

KINEMATICS-BASED MODEL FOR STOCHASTIC SIMULATION OF AIRCRAFT OPERATING IN THE NATIONAL AIRSPACE SYSTEM

Seamus M. McGovern, Seth B. Cohen, Minh Truong, U.S. DOT National Transportation Systems Center, Cambridge, Massachusetts

Gerard Fairley, EG&G Technical Services, Cambridge, Massachusetts

Abstract

Traditional six degree-of-freedom flight simulations provide a very accurate portrayal of aircraft motion allowing for many aspects of an aircraft's flight envelope, including those close to the edge of the envelope, to be accurately modeled. In some simulations, especially those that make use of aircraft as only one contributing component of the simulation and only in normal modes of aircraft operation, it may only be necessary to approximate the general motion of aircraft. In this case it is not necessary to study the many intricate forces that act on the airplane body or to solve the associated complex dynamical equations. In this situation, six degree-of-freedom aircraft simulators may add unnecessary complexity. However, it may still be of interest to accurately model different types of aircraft uniquely in order to enable the comparison of different performance and maneuver characteristics for different aircraft in the simulation. The original, kinematics-based model detailed here uses precise flight data collected by the Federal Aviation Administration and provides a satisfactory level of fidelity for a variety of aircraft types. It is especially accurate in representing different aircraft in normal flight regimes (i.e., within the flight envelope, non-emergency, standard operations). This paper presents the general mathematical aircraft formulation, a description of both the pilot and aircraft models and parameters, and an explanation of the concept for and design of a future control system.

Background

As part of the development of a Monte Carlo-based computer simulation capability to enable rigorous analysis of procedures, equipment, and airspace, the design of a variety of models is required. Specifically, it is desired that the simulation stochastically model many components of the National Airspace System (NAS) –

mechanical, electronic, and human – including navigation aids, surveillance systems (such as radar), pilots, aircraft, air traffic controllers, and weather; along with known, discrete models such as runway size and configuration, and obstacles. While high-fidelity flight models are desired for use in the simulation for the highest level of performance, due to a lack of the proprietary data for many types of aircraft and in order to speed development, a mathematically concise and computationally efficient aircraft model has been developed to stochastically model any aircraft type. ("Type" here refers to an aircraft model number; e.g., Airbus A320 versus simply a transport aircraft or an Airbus.) This original, kinematics-based model uses detailed maneuvering, takeoff, climb out, and approach data collected by the Federal Aviation Administration (FAA) and provides an acceptable level of fidelity, especially in normal flight regimes. The thrust produced from the engines is indirectly modeled kinematically. Airplane performance requiring increasing or decreasing thrust is modeled indirectly through the kinematic parameters. In addition, the pilot is modeled as a basic control system and includes reaction times (both the pilot's control inputs and reaction times are treated as stochastic using FAA-provided distributions).

Since the intent of the NAS simulation tool's flight simulator is to model the general motion (e.g., within the flight envelope, non-emergency, standard operations) of a real aircraft, it is not necessary to study the many forces that act on the airplane body or to solve the associated complex dynamical equations as would be performed by traditional six degree-of-freedom flight simulations. To approximately simulate an aircraft's motion without simulating its intricate movements a simpler, kinematic-based flight model is adequate. In normal flight regimes, this kinematic model accurately describes the desired output: movement of the aircraft's center of mass through space.

This paper describes the aircraft flight model portion of the simulation tool and its role in the NAS stochastic simulation tool.

Alternative Flight Simulation

There are generally considered to be four ways to mathematically model aircraft in flight [1]. This includes the use of basic kinematics, the small perturbation theory model, the total forces and moments method, and blade element theory. Each of these methods has its own advantages and disadvantages which impact ease of implementation as well as fidelity in various flight regimes. By choosing a suitable mathematical flight model, as well as appropriate coordinate systems and other assumptions, any modern computer system is typically able to easily solve these aircraft equations of motion with the respective accuracy. While a completely accurate computation that includes all possible flight is impractical due to the formidable problem of solving the complete Navier-Stokes equations, the difference in accuracy between this exact solution and the center of mass flight path for an aircraft using one of the four listed solution methods is generally small under the conditions of interest (i.e., laminar flow).

For the simplest simulator, a kinematic model consisting of only kinematic relations can be used. Kinematics describes an object's motion using non-dynamical equations that involve only positions, velocities, and accelerations. This method does not use advanced mathematics and therefore does not load the computer process very much; unfortunately, this usually results in lower-fidelity results for some flight conditions. Based purely in physics [2] and devoid of any type of aerodynamic or stability components, the kinematic flight simulator relies, with few exceptions, purely on the kinematic motion of the aircraft. Forces are generally not included nor are they necessary to calculate the aircraft's state. Benefits include simplified calculations enabling quicker design and software coding as well as faster calculations, no requirement for aircraft type-specific – and typically proprietary – data (e.g., lift and drag coefficients), and reduced computational workload.

Taking aircraft properties into account, the small perturbation method provides an incremental step in performance. Simplifying the nonlinear

aircraft equations of motion results in the formation of the aircraft stability derivatives; these derivatives are an intuitive way of describing aircraft motion around a certain equilibrium point. However, this method is limited by this; as its name implies, the model provides a description of motion, but only around an equilibrium point. For an aircraft in a steep dive or a wide speed range this method is unable to produce accurate results; however, this model is commonly used in aircraft development and test when limited aircraft data is available.

Using the total forces and moments method, the complete nonlinear aircraft equations of motion are solved. As with all of these models, the aircraft is assumed to be a rigid body; however, the aircraft now has six degrees-of-freedom for application of all forces (i.e., linear) and moments (i.e., rotational). The forces and moments are found for an entire aircraft using various methods, including table-lookup and polynomial representation. Using various common integration algorithms, the aircraft's position is calculated as a result of these forces and moments every time step. Although these formulations are quite detailed, there are still errors that arise from their use. If tabular data is used, it will not be smooth, but piecewise linear; this can prove to be an inadequate representation of smooth real-world data. A table also has a beginning and an end, past which there is no aircraft data. Some simulators hold the first and last values constant beyond these limits, while others extrapolate the data; either way this is not necessarily a precise representation of the actual aircraft. This method, however, is the most widely used in the simulation industry.

A final, lesser-used method employs the blade element theory, first derived for use with the analysis of helicopter rotors. In this method, the wing is split up into thin strips that are essentially two-dimensional, infinite-span airfoils. Since the aerodynamic properties of two-dimensional airfoils are readily calculated, the sum of these over the entire wing will produce the net forces on the wing and subsequently on the entire aircraft body. All of the force and moment calculations are based on the physical model described to the simulator; this is completely different than the other methods.

Kinematics is therefore much simpler to model and easier to run computationally, and since only

the center of mass flight path is desired it can produce results with the required accuracy for a simulation only requiring operation of aircraft in normal flight regimes.

Kinematic Flight Model

The NAS stochastic simulation tool requires that aircraft motion be accurately modeled in order to determine when aircraft are within close proximity. Using a complex aircraft simulator requires many more inputs than is desired or even necessary. If the required result is simply the aircraft's center of mass position, the entire aircraft state (i.e., the position of all control surfaces, all six aircraft accelerations and velocities, etc.) need not be known. A simpler model can be used to achieve the required result.

If the approximate general motion of an aircraft is to be effectively simulated, the description of each part of the aircraft and how they move individually need not be known. This general aircraft motion is described completely by a kinematic flight model and forces on the aircraft are generally not considered. Using aircraft and pilot stochastic data to provide items such as bank angle, roll rate, climb/descent rate, and climb/descent acceleration/deceleration, a type-specific flight simulator is formulated that describes the general motion of an aircraft type.

Parameters

The kinematic equations are called separately at each time step; therefore, at each step a number of inputs must be determined to ensure the proper position is calculated. The inputs include items such as the aircraft's state, its commanded state, and simulator clock time; all of these values must be saved by the main program since the kinematic equations are called independently.

The aircraft's state vector includes the following: current position (latitude, longitude, and altitude), magnetic heading, true airspeed, bank angle, roll rate, vertical speed, and vertical acceleration.

It is assumed that for a certain phase of flight (e.g., approach) the aircraft's indicated airspeed (IAS) remains constant; this is done through

implicit pitch adjustments. During each time step the indicated airspeed is first converted to a true airspeed (TAS). The conversion to true airspeed, V_{TAS} , involves the intermediate steps as shown in Figure 1.

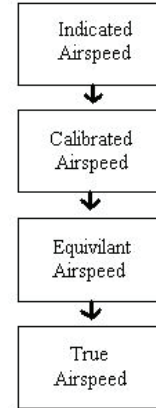


Figure 1. Airspeed Conversion Process

Calibrated airspeed (CAS) corrects for instrument error present in the pitot-static system. These errors are aircraft type-specific and can be found in aircraft operating manuals. Equivalent airspeed (EAS) is calibrated airspeed corrected for compressibility effects as the air passes into the pitot tube. Under 200 knots (i.e., in the approach phase of flight) this effect is judged to be negligible. True airspeed corrects equivalent airspeed for density variations with altitude, and is the actual speed that the aircraft is moving relative to the air. To calculate true airspeed from equivalent airspeed the following relation is used [3]

$$V_{TAS} = V_{EAS} \frac{1}{\sqrt{\sigma}} \quad (1)$$

The density ratio σ is the ratio of air density at altitude to the air density at sea level; with the standard atmosphere model being used here.

Hence the model takes as an input an initial vertical speed V_V and V_{TAS} . Prior to correcting for winds, the aircraft ground speed V_{GS} is set equal to V_{TAS} . Vertical speeds are calculated from within the kinematic equations as described in the section on vertical motion.

The commanded (as opposed to current) aircraft state must also be passed to the kinematic model during each run. These values are those

parameters that are commanded by the pilot and include the bank angle, vertical speed, and heading.

Aircraft properties are the component of the model that ensures different aircraft move differently through space for the same initial conditions. Through the FAA- or manufacturer-provided aircraft data (indicated airspeed, bank angle, vertical speed, roll rate, and vertical acceleration) different size aircraft will move based upon their predefined properties. Although the actual equations of motion are generic, the differences in these parameters provides for a dynamic aircraft simulator.

Separate from the aircraft model, the main simulation tool also allows for defining runway geometry and location, and appropriate navigational aids (e.g., the instrument landing system (ILS) and its localizer and glide slope properties) that are used during the simulation. The runway threshold's latitude, longitude, elevation, and crossing height are given, as is the magnetic runway heading. Intercept points are given for both the glide slope and localizer, as is the glide slope angle and localizer width and bearing. These combined data determine completely the intended flight path for the aircraft on approach.

Initialization

Initial values are generated to launch the aircraft model. The aircraft's starting position is determined before the model runs. The required initial values include latitude, longitude, altitude, bank angle (typically zero initially), indicated airspeed, and heading. Latitude, longitude, IAS and altitude are normally randomly initialized using statistical distributions.

Statistical Distributions

The kinematic model contains a mixture of deterministic (i.e., fixed) values and random values that describe the various airplane performance characteristics. Deterministic values are taken from actual aircraft data while random values are generated from distributions using numerical techniques. Five basic statistical distributions are used to determine variable error: uniform, triangle, Gaussian (i.e., normal), Johnson S_B , and Johnson S_L .

To compute the random error from these statistical distributions a functionally simple analytic expression for the error variate in terms of

a normally distributed random variable (determined from the Box-Muller technique) is found. For example the S_B Johnson probability distribution function (PDF; Figure 2) is used to model the time for an Air Traffic Controller (ATC) to react and direct the pilot.

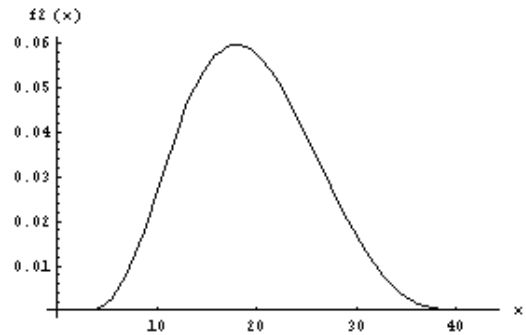


Figure 2. Johnson S_B PDF without Truncation

The S_L Johnson distribution (Figure 3) is used to model the human pilot reaction time delay error for responding to ATC instructions.

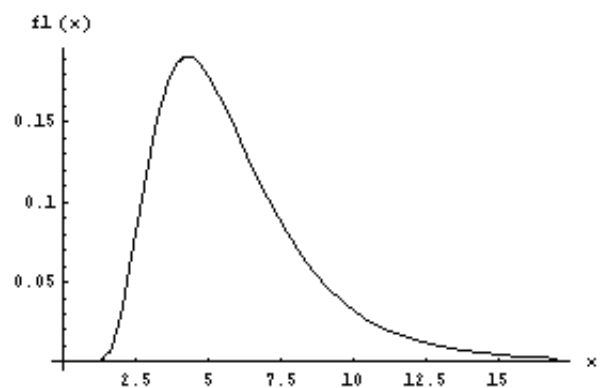


Figure 3. Johnson S_L PDF without Truncation

Where exact aircraft data is not known, a subject matter expert (SME), such as a line pilot in the particular aircraft type, can be used to provide an estimate of various aircraft parameters. These SMEs would be asked to provide minimum, maximum, and typical (i.e., mode) data for each parameter from their general experience. This is then incorporated into the stochastic model using a triangular probability distribution function (Figure 4). A triangular PDF is defined using x_{min} , x_{max} , and x_{mode} where $x_{min} < x_{mode} < x_{max}$. The triangle is formed by the intersection of a line of positive slope, a line of negative slope, and the x-axis.

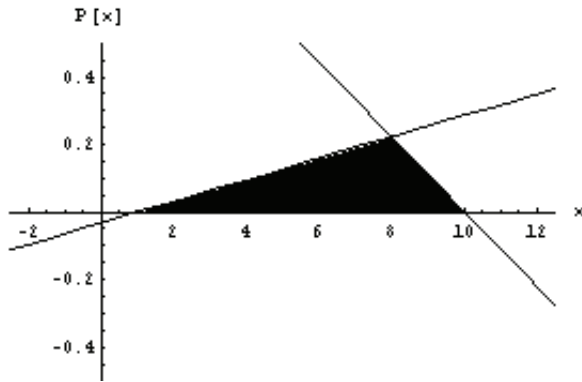


Figure 4. Triangular Distribution PDF

Finally, uniform and Gaussian distributions are used to model other variables (typically, various surveillance variables [4]).

Structure and Environment

Coordinate System

A set of two independent coordinate systems describes the aircraft's position. A two-dimensional coordinate system (everywhere tangent to a spherical earth's surface) is parameterized by the usual latitude and longitude angles for a sphere. Height, or altitude, is described by a one-dimensional coordinate system and is reported in feet mean sea level (MSL) with positive values above sea level and negative values below sea level. These coordinates were partly chosen to utilize real-world positions since airplane data typically includes reference to latitude and longitude. In addition, it may be desirable to travel long distances between landings and latitude and longitude are particularly well-suited coordinates for this. Finally, the use of knots implies a round earth frame of reference.

The coordinate systems are necessarily independent because the model velocities are independently specified for motion in the vertical direction and motion parallel to the earth's surface. For example, V_{TAS} applies tangentially to the earth's surface; adding the vertical velocity and the velocity parallel to the earth's surface is mathematically, and physically, undefined since they reside in separate spaces. Thus the motion of a descending plane, for example, is described by a path of decreasing altitude and a path along the earth's surface with changing latitude and longitude. These two paths are coupled in the sense that the plane moves in tandem along each

simultaneously. These two motions can be graphically merged into what appears as movement in a general three-dimensional spherical system.

The current coordinate system is also advantageous since it could be rigorously merged into a standard three-dimensional spherical coordinate system. This would allow, for example, the addition of radial and tangential velocities and hence the coupling of horizontal and vertical motions.

The aircraft ground speed V_{GS} is resolved into its north and east components based on aircraft heading Ψ ; this simplifies change in position calculations as shown below in Figure 5 and Eq. (2). Positive values indicate north or east movements, while negative values indicate south or west movements, respectively. All velocities are relative to the assumed non-rotating earth.

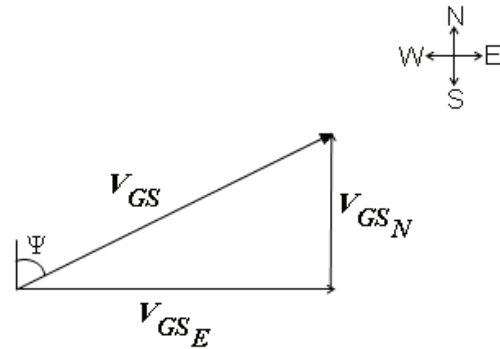


Figure 5. North/South and East/West Ground Speed Components

$$\begin{aligned} V_{GS_N} &= V_{GS} \cos(\Psi) \\ V_{GS_E} &= V_{GS} \sin(\Psi) \end{aligned} \quad (2)$$

Vertical velocity and acceleration are positive if upwards and negative if downwards.

The aircraft's bank angle ϕ is positive if banking to the right and negative if banking to left. This also applies to the aircraft's turn rate $\dot{\phi}$. This ensures that a positive bank angle leads to a positive (i.e., increasing) change in heading.

Since almost all aviation data is given in magnetic headings, the aircraft model performs all processing using data in this format. Unfortunately, cartography is generally performed using true north.

Therefore, all magnetic information is converted to true using the known magnetic variation just prior to calculation of the aircraft's updated latitude and longitude each time step since the latitude and longitude grid is true by construction.

Simulator Time Step

The necessary simulator time step for adequate fidelity is determined through verification and validation of the simulator. For example, selection of a time step sufficiently small to produce accurate latitude and longitude. The main simulation program sends the kinematic model the current simulator time and the time between the previous and current time is calculated (this number is in fact the only number necessary). This current time step Δt is used to determine the change in position and, ultimately, the aircraft's subsequent position. The current time step in the model is set at 0.02 seconds (i.e., 50Hz).

Assumptions

Assumptions must be made in order to simplify the flight model; however, this results in some restrictions on the use of the kinematic model as a flight simulator.

Engine data is not provided for the aircraft, but vertical velocity and acceleration are; this implies that thrust is assumed to implicitly change in order to produce the desired vertical rates. The kinematic method for changing the thrust is left to the pilot/autopilot "black box," but it is done in such a way to produce the desired result.

For simple kinematic motion, the horizontal and vertical components of motion are decoupled. For this to occur, it has to be assumed that while turning but remaining level, adequate thrust is maintained to keep the aircraft level. If the aircraft is turning and climbing or descending, adequate thrust is maintained to keep the assigned bank angle and turn rate as well as the assigned vertical rates.

One result of these assumptions is that there are various flight regimes that cannot be modeled. As an aircraft stalls, aerodynamics become more complicated and the use of a more detailed flight model is required. Just as in a stall, when an aircraft reaches a high bank angle or a high speed, the aerodynamics become more complicated and compressibility needs to be taken into account when calculating aircraft dynamics.

Wind Inclusion

A simple two-dimensional wind is included in the simulation. This wind consists of a wind speed V_W and direction at each altitude Ψ_{wind} (Eq. (3)). The wind speed and direction may be a function of altitude, creating a quasi-three-dimensional wind. The wind is resolved into its directional components and therefore is vectorally added to the aircraft's velocity vector.

$$\begin{aligned} V_{WN} &= V_W \cos(\Psi_{wind}) \\ V_{WE} &= V_W \sin(\Psi_{wind}) \end{aligned} \quad (3)$$

Pressure and Temperature

The pressure and temperature are modeled as varying with altitude in accordance with the U.S. Standard Atmosphere [5]. The boundary conditions for temperature and pressure are specified at the airport at ground level. It is assumed that isothermal and isobaric lines are everywhere tangent to the earth's surface. For example, the temperature is linearly modeled as decreasing 2°C for every 1,000 foot increase in altitude.

Pilot Control

A Johnson distribution S_L models the human pilot reaction time delay error for acting on ATC directives; other parameters can be seen in Table 1. The data in this table models a human pilot flying a Boeing 777 on approach. These are commanded by the simulation as inputs to the kinematic flight model.

Table 1. Human Pilot Flying Parameters

Type of Data	Pilot
Roll rate (degrees/second)	5°/s
Angle of bank (degrees)	Bounded normal distribution: $\sigma = 3^\circ$, $\mu = 17^\circ$, min = 15°, max = 20°
Rate of vertical speed, i.e., acceleration (feet/minute/second)	Johnson S _B distribution: $\gamma=0.225$, $\delta=0.927$, $\lambda=394.848$, $\varepsilon=88.072$, min = 123.6, max = 462.6
Vertical speed (feet/minute)	Johnson S _B distribution: $\gamma=2.592$, $\delta=2.185$, $\lambda=4526.482$, $\varepsilon=1400.0$, min = 1442.8, max = 3108.9

Horizontal Motion

As stated under assumptions, the aircraft's horizontal and vertical motions are decoupled. Further simplifying the motion, there are two sub-components of horizontal flight: straight flight and turning flight. For our model, horizontal refers to constant altitude flight.

Straight Flight

In straight flight the aircraft's bank angle is exactly zero and its heading remains constant between time steps; in turning flight the aircraft's bank angle is nonzero and its turn rate may or may not be zero. Each case is described below.

In horizontal flight the aircraft's horizontal velocity V_{GS_N} must be added to the wind velocity V_W at the current altitude to obtain the true ground speed in the north/south and east/west directions as shown in Eq. (4).

$$\begin{aligned} V_N &= V_{GS_N} + V_{W_N} \\ V_E &= V_{GS_E} + V_{W_E} \end{aligned} \quad (4)$$

The aircraft's tangential speed V with respect to the ground is given by

$$V = \sqrt{V_N^2 + V_E^2} \quad (5)$$

During straight flight the aircraft's bank angle is exactly zero and its heading does not change (note that the commanded bank angle may or may not be zero). The motion is simply a straight line with distance equal to the aircraft's velocity times

the time of the current step. The change in position is given by

$$\begin{aligned} \Delta N &= V_N \Delta t \\ \Delta E &= V_E \Delta t \end{aligned} \quad (6)$$

The time scales for each time step must always be sufficiently small such that these positions (given in distance units) can be accurately converted to degrees of latitude and longitude.

Turning Flight

Turning flight is achieved when the aircraft's bank angle is nonzero. This occurs when the commanded bank angle is changed (either to zero or to a nonzero value) and the aircraft must bank to achieve the new commanded value. If it is determined that the aircraft was in fact turning between the last time step and the current one, the new bank angle is calculated given the aircraft's turn rate as shown in Eq. (7).

$$\phi_{n+1} = \phi_n + \dot{\phi}(\Delta t) \quad (7)$$

The average value of the last and current bank angle is used for calculating the position change when in a turn. This ensures that the bank angle is weighted evenly in case the aircraft was not at a constant bank angle during the last time step. The aircraft's interpolated bank angle ϕ_{calc} is given by

$$\phi_{calc} = \frac{\phi_{n+1} + \phi_n}{2} \quad (8)$$

If during the last time step the aircraft's bank angle exceeded the commanded bank angle, the

bank angle used for the calculations is set to equal the commanded bank angle. For simulation purposes the aircraft will never exceed its commanded bank angle. The same is done to the “actual” bank angle, shown above in Eq. (7), but the turn rate $\dot{\phi}$ is also set to zero.

To turn the aircraft to a commanded heading, a logic system is used that is analogous to a proportional integral (PI) controller (i.e., a proportional integral control loop feedback mechanism). When a turn is required, the maximum bank angle of the turn is determined by the pilot model. If the aircraft only needs to change heading by 10° , for example, and the aircraft bank angle as selected using the angle of bank PDF is 15° , the aircraft will only turn to $\frac{1}{2}$ the required heading change (proportional to the heading change) as described in the FAA instrument flight handbook [6]; in this case the new commanded angle of bank would be 5° .

As the aircraft approaches its intended course, it begins to roll out prior to the commanded heading; this is the “integral” portion of the PI controller. This stage helps to ensure that the aircraft does not overshoot its commanded course. A typical rule-of-thumb is that the aircraft begins to roll out of the turn when the difference between the current and commanded heading is less than half of the current bank angle [6]. For example, if the aircraft were turning right from a heading of 90° to 120° with a bank angle of 14° , the aircraft initiates its roll to wings level when arriving at a heading of 113° (one-half of 14° is 7° , and $120^\circ - 7^\circ$ is 113°). The rate for rolling out of a turn can also be controlled by a predefined ratio. The pilot might use a smaller roll rate to come out of a turn than was used to roll into the turn. For example, if the typical roll rate of the aircraft is $3^\circ/\text{s}$ and the roll rate multiplier is 0.5, the aircraft would roll out of the turn at a rate of $1.5^\circ/\text{s}$. By tuning this part of the pilot model the effect of “smoothing” the aircraft motion on rollout can be achieved. Additionally, if the difference between the current and commanded heading is less than a predefined error (e.g., 0.5°) the aircraft will not be commanded to turn; this decreases the small corrections that would otherwise always be present in the aircraft’s motion. This also models to some extent the limits of error

that a pilot would use to keep a constant heading or follow a predefined course.

Once the bank angle for the current time step is found, the change in horizontal position during the current time step can be found. This is done by using simple circular motion, and this part of the simulation is the one part that does require the use of aircraft forces (note, these forces are calculated and no aircraft type-specific data is required). When the aircraft’s bank angle is nonzero, there is a nonzero component of lift in the horizontal plane (Figure 6 and Eq. (9)).

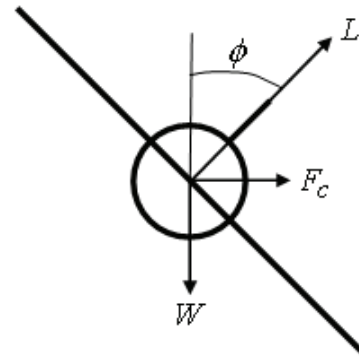


Figure 6. Lift Force During Banks

$$\begin{aligned} F_c &= ma_c \\ W &= mg \\ \tan \phi &= \frac{F_c}{W} \end{aligned} \quad (9)$$

The remaining vertical component of the lift is used to keep the aircraft in the air (i.e., counter the aircraft weight W ; recall that thrust is considered to be adjusted to do so), but the horizontal component forces the aircraft to turn in a circle of radius R as described by Eq. (10).

$$a_c = \frac{V^2}{R} \quad (10)$$

The acceleration is found after finding the horizontal component of lift, which is a direct function of the aircraft’s bank angle ϕ shown below in Eq. (11).

$$\tan(\phi) = \frac{a_c}{g} \quad (11)$$

This results in the aircraft's turn radius R as a function of the averaged bank angle and the speed V shown below in Eq. (12).

$$R = \frac{V^2}{g \tan(\phi_{calc})} \quad (12)$$

The turn radius can then be used to calculate the aircraft's change in heading during the current time step. Knowing that

$$\frac{\Delta\Psi}{\Delta t} = \frac{V}{R}$$

Eq. (13) is obtained (see also Figure 7).

$$\Delta\Psi = \frac{V\Delta t}{R} \quad (13)$$

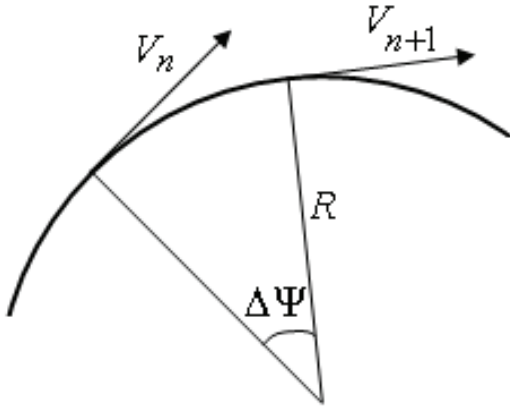


Figure 7. Circular Motion

The aircraft's new heading is the sum of Eq. 13) and the previous heading. Since the simulation time steps are small, it can be assumed that the change in aircraft position is simply the time step multiplied by the aircraft's velocity at the beginning of the step. The new position is then found using Eq. (5).

Vertical Motion

Just as in the horizontal motion case, motion in the vertical plane can be further simplified. When considering the climbing and descending of an aircraft it is seen that the two distinct cases are when the vertical speed is constant (i.e., moving in a "straight line") and when the rate of change of vertical speed is not zero (i.e., "turning").

The maximum value for the vertical speed, and the rate of change of vertical speed, is a function of the aircraft being simulated and are given at the beginning of the simulation.

These vertical speeds are a result of the pilot/autopilot adjusting the implicit engine thrust. Changes in "thrust" will maintain the vertical speed and altitude, as required; this implicitly takes into account whether or not the aircraft is banking (when part of the aircraft's lift is in the horizontal plane). Again, it is assumed that thrust is changed to maintain the required vertical rates and horizontal motion.

Although vertical "turning" is analogous to horizontal turning there are differences in the way the model is designed. In the horizontal plane a particular course is commanded, and the aircraft's bank angle is changed to achieve this heading. In the vertical plane, vertical speed itself is commanded and changed to maintain an altitude. This concept is appropriate since it more accurately reflects the actions of pilots and the information that is provided to them in the cockpit. This defined commanded vertical speed is used during each time step to determine the current vertical speed that is used in calculating the aircraft's altitude. As in the horizontal case, if the previous vertical speed is not equal to the commanded vertical speed, then the aircraft is vertically turning. The actual vertical speed for the current time step is shown in Eq. (14).

$$V_{V_{n+1}} = V_{V_n} + \dot{V}_V (\Delta t) \quad (14)$$

Similar to the horizontal motion of the aircraft, a controller akin to a PI control system is used to turn the aircraft vertically. If the aircraft's vertical speed does not match the commanded value (and this difference is outside the predefined error range; 25 ft/min, for example) then the aircraft commands a vertical turn. Since vertical speed is the parameter that is being controlled, only the rate of change of vertical speed (as selected by a Johnson S_B PDF) is varied to change the aircraft's altitude and achieve the commanded vertical speed.

As the aircraft gets closer its commanded vertical speed it begins to "roll out of the turn." A general rule is to begin leveling off (or decreasing the rate of change of vertical speed to zero) when the difference between the flight path altitude and the aircraft altitude is less than 10% of the current

vertical speed. This also is a tuning parameter of the pilot model. For example, if the aircraft was supposed to be at an altitude of 5,000 ft (note that altitude is not being controlled directly) climbing at 500 ft/min, it would begin to level-off at an altitude of 4,950 feet. The leveling-off will not be at the aircraft's maximum rate of change of vertical speed, but is at a predefined constant times this value (0.4, for example).

Since a constant vertical acceleration is assumed (i.e., the rate of change of vertical speed is an aircraft property) the equations for linear motion can be used to find the change in altitude Δh during a time step.

$$\Delta h = \frac{1}{2} \dot{V}_v (\Delta t)^2 + V_v \Delta t \quad (15)$$

The result of Eq. (15) is added to the previous altitude to obtain the aircraft's altitude at the current time step. Note also that this equation is valid for both cases of vertical flight (i.e., climbs and descents). When the rate of change of vertical speed is zero, a linear change in altitude occurs solely due to the $V_v \Delta t$ term (Δh would be exactly zero only if V_v was also zero). If the aircraft is "turning," then the first term adds the effect of the changing vertical speed.

Model Outputs

Model outputs include latitude, longitude, altitude, time, heading (to allow for visualizing

aircraft orientation), and angle of bank (to allow for visualizing the aircraft banking).

To display this model, several methods have been employed including the printing of data values for quantitative analysis as well as visualization using Google Earth™ and open source aircraft wire frame models.

Future Efforts

Several efforts are currently underway to further expand the model's capabilities while preserving its ease of implementation. One of the main efforts is the creation of vertical and horizontal "comfort cone" to mimic the allowable error and corrective actions of a pilot or flight management system (FMS) when navigating on an airway (using, for example, VHF omni-directional range, or VOR, nav aids) or while executing an approach (using, for example, an ILS). In the horizontal plane, the computer pilot makes no heading corrections when in the comfort cone, but makes proportionate corrections in accordance with the FAA instrument flight manual [6] once a lateral boundary is crossed. The process is similar in the vertical plane. Further details can be seen in Figures 8 and 9.

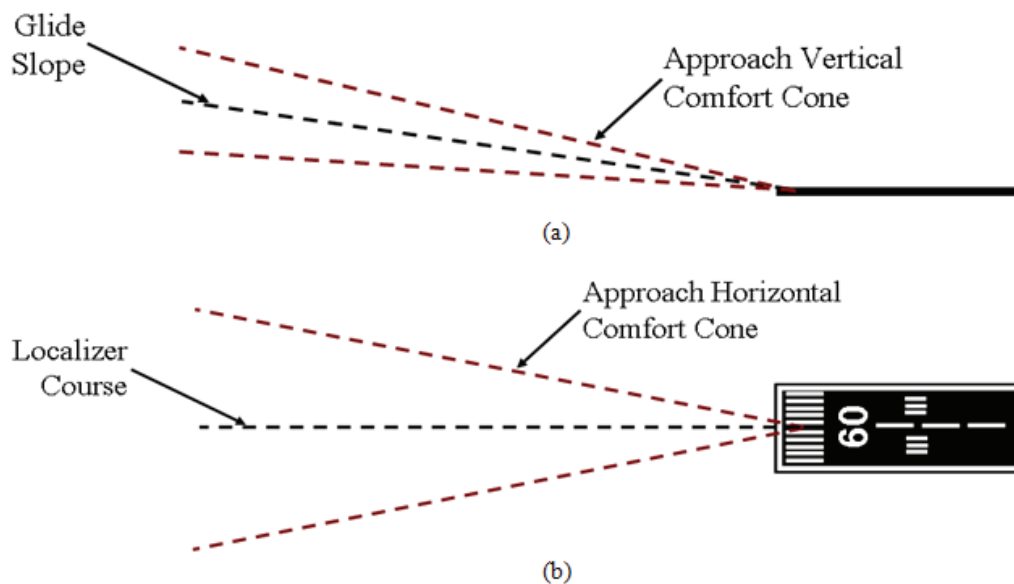


Figure 8. (a) Vertical Comfort Cone and (b) Horizontal Comfort Cone as used with an ILS Approach

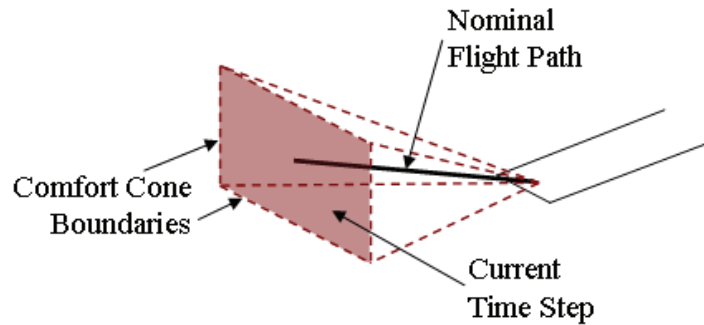


Figure 9. Approach Comfort Cone at One Time Step

Summary

If an aircraft model is to be used exclusively in normal flight regimes and simplicity is desired, a kinematic model may be adequate. Here, a type-specific aircraft model described completely by kinematics was presented. This aircraft simulator uses actual, but minimal and generally readily available, aircraft data and statistical distributions to accurately mimic real aircraft and pilots while allowing for the varying of parameters as would be desired in a Monte Carlo-type simulation.

Using a kinematic model requires making certain assumptions. Thrust is not explicitly modeled and it is assumed that it is implicitly modeled through the command of various kinematic parameters via the pilot. It is also assumed that the horizontal and vertical motions are decoupled, greatly simplifying calculations and allowing for a simplified coordinate scheme.

Because this kinematic flight model allows simple generation of stochastic representations for key parameters, it is easily incorporated into a variety of simulations with these representations easily changed to reflect empirical data describing the performance of various aircraft.

References

[1] McGovern, S. M. and S. B. Cohen, 2007, "Survey of Contemporary Aircraft Flight Dynamics Models for Use in Airspace Simulation," 6564-16, Proceedings of the 2007 SPIE Defense and Security Symposium, International Conference on Modeling and Simulation for Military Operations II, Orlando, FL, SPIE.

[2] Halliday, D., R. Resnick, and J. Walker, 1993, Fundamentals of Physics, New York, John Wiley & Sons, Inc.

[3] National Test Pilot School, 1995, Introduction to Performance and Flying Qualities Flight Testing, Second Edition, Mojave, CA, National Test Pilot School.

[4] McCartor, G. R. and S. Ladecky, 2005, Safety Study Report on Triple Simultaneous Parallel Instrument Landing System and Area Navigation/Required Navigation Performance Approaches at George Bush Intercontinental Airport, DOT-FAA-AFS-440-16, Oklahoma City, OK, FAA.

[5] White, F., 1979, Fluid Mechanics, McGraw-Hill.

[6] Federal Aviation Administration Flight Standards Service, 2001, FAA-H-8083-15, Instrument Flying Handbook, Washington, D.C., U.S. Government Printing Office.

Acknowledgements

This material is based on work supported by the Federal Aviation Administration Flight Standards Service, Flight Technologies Division, Flight Operations Simulation and Analysis Branch under RITA/VNTSC #FA7G DD972. Any opinions, findings, and conclusions or recommendations expressed in this paper are those of the authors.

*26th Digital Avionics Systems Conference
October 21, 2007*