An interpretation of scalars in SO(32)

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Abstract

We propose an interpretation for the adjoint representation of the $SO(32)$ group to classify the scalars of a generic Supersymmetric Standard Model having just three generations of particles, via a flavour group $SU(5)$. We show that this same interpretation arises from a simple postulate of self-consistence of composites for these scalars. The model looks only for colour and electric charge, and it pays the cost of an additional chiral $+4/3$ quark per generation.

Keywords: supersymmetry, flavour

1. Introduction

While highly relevant in string theory and supergravity, $SO(32)$ group is not a good unification group as it doesn't have complex representations [12]. But it stills get an interesting family group when decomposed. In this letter, we first review the decomposition, interpret is as a group symmetry on scalars that could be supersymmetry partners of the Standard Model fermions, and then we present an interesting reconstruction of such scalars as composites. Besides, the interpretation has an uniqueness that limits the number of generations for the SM group.

This reconstruction could have some application when considering open string theory and their branes, or could be used as basis for other GUT-flavour models. Considering this, we include a pair of sections with some separate discussion on other related groups.

2. The flavour group in $SO(32)$

The authors of [11] classify decomposition of groups having explicitly a $SU(3)$ colour subgroup, giving candidate representations as well as the decomposition of the adjoint representation in all the cases. Groups $SO(2n)$ are case 4 of this classification, where they obtain the decomposition $SO(n_1) \otimes SU(n_2) \otimes SU(3) \otimes U_1(1)$ with $2n = n_1 + 6n_2$. Our case of interest is $SO(32)$ with the maximal $SU(n_2)$, this is $n_2 = 5$. The representations intended for fermions are not very useful, as the group is of kind $SO(4k)$, without complex representations. But we are interested on the adjoint as a place for scalars. The stated result gives us

$$\begin{array}{llll} 496 = & & & \\ (\mathbf{1}, \mathbf{24}, \mathbf{1^c}) & +[\mathbf{1}, \mathbf{15}, \bar{\mathbf{3}}^\mathbf{c}] & +[\mathbf{1}, \overline{\mathbf{15}}, \mathbf{3^c}] & + \\ 1, 24, 8^c & +[1, 10, \bar{6}^c] & +[1, \overline{10}, 6^c] & + \\ (1, 1, 8^c) & & & + \\ & (2, 5, 3^c) & +(2, \bar{5}, \bar{3}^c) & + \\ & (1, 1, 1^c) & +[1, 1, 1^c] & \end{array} \tag{1}$$

And our components of interest are the three first ones, that we have stressed with boldface. The explicit $U_1(1)$ group provides an hypercharge that counts the number of coloured representations and is zero for colour singlets, so we can assign respectively $Y_1 = 0, +1, -1$ to the above 1^c, $\bar{3}^c$ and $\bar{3}$.

To get a second hypercharge, we can consider $SU(5)$ as the flavour group and decompose it [18] down to multiplets in $SU(3) \times SU(2) \times U_2(1)$

$$15 = (1, 3)_{-6} + (3, 2)_{-1} + (6, 1)_4 \tag{2}$$

$$24 = (1, 1)_0 + (1, 3)_0 + (3, 2)_5 + (\bar{3}, 2)_{-5} + (8, 1)_0 \tag{3}$$

Now from the two hypercharges we can produce a charge

$$Q = \frac{1}{5}\left(\frac{2}{3}Y_1 - Y_2\right) \tag{4}$$

and check that the resulting decomposition includes content corresponding to the scalars of a minimal, three generations, supersymmetric standard model.

$$\begin{array}{llrrr} & & Y_1 & Y_2 & Q \\ (1, 15, \bar{3}) & (3, 2) & 1 & -1 & +1/3 \\ & (6, 1) & 1 & 4 & -2/3 \\ (1, \overline{15}, 3) & (3, 2) & -1 & +1 & -1/3 \\ & (6, 1) & -1 & -4 & +2/3 \\ (1, 24, 1) & (1, 1) & 0 & 0 & 0 \\ & (1, 3) & 0 & 0 & 0 \\ & (8, 1) & 0 & 0 & 0 \\ & (3, 2) & 0 & 5 & -1 \\ & (\bar{3}, 3) & 0 & -5 & +1 \end{array} \tag{5}$$

Plus an extra content

$$\begin{array}{llrrr} & & Y_1 & Y_2 & Q \\ (1, 15, \bar{3}) & (1, 3) & 1 & -6 & +4/3 \end{array} \tag{6}$$

We can arrive to the same result by chaining some branchings. A straighforward way is $SO(32) \supset SU(16) \times U(1)$,

$$496 = 1_0 + 120_4 + \overline{120}_{-4} + 255_0 \tag{7}$$

and then $SU(16) \supset SU(15) \times U(1)$ and $SU(15) \supset SU(5) \times SU(3)$, to finish applying (2),(3). In this way the quarks come from the initial 120s, while the leptons are from the 255. Or respectively in $SU(15)$, from the 105s and the 224.

3. $SO(32)$ from postulates

Once we know that our aim is to get not the fermions but just the scalar partners of a Susy Standard Model, we can wonder if there is some set of postulates that isolates directly the flavour group, or at least the number of generations it has. It turns up, there is an amusing set of requirements that force this result.

The clue is the "recursive" property of colour: we can get the $\mathbf{3}$ colour triplet out of $\bar{\mathbf{3}} \times \bar{\mathbf{3}} = \mathbf{3} + \mathbf{6}$. And also we can get singlets, from $\mathbf{3} \times \bar{\mathbf{3}} = \mathbf{1} + \mathbf{8}$.

And adding to this hint, we notice that one quark with an antiquark allows to build particles of electrical charges +1, 0, and -1, but not only that: also we can build a charge +2/3 with two antiquarks of down type, and a charge -1/3 with one antiquark down plus other antiquark down. This was in fact the spirit of the above decomposition of $SU(5)$ flavour, but it is even more interesting when starting from particles and going later to groups.

3.1. Turtles and elephants

We consider scalars as composites either of pairs of quarks, as a colour triplet, or of pairs quark anti-quark, as a singlet. Furthermore, we divide the quarks in two classes: *turtles* and *elephants*, and add a rule: only *turtles* can combine into composites.

We assume there are N up-type quarks, of these k_u *turtles*, and N down-type quarks, of which k_d *turtles*.

We ask for what values of N, k_u, k_d the number of scalars of each type is exactly $2N$, as required in supersymmetry models. This gives two equations for squarks up and down:

$$2N = k_u k_d \tag{8}$$
$$2N = k_d(k_d+1)/2 \tag{9}$$

So $N \geq 3$ (actually, N must be half of an hexagonal number) and $k_d = 2k_u - 1$. If we add other two conditions, for sleptons charged and neutral

$$2N = k_u k_d \tag{10}$$
$$4N = k_u^2 + k_d^2 - 1 \tag{11}$$

then the solution is unique, $N = 3$, $k_u = 2$, $k_d = 3$. There are five *turtles* and one *elephant*, that we can name as the top quark.

However, note that if we consider all the combinations of *turtles* we find that we get three extra "squarks" of charge +4/3, and their opposites.

3.2. Colourless and coloured flavour groups

The extra "squarks" look as a penalisation but group theoretically they are the ones that allow to complete the flavour supermultiplet into a 15 of $SU(5)$

At this level and without colour, we could consider that the flavour is organized in the 54 of $SO(10)$, and then break it down to $SU(5) \times U(1)$

$$54 = 15_4 + \bar{15}_{-4} + 24_0$$

where again the hypercharge from this U(1) can be combined with the one on 2,3 to reproduce the electric charge.

If we want to incorporate colour and unify colour-flavour, our minimal candidate is $SU(15)$. From here we can go up to $SO(30)$ and then to $SO(32)$ adding singlets, or substituting colour $SU(3)$ by $U(3)$.

4. On $SU(15)$

For the group decomposition, similar results could be obtained with only $SO(30)$ or $SU(15)$ as coloured flavour group, or $SO(10)$ or $SU(5)$ as colourless flavour groups, or even with $Usp(32)$.

$SU(15)$ was considered as a GUT group by [1] and [9]. The first reference notes that it is a subgroup of $SO(32)$ Both references embed a full generation

$$(l_L, l^c_L, \nu_L, u_{rgb,L}, u^c_{rgb,L}, d_{rgb,L}, d^c_{rgb,L})$$

inside the fundamental of $SU(15)$. On the other hand, our approach embeds the $(2,1)+(1,3)$ *turtles* of our $SU(5)$ flavour:

$$(u_{rgb}, c_{rgb}, d_{rgb}, s_{rgb}, b_{rgb})$$

and we use, as noted above, the $105, \bar{105}$ and 224 representations.

Recently [7, 8] have considered $SU(15)$ in the context of the standard model extended with bifermions, so they naturally use these representations. They consider the particles to be elementary, so "biquarks" instead of "diquarks" or mesons, but this distinction blurs away when we consider an interpretation as open string terminated in quark labels. More importantly, they still keep having leptons in the fundamental representation, so it is possible to get a lepton number in the 15×15 and $15 \times \bar{15}$ products.

The difference with our model is due to option for the breaking path $SU(15) \supset SU(12) \times SU(3)_l \times U(1) \supset SU(6)_L \times SU(6)_R \times SU(3)_l \times U(1) \times U(1)$, that allows to put a whole generation of the SM without right neutrinos in the decomposition of the 15, at the cost of some delicate surgery [1, 9]. The first extracted $SU(3)_l$ group has the goal of joining all the leptons of each generation in a single multiplet; if we want an extra ν^c_L neutrino it must be expanded to $SU(4)_l$ and then the whole group to $SU(16)$

5. On $SU(8)$

This section and the next one are explorative work, the main theme being if representations of other groups from supergravity and string theory can benefit of a similar interpretation as scalars of some supersymmetric standard model.

$SU(8)$ appears directly because an alternate chain down from $SO(32)$ is to take the detour $SU(16) \supset SO(16) \supset SU(8) \times U(1)$

$$\begin{aligned} 496 = &\quad 1 + 120_4 + 120_4 + 120_0 + 135_0 \\ = &\quad 1 + 3(1_0 + 28_2 + \bar{2}8_{-2} + 63_0) + \\ &\quad +36_2 + \bar{3}6_{-2} + 63_0 \end{aligned} \tag{12}$$

And then we can go for the group theory of $SU(8) \supset SU(5) \otimes SU(3) \otimes U(1)$ but with a lot more of hypercharge assignments (usually uglier, but worth a glance).

Family GUT unification with $SU(8)$ was examined with some detail in 1980, see for instance the references in the recent revisit of [3]. Typically three families of standard model fermions were expected to be in the summed complex representation $\bar{8} \otimes \bar{2}8 \otimes 56$ and some criteria was used to select the hypercharge assignments.Most models preferred to interpret for flavour the first $SU(3)$ in $SU(8) \supset SU(5) \otimes SU(3) \otimes U(1)$ instead of leaving it for colour as [11]. Both approaches differ only in the algebra of $U(1)$ charges for the multiplets. The fundamental decomposes as a colour triplet, a $SU(2)$ horizontal doublet, and a $SU(3)$ horizontal triplet.

$$8 = (1,1,3)_{0,-5} + (1,2,1)_{-3,3} + (3,1,1)_{2,3}$$

Note it went first to

$$8 = (1,3)_{-5} + (5,1)_3$$

and while in the first approach $SU(5)$ is flavour-colour, in the second it is just two horizontal symmetries and the colour triplet is explicit. So we prefer this later way because so all the irreducible representations of $SU(8)$ have an interesting interpretable descent. The decomposition of the 28 has a quark content that looks very much as our division in five turtles and one elephant,

$$28 = (1,\bar{3})_{-10} + (5,3)_{-2} + (10,1)_6$$

but it is different to the $SO(32)$ case. To ilustrate a particular assignment, if we think of the fundamental as "half-charged preons" of charges $\pm 1/2$, $1/6$, then:

- $(1,\bar{3})$ is one anticoloured particle of charge $+1/3$
- $(5,3)$ are coloured particles, three of charge -1/3, two of charge $+2/3$
- $(10,1)$ contains six particles of charge 0, three of charge -1 in an horizontal "antitriplet"... and one of charge $+1$

So this content doesn't allow for our "recursive" interpretation of the interplay between the 32 and the 496 of $SO(32)$

We can play also with content from extra representations. The 36 somehow complements the 28, and the 63 can provide a full uncoloured $(24,1)$ to break into different charges. Besides, in this path, the fundamental of $SO(32)$ appears in $SU(8)$ as

$$32 = (8_{1,2} + \bar{8}_{-1,2}) + (8_{1,-2} + \bar{8}_{-1,-2}) \tag{13}$$

and so it provides extra $U(1)$ charges and extra particles; one needs a good motivation to justify a particular pick. We can explore one hundred weightings to extract the electric charge Q of each representation, most of the combinations offering extra quark and lepton content, including some $+4/3$ quarks.

We could also use the process via via $SU(5) \supset SU(2) \otimes SU(3)$ to assign weak and colour multiplets as usual. On our point of view, both $SU(2)$ and $SU(3)$ here are horizontal groups.

One can observe that (13) meets the condition asked in [11] of having only singlets and triplets of colour, and so wonder what reasons, besides simplicity, motivate the exclusion from the listing.

We could also consider first a regular descent, via $SO(16)$ to $SU(8) \times SU(8)$

$$120 = (8,8)_0 + (28,1)_2 + (1,28)_{-2}$$

$$255 = (1,1)_0 + (8,\bar{8})_2 + (\bar{8},8)_{-2} + (63,1)_0 + (1,63)_0$$

6. On $E_8 \times E_8$

Exotic approaches to flavour are not unknown in supergravity, a good example being the diagonal $SU(3)$ from Gell-Mann, that also ignores electroweak charge [16]. And as $SO(32)$ is relevant to 10D sugra, and all the 10D supersymmetric theories are related via string dualities, it is interesting to speculate if other corner of this space, the $E_8 \otimes E_8$ group, can present a similar mix.

We can examine this possibility starting from the conclusions of the above sections, albeit at the moment the discussion will be very light, and inconclusive, if not disappointing.

E_8 is not considered in [11] because the authors apply a "colour restriction" in their selection of groups, asking for decomposition of the fundamental representation having only singlets and triplets of $SU(3)$. It is more particularly reviewed by [2], who enumerates the problems to use it as a group GUT and also considers decomposition with explicit family group $SU(3)_F$. A separate approach with explicit colour group $SU(3)_c$ and then mixed electroweak-flavour $SU(6) \times U(1)$ was done in [5] via an initial breaking into $SU(9)$. Generically, E_8 has an industry of its own for pure algebraic approaches, linked to Clifford algebras, and full of interesting observations, but reviewing it is out of the scope of this letter.

Both $SO(32)$ and $E_8 \otimes E_8$ have a subgroup $SO(16) \otimes SO(16)$. The branching of $SO(32)$ to this subgroup is

$$496 = (120, 1) \oplus (1, 120) \oplus (16, 16)$$

very similar to the branching we have used in (7)

Isolately, each E_8 branches to $SO(16)$ as

$$248 = (120) \oplus (128')$$

What we suspect is that quark and lepton parts have different roles, the quark part coming from 120; one of the 120s will provide the quark-like charges, the other will provide the antiquark ones. The lepton part can be extracted from the 28 of $SU(8)$ but it could also come from the 63, and then we should investigate the $(128')$ irrep.

Remember that in the initial sections the critical part has been to obtain a 15 representation of $SU(5)$ associated to a triplet 3 of $SU(3)$, as well as a 24 associated to the singlet. And here is the problem: any further factorisation of $SO(16)$ fails to get representations as big as the 15. We are down to fives and tens too soon. Amusingly, we could also consider a directly branching $E8 \supset SU(5) \otimes SU(5)$; this is exploited in model building, for instance [6, 3], but with different assignments to colour and flavour. If we use this kind of decomposition and we accept the irreps 5 and 10 instead of the 15, it amounts to exchange some of the $\pm 4/3$ and $\pm 2/3$ charges by an excess of $\pm 1/3$ charges.

7. Discussion

The postulate *It is turtles all the way down*[1] applied solely to squarks already fixes the number of generations to be greater or equal than three. Adding a reasonable condition on the building of neutral sleptons, it fixes uniquely $N = 3$ and then also the separation between five light quarks and one heavy one that does not participate in the composites. Of course this uniqueness is not seen when going directly from the $SO(32)$ group down to flavour times colour, but even in this case there is a separation between five "turtles" making the fundamental of $SO(32)$ and a non-participant "elephant".

While eventually all the extant multiplets of the decomposition should be explained, the $(1, 3)$ squarks, of charge $\pm 4/3$, are specially puzzling. They can not be organised as three generations of partners of four-component Dirac quarks. Still, they have a role in the flavour multiplet, and they could exhibit their singularity if chirality is introduced back in the game.

The symmetry between quarks and diquarks or its hadronic equivalent is known to be one of the historical origins of supersymmetry [15, 10] and it is used in hadronic phenomenology. But a concrete hadronic construction of our scalars as real diquarks produces the ones of odd parity, that are excluded of phenomenological discussions as they do not survive the 'single mode approximation' [14]. Thus the composite "squarks" and "sleptons" bound here should be not the ones found at QCD scale, but it is intriguing that they are similar in number and mass.

We can justify the uplift from $SU(15)$ to $SO(32)$ by asking particle colour to be in a slightly greater group, such as $U(3)$. This could be a hint of the difference between the binding mechanism needed here, that should happen at high energy scale, and the usual binding of mesons and diquarks. Observe that the usual binding shares some properties: the top quark, our *elephant*, does not bind into mesons -because it disintegrates before-, and the masses of mesons and diquarks are in the same range of energies that the SM fermions, as expected of a slightly broken supersymmetry.

One must recognise that the motivation to use $SO(32)$ is not only to produce one hypercharge and the adequate multiplets in the decomposition, but also because of its role in string theory. The postulates of composition need a pairing that looks similar to labels in terminated open strings. The composition process from the point of view of a terminated "QCD string" bears some similarity to the techniques of [4] using "planar orientifolds".

The focus on scalars, and thus in pairs of fermions, makes the results to differ from preon constructions with the same groups. To get again a fermion, one should consider an extra object, particle or string, providing an 1/2 spin.

8. Conclusions

In conclusion, lets review what we have got. We offer a novel interpretation of the $SO(32)$ group within the context of supersymmetric models, emphasizing its potential as a flavour group. The decomposition and hypercharge assignment that allows to recover three generations has not been presented in the literature explicitly. This is for the obvious reason that it recovers scalars, not fermions. But on the other hand, to look for scalars avoids to address the problem of the lack of chiral fermions on SO(32).

Besides, we offer a composite explanation for scalars of the SSM, that fixes the number of generations and limits the possible groups that can be used to generate flavour with a separate colour factor. In the list of possible groups, $SO(32)$ stands up.

Our postulate is, certainly, exotic: it suggests that while SSM fermions could be -or not- elementary, the SSM scalars are composites, with their preons being a subset of the fermions. Far fetched as this postulate looks, it reproduces the $SO(32)$ decomposition and fixes the number of possible generations.

The decomposition seems to imply that each generation also includes two extra "scalar quarks" of charge $\pm 4/3$. It is unclear if such scalars could have an associated fermion, as it should be of Weyl type, not Dirac.

[1] I first heard this idiom in a talk from Alvaro de Rujula in 1986

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