dashed-line stands for $y_1|_{(d_0=0.4)}$. Then choose $d_0=5.0$ for Equation 10 and plot $y_1|_{(d_0=5.0)}$ in the same figure represented by the dash-dotted-line. It shows that $y_1|_{(d_0=0.4)}$ and $y_1|_{(d_0=5.0)}$ have different results. This implies that the single logarithmic model is sensitive to d_0 .

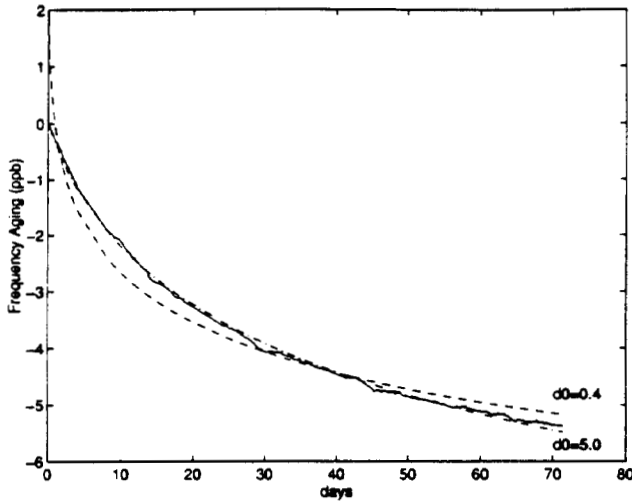


Figure 2 Frequency Aging Described by a Logarithmic Function with Different d_0

Experiment 2:

We estimate the frequency of Oscillator #1, in Experiment 1, by using the new model. First, choose $d_0=0.4$ for Equation 11 and plot $y_7|_{(d_0=0.4)}$ in Figure 3, where, the solid-line stands for frequency measurements, and the dashed-line stands for $y_7|_{(d_0=0.4)}$. Then choose $d_0=5.0$ for Equation 11 and plot $y_7|_{(d_0=5.0)}$ in the same figure

represented by the dash-dotted-line. The plots of $y_7|_{(d_0=0.4)}$ and $y_7|_{(d_0=5.0)}$ in Figure 3 are virtually identical. Both curves fit data $z(k)$ very well. The difference $y_7|_{(d_0=0.4)} - y_7|_{(d_0=5.0)}$ is plotted in Figure 4 together with the difference $y_1|_{(d_0=0.4)} - y_1|_{(d_0=5.0)}$, where the dashed-line represents $y_1|_{(d_0=0.4)} - y_1|_{(d_0=5.0)}$ and the solid-line represents $y_7|_{(d_0=0.4)} - y_7|_{(d_0=5.0)}$. It shows that in multiple logarithmic functions case the difference of $y_7(k)$ by using different d_0 is negligible. Since $y_7(k)$ has seven logarithmic terms, the change in d_0 will be easily compensated for adjusting parameters a_0, a_1, \dots, a_7 .

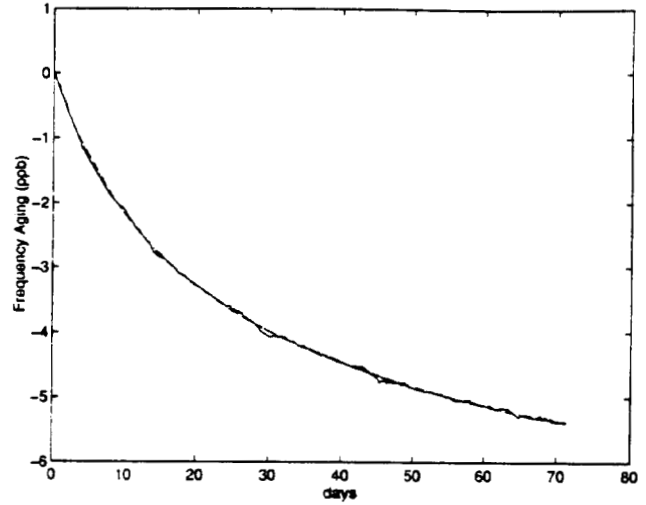


Figure 3 Frequency Aging described by Multiple Logarithmic Functions with Different d_0

Experiment 3:

The sensitivity of the shift-step d can be studied in the same manner. We repeat Experiment 2 using

$$y_7(k) = \sum_{j=1}^7 a_j \log(t(k) + 0.4 + (j-1) \cdot d) + a_0$$

Firstly, we choose $d=0.2$ and plot $y_7(k)$ in Figure 5. Then, we choose $d=1.0$ and plot $y_7(k)$ in the same figure, where the solid-line stands for frequency measurements of Oscillator #1, the dashed-line stands for the $y_7|_{(d=0.2)}$, and the dash-dotted-line stands for $y_7|_{(d=1.0)}$. It shows that both $y_7|_{(d=0.2)}$ and $y_7|_{(d=1.0)}$ match the measurements very well, and the difference between $y_7|_{(d=0.2)}$ and $y_7|_{(d=1.0)}$ is hardly noticeable. Thus, the multiple logarithmic model is insensitive in choosing shift-step d .

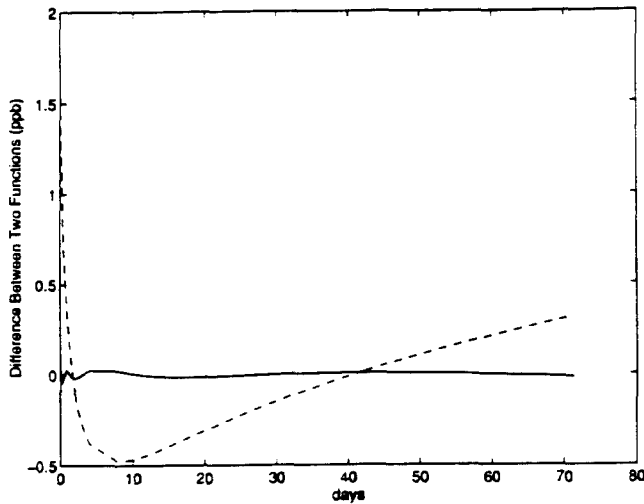


Figure 4 Comparison of Single Logarithmic Function Model and Multiple Logarithmic Functions Model

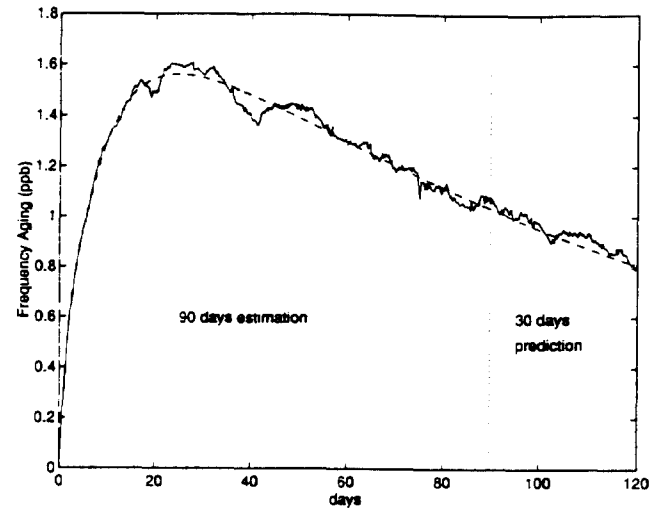


Figure 6 Frequency Aging which is not Described by a Logarithmic Function

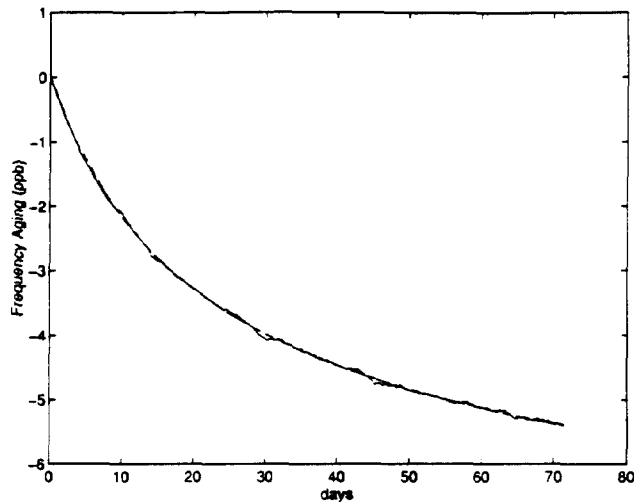


Figure 5 Frequency Aging Described by Multiple Logarithmic Functions with Different d

Experiment 4:

In this example we repeat Experiment 2 by reducing the parameter M from seven to five. That is

$$y_7(k) = \sum_{j=1}^5 a_j \log(t(k) + 0.4 + (j-1) \cdot 0.2) + a_0$$

The experiment result shows that the frequency aging plots of $y_7|_{(M=5)}$ and $y_7|_{(M=7)}$ are virtually identical. It

implies that the estimation is not sensitive to M in this experiment.

In short, the new model in Equation 3 or 5 consists of a sum of logarithmic functions with parameters: a_j , d , d_0 , and M . Where, a_j is computed recursively by estimation algorithm, d , d_0 , and M are pre-selected constants. Based upon our experiment results, we suggest to choose d between 0.1 and 1.0, M between 5 and 10, and $d_0 = 1 - 0.5 \cdot d \cdot (M - 1)$. It is remarkable that parameters d , d_0 , and M are insensitive in all our experiments so that the selection of d , d_0 , and M is not restrictive.

Experiment 5

Occasionally, the frequency curve of a crystal oscillator may not be matched by a logarithmic function in short-term, as shown in Figure 6. Where the solid-line stands for frequency measurements of Oscillator #2. In this case, the frequency model described in Equation 2 is no longer valid. However, the new model can overcome such problem successfully by combining several logarithmic functions with different shift values. We choose the same model discussed in Experiments 2 and 3, which is

$$y(k) = \sum_{j=1}^7 a_j \log(t(k) + 0.4 + (j-1) \cdot 0.2) + a_0 \quad (12)$$

and assume that the frequency measurements after day 90

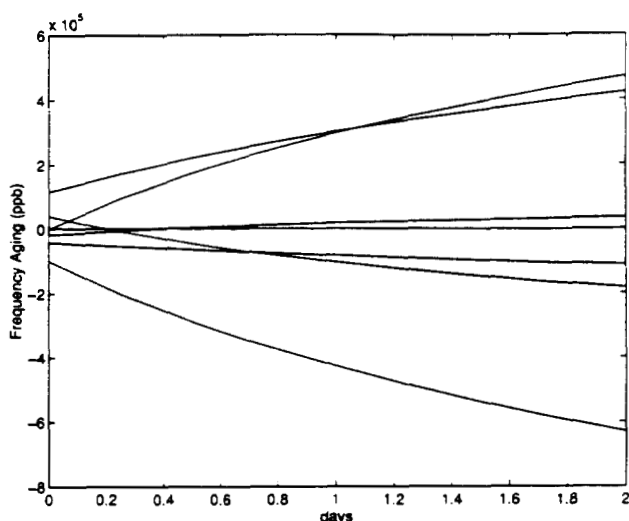


Figure 7 A Family of Logarithmic Functions

are not known in advance. Thus, the parameters a_0, a_1, \dots, a_7 will be estimated by using 90 days of input data. As shown in Figure 6, the frequency estimation curve matches frequency measurements perfectly for 90 days. Where, the solid-line represents frequency measurements, the dashed-line starting from day 1 though day 90 represents the frequency estimation. After 90 days, the frequency has to be predicted based on

$$y(n+l) = \sum_{j=1}^7 a_j \log(t(n+l) + 0.4 + (j-1) \cdot 0.2) + a_0 \quad (13)$$

Comparing the predicted frequency, the dashed-line from day 90 though day 120, with actual frequency measurements, the solid-line in Figure 6, the prediction result is satisfactory. After frequency prediction, frequency errors are obtained by

$$f(n+l) - f(1) = f(1) \cdot y(n+l) \\ l = 1, 2, \dots$$

which can be used for the frequency correction of the oscillator

We have shown that the frequency measurement of Oscillator #2 does not match logarithmic function. However, it matches the sum of logarithmic functions, $a_1 \log(t+0.4)$, $a_2 \log(t+0.6)$, $a_3 \log(t+0.8)$, $a_4 \log(t+1.0)$, $a_5 \log(t+1.2)$, $a_6 \log(t+1.4)$, and $a_7 \log(t+1.6)$, as shown in Figure 7. The relation between parameters a_0, a_1, \dots, a_7 and parameters $d_0, d_0+d, \dots, d_0+6d$ is illustrated in Figure 8.

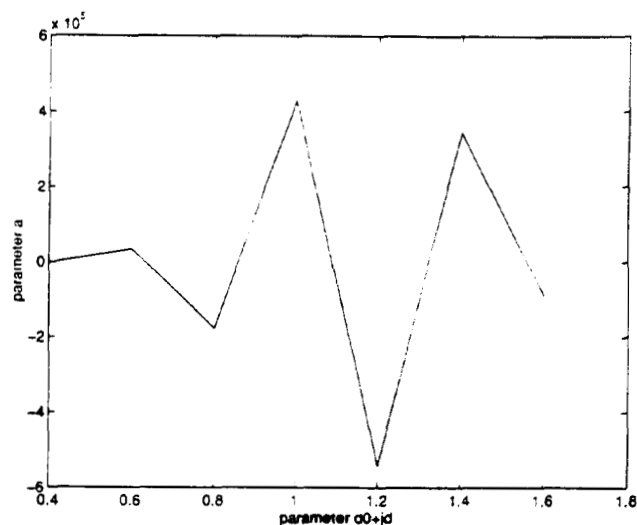


Figure 8 Relation Between Model Parameters a_j and d_0+jd

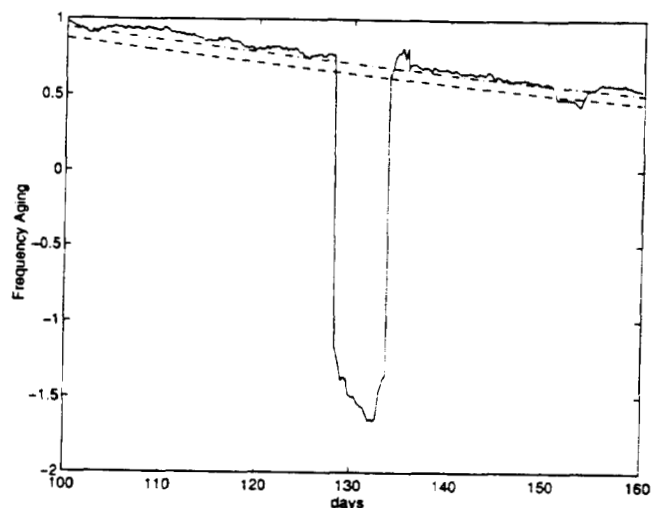


Figure 9 Frequency Estimation with Outlier Points

Notice that $\log(t+1)$, $\log(t+1.2)$, and $\log(t+1.4)$ are weighted heavily which are dominant terms in Equation 13.

Robustness Estimation of Parameters

The term robust here means insensitive to data departures from the idealized model for which the estimator is optimized. Frequency aging usually varies very slowly so that the frequency measurement is in general a smooth curve with small fluctuations. If a sudden departure (difference between present point and last point)

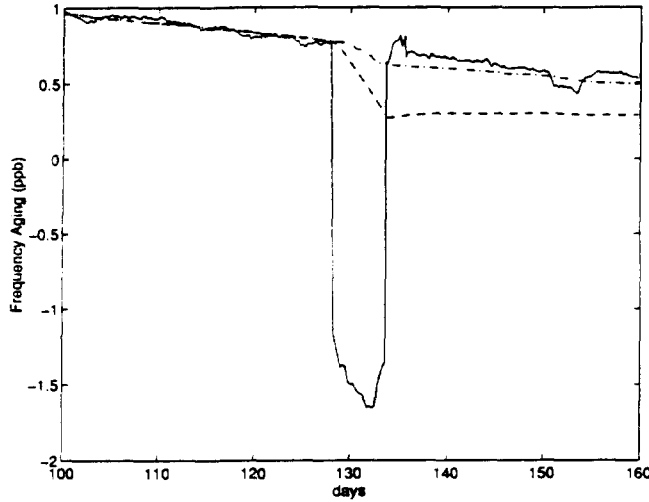


Figure 10 Frequency Prediction with Outlier Points

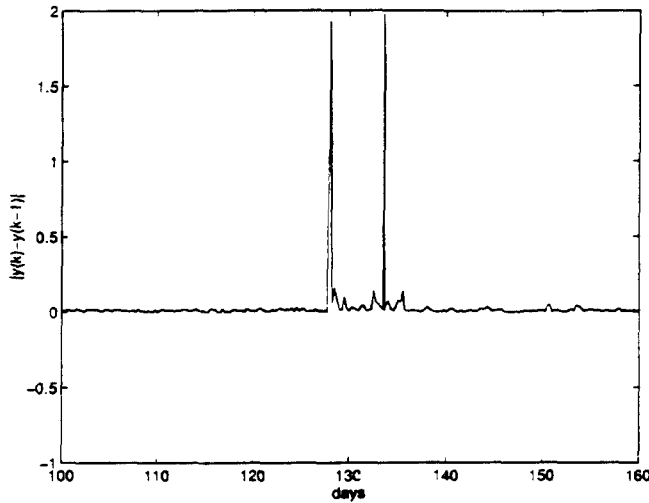


Figure 11 Absolute Value of Frequency Differences

occurs in frequency measurements, as the solid-line shown in Figure 8, it is most likely related to other problems, such as power failure, mechanical strokes, or wrong measurements, rather than frequency aging effect. A correct frequency aging estimation and prediction should not be affected by those outlier points. However, if $w(k)$ in Equation 4 is a constant value, as in previous experiments, outlier points will be weighted equally in the merit function. As a result, the frequency estimation curve is shifted away from the expected course towards the outlier points as illustrated in Figure 8. Where, the solid-line is frequency measurements of Oscillator #3 between day 100 and 160,

and the dashed-line is the frequency estimation for 160 days by using $y(k)$ in Equation 12. Frequency prediction error can be observed in Figure 10, where the dashed-line is the one-step frequency prediction, and the solid-line is the frequency measurements of Oscillator #3 between day 100 and day 160. Although the effect of outlier points will become smaller and smaller as the data set becomes larger and larger, the recovery process may take a considerably long time. To solve this problem, a robust estimation algorithm is used by choosing a time-varying weight

$$w(k) = e^{-|z(k) - z(k-1)|} \quad (14)$$

$$z(0) = z(1)$$

Then, the merit function in Equation 4 becomes

$$J = \sum_{k=1}^n e^{-|z(k) - z(k-1)|} [z(k) - y(k)]^2 \quad (15)$$

The idea of robust estimation is to assign smaller weights to outlier points so that the outlier points will have less influence than regular points in optimal estimation. The outlier point is detected by a high-pass filter $z(k) - z(k-1)$. The larger the difference between $z(k)$ and $z(k-1)$, the smaller the weight $w(k)$. Referring to Figures 11 and 12, a large departure in Figure 11 yields a small weight in Figure 12.

Alternative time-varying weight $w(k)$ can be used, such as

$$w(k) = e^{-(z(k) - z(k-1))^2}$$

$$z(0) = z(1)$$

or

$$w(k) = e^{-(z(k) - 2z(k-1) + z(k-2))^2}$$

$$k = 3, 4, \dots$$

After using robust estimation, the offset of frequency estimation $y(k)$ is significantly eliminated as plotted in dash-dotted line shown in Figure 9, and the frequency prediction $y(k+1)$ is largely improved as plotted in dash-dotted line shown in Figure 10. In both cases, the merit function in Equation 15 is used.

Conclusion

Frequency aging of a quartz crystal oscillator can be described by a family of shifted logarithmic functions. Frequency aging estimation can be conducted by a recursive computation based on the present data inputs and the last step data outputs. Massive data storage of aging data is not need. Frequency aging prediction can be conducted by

using the best estimated parameters. Outlier points in frequency measurements can be filtered by using robust parameter estimation. Unlike previous methods, this approach does not require knowledge of the error covariance matrices, and it is much less sensitive in parameter selection.

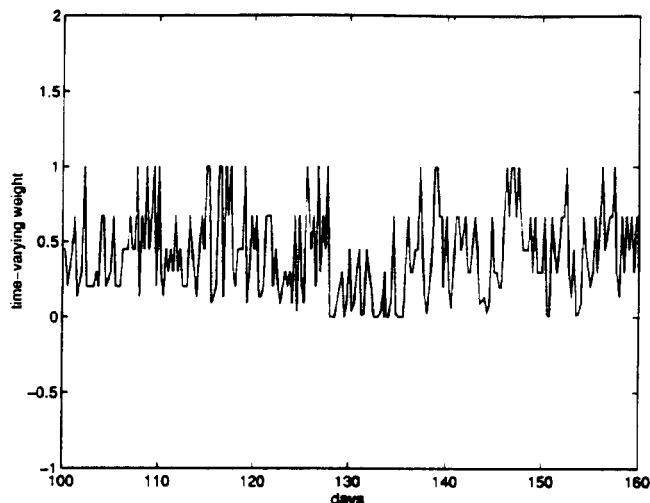


Figure 12 Time Varying Weights

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