Chapter 7

A Peircean turn in linguistics: Syntactic-semantic composition as logical inference

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In the late 19th century Charles Sanders Peirce proposed what can be seen as a model of natural language in which the combinatoric affinity of lexical items – which he characterizes as their respective valence – drove the composition of sentences. In this paper I argue that Peirce's conception of valence as the basis of linguistic composition, incorporated into a logic of types in which valence is interpreted as implication, finds its formal realization in a species of categorial grammar. I further show the power of this conception in capturing a complex interaction of filler-gap connectivity with ellipsis, which has been claimed to be one of the strongest pieces of evidence for covert structure analyses of ellipsis patterns. The type-logical treatment of this supposed pattern of extraction from ellipsis sites undercuts such claims, and reinforces Joachim Lambek's invocation of Peirce as perhaps the earliest intellectual ancestor of modern type-logical approaches to natural language architecture.

1 Peirce and valence

The work of Charles Sanders Peirce – a long-time research focus of Dan Everett, whom this festschrift honors – spans a range of interests in, and major contributions to, a variety of mathematical and scientific domains that may well be unique in the history of human accomplishment. Peirce's work is widely recognized as seminal in mathematics, logic, the philosophy of knowledge, chemistry, astronomy and many other fields, but it is not generally recognized that he was the

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source of an analytic concept, *valence*, which has become a foundational tool in linguistic theory. It is only recently that Peirce has been credited with his identification of the combinatorial potential of lexical items as one of the key drivers of linguistic form, or for being the first scholar to use the term "valence", which he borrowed from chemistry as a close analogue of this linguistic concept, and it is still common to see linguistic applications of valence as having been "founded in 1959 by Lucien Tesnière" (Höllein 2022). There were, as noted in Przepiórkowski (2018), a number of earlier invocations of the metaphor referencing the electrons needed by atoms of an element in order to attain a stable state. But the earliest such appeal to this metaphor, as Askedal (1991) and Przepiórkowski document, was Peirce (1897), where the word *gives* is explicitly identified as having the same number of "unsaturated bonds" as the nitrogen atom, which combines with three hydrogen atoms to form the ammonia molecule NH_3, and Przepiórkowski considers it likely that Tesnière and the others who introduced valence into the parlance of grammarians were all influenced by Peirce's original invocation of the concept. Przepiórkowski (2018: 155) notes that "four linguists working in four different countries independently came up with the valence metaphor", within the space of a single decade, and suggests that the common source for their exposure to Peirce's metaphor was not Peirce himself, but Roman Jakobson, who was probably the earliest grammarian of the modern era to recognize the depth of Peirce's insights on natural language, particularly Peirce (1897), and actively promoted Peirce's work in conversations and international gatherings, such as the 1948 International Congress of Linguists in Paris, among other venues.

Those who have studied Peirce's work as it bears on natural languages generally concede that his perspective was primarily rooted in their semiotic capabilities, as systems of signs. But as Nöth observes, for Peirce, "the key to syntactic structure is the predicate and its valence" (2000: 7). Peirce seems to have regarded the valence of sentences in both a syntactic and semantic way: on the one hand, the places in which names can appear (whose occupants he called "subjects") and, on the other, as the parts of propositions which the predicate sets into the relationship that the predicate denotes, and which point to particular individuals – the referents of the names themselves.

There are a few aspects to this conception of syntax which deserve some amplification, because they bear directly on what I believe amounts to a specific development of Peirce's ideas. Peirce clearly did not adopt the widespread contemporary view that syntactic categories are to be regarded as projections of lexical categories; that e.g. NPs are in effect just nouns with various other encrusted bits – adjectives, determiners and so on that are attached to the Ns that are the "head" of the NP. Rather, his perspective appears to have been based much more on the conceptual burden of the items corresponding to the parts of the proposition conveyed by the sentence. But in principle, if there were a one-to-one relationship between the way in which syntactic valence is satisfied and the way in which semantic meaning is assembled, then the conceptual construal of valence and the syntactic combinatorics of language would essentially mirror each other.

Contemporary phrase structure approaches, of course, do not adhere to the analytic program such a unified view of syntax and semantics imposes. Typically, we find a set of lexical (and, in certain approaches, morphological) elements that represent the lowest tier of syntactic objects, corresponding to the terminal nodes in phrase structure trees, and more complex objects that these element compose into, which satisfy some set of criteria – typically based heavily on distributional possibilities, displaceability chief among them. These elements combine by rules which license hierarchical structures that represent the syntactic form of a sentence as the record of all the combinatorial steps that had to apply to derive that sentence. But there is a alternative approach available, one in which lexical items are regarded as inhabitants of different TYPES, representing what is in effect the combinatorial "destiny" of the words inhabiting that type, and in which the mode of syntactic composition and the mode of semantic composition are at a more abstract level the same operation. Such a theoretical architecture represents, in my view, one possible way in which contemporary formal linguistics reflects a Peircian turn, although one perhaps rather different from what Peirce himself had in mind.

In a sense, it seems a bit of truism to describe any particular framework as "valence-based"; virtually all major theories or "programs" utilize some notion of valence as a central feature in licensing sentences. But it is not often appreciated how much mileage is possible by driving an approach in which the combinatoric possibilities of individual words can determine quite complex patterns and effects, including arbitrarily non-local dependencies and interactions amongst such dependencies. In the following sections I outline a framework based on this architecture – as first envisaged and articulated by Peirce – and show how it allows us to formulate alternatives to standard phrase structure analyses that do not require us to posit elaborate machinery altering the hierarchical arrangement of structures that have already been formed, but nonetheless capture a particularly intricate relationship between long-distance dependencies and ellipsis strictly on the basis of lexical argument structure.

2 Argument structure and labeled deductive systems

Theories of syntactic structure of the sort alluded to in the final paragraph of the preceding section belong to a family of frameworks that represent different versions of type-logical categorial grammar. The essential premise shared by the frameworks is that the rules of syntactic composition are stated as a deductive calculus formally equivalent to at least the implicational fragment of one or another standard truth conditional logic, with inference from valid type(s) to valid type in place of inference from true premise(s) to true premise. In the framework described below, each linguistic sign comprises a phonological and semantic annotation which is said to *label* the sign's syntactic type. The compositional rules of the grammar are homologous to the implicational subsystem of substructural intuitionistic propositional logic (SIPL), i.e., IPL lacking rules of permutation, contraction or weakening, with implication corresponding to types of the form Y/X, $X\backslash Y$ and $Y{\upharpoonright}X$. The first of these can be thought of as something like, "give me a sign of type X on my right and you'll get back a sign of type Y", and the second is the same with "left" in place of "right". The third is a bit more complex: it tells you that, if there is a sign of type X it can be realized in a certain designated position "within" the sign typed $Y{\upharpoonright}X$. I refer to inhabitants of slashed types as *functional* terms, in view of their semantics, as discussed below.

What are the syntactic types that can instantiate Y and X? For our purposes, we can posit three atomic types, which are in a one-to-one relationship with basic semantic types:[1]

(1)

Type	Semantic object	Semantic type
S	proposition	t
NP	referring expression	e
N	property	$\langle e, t\rangle$

Clauses then correspond to propositions, and NPs to Peirce's "subjects", so that in (2a), for example, we would assign *give* the type (NP\S)/NP/NP:[2]

(2) a. John gave Mary the manuscript.
   b. gave; **give**; (NP\S)/NP/NP

[1]Here and in what follows, I used the standard angled bracket notation $\langle \tau_1, \tau_2\rangle$ to indicate a function from some object of semantic type τ_1 to an object of semantic type τ_2.

[2]We will also take PP to be a basic type, although here matters are a bit more complex: typically, inhabitants of the type PP have the same semantic type as those typed NP.

The rules under which (2b) composes with its argument terms to yield the sentence in (2a) are, as noted, the elimination and introduction rules for implication of intuitionistic propositional logic, where implication takes the three forms noted earlier. There are a number of different formats for logical rules; the system I introduce here belongs to a subfamily of type-logical frameworks which uses the Natural Deduction conventions. In the Prawitz notation followed below, the ordinary IPL rule would take the form in (3b):

(3) a. $\dfrac{\phi \supset \psi \qquad \phi}{\psi} \supset \text{Elim}$

b. $\dfrac{\begin{array}{c}\phi \\ \vdots \\ \psi\end{array}}{\phi \supset \psi} \supset \text{Intro}$

(3a) is nothing other than the ancient principle of modus ponens, where there is an antecedent (ϕ) and a consequent (ψ), such that the truth of ϕ is a guarantor of the truth of ψ (or, under the more appropriate intuitionistic interpretation, a proof of ψ follows from a proof of ϕ). (3b) is the slightly less transparent rule of hypothetical reasoning: if, in some context of established results, introducing an hypothesis ϕ allows us to deduce ψ, then in that same context, we know that the implication $\phi \supset \psi$ follows.[3] In a nutshell, if we assume a certain premise that allows us to deduce a certain result, we know that, mutatis mutandis, if that premise were true, the result would then follow.

But translating these rules into the type-logical domain requires a good deal more than just inference rules for types. Linguistic signs do not just inhabit types; they also carry phonological and semantic information. Unlike the propositions that combine under intuitionistic rules of inference, the word(sequence)s that are the corresponding type-logical objects are ordered linearly in sentences – a property we take to be a prosodic, not syntactic fact, reflecting our partial adoption of the tectogrammatical/phenogrammatical distinction advanced in Curry (1961). Similarly, syntactic composition and inference are exactly mirrored in the semantic combinatorics, as will become evident from the full statement of the type-logical rules of inference given in (4), corresponding to (3), assumed throughout this paper. In (4) and hereafter, I take a sign to be a tripartite object with a prosodic sector, a semantic sector and a type value, presented in that order.

[3] Intuitionistic implication differs from classical implication in that Peirce's Law – $((\phi \supset \psi) \supset \phi) \supset \phi$ – holds for the latter but not the former, since on intuitionistic assumptions there is no way to deduce the consequent ϕ from the antecedent $(\phi \supset \psi) \supset \phi$. This is as it should be so far as our type logic is concerned, since translation of Peirce's Law into type logic results in a generally false prediction about argument structure.

(4)

Connective	*Introduction*	*Elimination*
/	$\dfrac{\dfrac{\vdots\ \vdots\quad \dfrac{[\varphi; x; A]^1}{\vdots\ \ \vdots}\quad \vdots\ \vdots}{b \circ \varphi; \mathcal{F}; B}}{b; \lambda x.\mathcal{F}; B/A}\ /\mathrm{I}^n$	$\dfrac{b; \mathcal{P}; B/A \qquad a; \alpha; A}{b \circ a; \mathcal{P}(\alpha); B}\ /\mathrm{E}$
\	$\mathrm{x}\ \dfrac{\dfrac{\vdots\ \vdots\quad \dfrac{[\varphi; x; A]^1}{\vdots\ \ \vdots}\quad \vdots\ \vdots}{\varphi \circ b; \mathcal{F}; B}}{b; \lambda x.\mathcal{F}; A\backslash B}\ \backslash\mathrm{I}^n$	$\dfrac{a; \alpha; A \qquad b; \mathcal{P}; A\backslash B}{a \circ b; \mathcal{P}(\alpha); B}\ \backslash\mathrm{E}$
↾	$\dfrac{\dfrac{\vdots\ \vdots\quad \dfrac{[\varphi; x; A]^1}{\vdots\ \ \vdots}\quad \vdots\ \vdots}{b; \mathcal{F}; B}}{\lambda\varphi.b; \lambda x.\mathcal{F}; B{\upharpoonright}A}\ {\upharpoonright}^n$	$\dfrac{b; \mathcal{P}; B{\upharpoonright}A \qquad a; \alpha; A}{b(a); \mathcal{P}(\alpha); B}\ {\upharpoonright}\mathrm{E}$

In (4), the vertical ellipses surrounding the variable and its composition into the proof denote the proof history subsequent to the introduction of the variable. *a*,*b* are metavariables over strings – lexical items or sequences of lexical items – while φ is a variable, supplied not by the lexicon, or as a stand-in for some actual string whose value is irrelevant in the context of the rule. Rather, variables are part of the logic itself, representing, in effect, a space in a prosodic or semantic expression that could be occupied by any term of the same type as the variable. Each variable sign is introduced with a specific index, and each application of an introduction rule is keyed to the index of the variable which is removed in the introduction of the directional slashes or λ-bound in ↾ introduction. The elimination rules shown are, again, different avatars of (3a): a slashed term seeks a term of the antecedent type to give us a consequent type, and the result of composing the slashed term with the antecedent term is necessarily a term of the consequent type. One can see these inference rules as inversions of ordinary context-free PS rules; for example, taking VP to be an abbreviation for NP\S – a clause modulo an NP term on its left edge – we have S → NP NP\S on the one hand and a deduction

$$\dfrac{\mathrm{NP} \qquad \mathrm{NP}\backslash\mathrm{S}}{\mathrm{S}}\ \backslash\mathrm{E}$$

on the other.[4] The prosodic and semantic sectors combine in lockstep with the type composition: the prosody of directionally slashed types – Y/X and $X\backslash Y$ – reflects the direction of the slash: the former precedes the prosody of its type X argument, the latter follows it.[5] The semantics, however, does not reflect the

[4]However, as noted below, this view leaves the nature of a type-logical proof open to a foundational misinterpretation.

[5]I defer discussion of vertically slashed terms till we get to the introduction rules.

direction of the slash: for all functional types, the semantic term is a function which takes the denotation of the syntactic argument as its own arguent.

With this much in hand, we can now provide a complete proof of an English sentence that illustrates the ways in which type, prosody and semantics collaborate to derive the sentence in (5) as, in effect, a theorem. We start with a lexicon, as in (6):

(5) John sent those documents to that committee over the weekend.

(6)

john; **j**; NP	sent; **send**; $VP/PP_{to}/NP$
those; ι; NP/N_{pl}	documents; **docs**; N_{pl}
to; $\lambda x.x$; PP_{to}/NP	that; ι; NP/N
committee; **comm**; N	over; **over**; $(VP\backslash VP)/NP$
the; ι; NP/N	weekend; **wknd**; N

Lexical entries are axioms of the type logic (though other axioms are possible, including axioms which incur some kind of penalty, and license proofs whose output is not fully acceptable, allowing us to incorporate a range of gradience effects into the framework). A few comments on (6) are in order: PP_{to} is a subtype of PP, derived via the the unique prepositional type PP_{to}/NP, whose semantic interpretation is an identity function, yielding a denotation identical to that of its argument. Ns have subtypes N_{sg} and N_{pl}, with some determiners targeting one or the other. Finally, *over*, despite its standard identification as a preposition, is in type-logical terms a function composing with an NP to yield a function which applies a temporal semantics to a property, corresponding to a restriction of the event instantiating that property.

The rules of the logic apply to the lexical axioms to yield the proof in Figure 1. This proof can be seen as a realization of Peirce's emphasis on argument structure, and its satisfaction, as the "engine" of syntactic combinatorics. As noted at the beginning of this chapter, the types associated with strings – either in the lexicon or via composition in the course of the proof – do not reflect the standard parts of speech inherited from the classical grammarians, but rather their combinatorial affinities, determined in part by the nature of their contribution to the formation of the proposition conveyed by a declarative sentence, or of the more complex semantical object denoted by questions, and so on. Proofs proceed purely on the basis of logical inference driven by type specifications, with semantic composition mirroring the composition steps determined by the inference rules given in (4), and the rules themselves reflecting standard truth-conditional deductive systems.

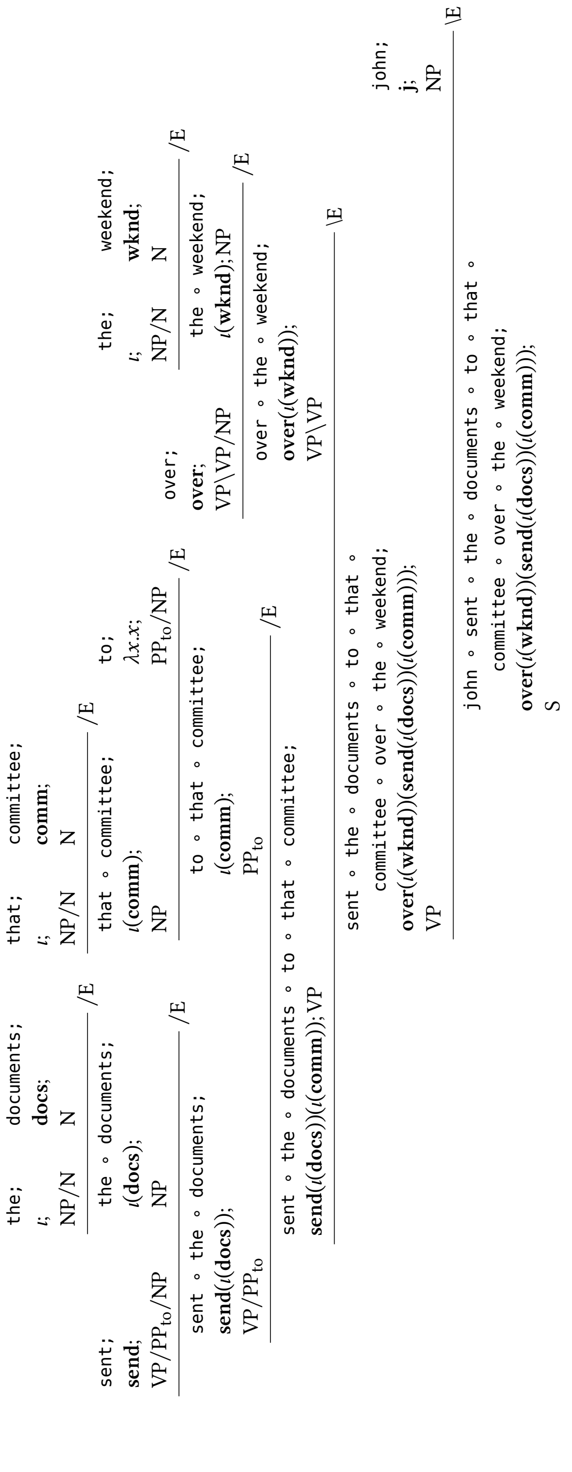

Figure 1: Proof for *John sent the documents to the committee over the weekend.*

On the other hand, one might take the view that such a proof is nothing more than a recasting of a standard hierarchical representation licensed by context-free phrase structure rules. This would be however a fundamental error: note that, in contrast to the hierarchical realization of phrase-structure rules in branching tree representations of constituent syntax, the proof steps in Figure 1 have no representational status so far as the structure of the sentence is concerned. Indeed, strictly speaking there *is* no such structure: what we have in Figure 1 is a demonstration that the closure of the axioms of the system – the English lexicon – under the inference rules of the logic allows a valid inference of a prosodic string john ∘ sent ∘ the ∘ documents ∘ to ∘ that ∘ committee ∘ over ∘ the ∘ weekend which signifies a proposition **over**(ι(**wknd**))(**send**(ι(**docs**)))(ι(**comm**))(**j**) and that the linguistic expression which has those prosodic and semantic values is a sentence. The steps involved in the proof have no representational status, any more than, given a set of premises Γ, the steps in the proof of $\Gamma \vdash \phi$ in some standard logic have any bearing on the content of ϕ.[6]

The difference between the logical composition of terms in Figure 1 and a tree representation of (5) under a set of phrase structure rules becomes far more stark when we turn from the elimination rules, which are the only ones in play in Figure 1, to the introduction rules shown in (4). There is nothing in phrase structure grammmar which corresponds to the introduction rules, and here the advantages of the proof-theoretic framework become apparent. So-called non-constituent coordination patterns such as Right Node Raising in (7a) and Dependent Cluster Coordination in (7b) are pointed examples:

(7) a. John bought, and Bill baked, the pizza margherita.
    b. John sent that message to Bill on Thursday and Mary on Saturday.

Both of the patterns in (7) are essentially embarrassments to frameworks based on phrase structure configurations, requiring either transformational grammar's complex arrangements of structure-altering operations, including movement and/or deletion (along with the purely stipulative constraints on the linear output of these operations required to get the facts right), or essentially stipulative constructional templates, as in later developments of HPSG (for detailed critiques of these approaches, see Levine 2011 and Kubota & Levine 2015, 2020). For proof-theoretic approaches, on the other hands, where valence satisfaction is driven by the inference rules of standard logics, the data in (7) are almost trivial to obtain with the correct semantics, once we've generalized the system based only on the

[6]For example, there are any number of ways to prove that $\vdash \phi \supset (\neg\phi \supset \psi)$ in classical logic, but the content of the implication is altogether independent of proof narrative.

elimination rules to the introduction rules that are their logical duals, as in all Natural Deduction systems. For example, we have the following straightforward application of the / Elimination rule:

(8)
$$\dfrac{\dfrac{\dfrac{\text{bought; } \mathbf{buy}\text{; VP/NP} \quad \varphi;\ x\text{; NP}}{\text{bought} \circ \varphi;\ \mathbf{buy}(x)\text{; VP}} \quad \text{john; } \mathbf{j}\text{; NP}}{\text{john} \circ \text{bought} \circ \text{p; } \mathbf{buy}(x)(\mathbf{j})\text{; S}}}{\text{john} \circ \text{bought; } \lambda x.\mathbf{buy}(x)(\mathbf{j})\text{; S/NP}}$$

The / Elimination rule allows us to obtain what in standard phrase structure approaches would be characterized as a partial constituent (although, in the framework adopted here, it is no more "partial" than VPs, i.e., signs inhabiting the type NP\S). A completely parallel proof will derive the sign in (9):

(9) bill ∘ baked; $\lambda u.\mathbf{bake}(u)(\mathbf{b})$; S/NP

Application of the standard generalized conjunction operator ⊓ introduced in Partee & Rooth (1983), which we take to be the denotation of *and*, with the type (*X**X*)/*X*, will then lead to the inference in (10):

(10) john ∘ bought ∘ and ∘ bill ∘ baked; $\lambda w.\mathbf{buy}(w)(\mathbf{j}) \wedge \mathbf{bake}(w)(\mathbf{b})$; S/NP

The final step in the proof will then be (11):

(11)
$$\dfrac{\begin{array}{c}\text{john} \circ \text{bought} \circ \text{and} \circ \\ \text{bill} \circ \text{baked;} \\ \lambda w.\mathbf{buy}(w)(\mathbf{j}) \wedge \mathbf{bake}(w)(\mathbf{b}); \\ \text{S/NP}\end{array} \quad \begin{array}{c}\vdots \quad \vdots \\ \text{the} \circ \text{pizza} \circ \text{margherita;} \\ \iota(\mathbf{pzzmarg})\text{; NP}\end{array}}{\begin{array}{l}\text{john} \circ \text{bought} \circ \text{and} \circ \text{bill} \circ \text{baked} \circ \text{the} \circ \text{pizza} \circ \text{margherita;} \\ \mathbf{buy}(\iota(\mathbf{pzzmarg}))(\mathbf{j}) \wedge \mathbf{bake}(\iota(\mathbf{pzzmarg}))(\mathbf{b}); \\ \text{S}\end{array}}$$

(7b) can be similarly derived via a somewhat tedious but straightforward subproof that yields *Bill on Thursday* and *Mary on Friday* as inhabitants of the type (PP_{to}/NP)\(NP\(PTV\VP)), where PTV is an abbreviation for the type VP/PP_{to}/NP. The conjunction of the two is therefore also of this same type, so that *Bill on Thursday and Mary on Saturday* combines to its left first with a PP_{to}/NP sign (i.e., *to*), then an NP (*the message*), then a PTV sign (*sent*), and finally VP, i.e., NP\S, which picks up *John* to give us (7b).

In a nutshell, in both of the patterns exhibited in (7), the interplay of the elimination and introduction rules allows us to compose each of the conjoined "non-constituents" into an S as arguments of a variable, with all other components of the S realized as variables, and then eliminate the variable terms by successive applications of the relevant introduction rules. The result is that the apparent nonconstituent prosodic elements are assigned a type, with a corresponding semantics corresponding to the application of abstraction operators at each elimination step. They are thus, in our terms, full constituents, now with the status of functional terms, and can then be conjoined. The resulting conjunction, possibly with a rather elaborate valence as in the case of (7b), then composes with its arguments to form the coordination. No structural operations, or indeed any structures at all are involved; the proofs given do nothing more than verify the association of the prosody of the specific conjunctions with a certain valence, or argument structure, and a corresponding semantics. This kind of operation is often characterized as type-raising, but in the deductive system embodied in (4), it is simply a by-product of the logic of implication elimination and introduction.

This leaves the rule for ↾ introduction to be considered. ↾ introduction differs from directional slash introduction in one foundational respect: rather than simply removing φ from the prosodic string, the variable becomes bound by an abstraction operator. This makes the resulting prosodic object a function, not a string, and when the prosody of a sign typed $Y \upharpoonright X$ composes with the prosody of a type X sign, the former takes the latter as an argument (as aptly illustrated by Figure 2). It is worth noting that the introduction rules represent a formal expression of Peirce's observation, quoted in Nöth (2000: 8): "in the proposition 'Anthony gave a ring to Cleopatra', Cleopatra is as much a subject of what is meant and expressed as is the ring or Anthony. A proposition, then, has one predicate and any number of subjects.". The significant insight here – that a sentence expressing a proposition can be composed as the ascription of some property to *any* of the argument terms – corresponds exactly to the possibility of deriving a predicate by composing a predicate with one variable term, with constants for all the other arguments, and then abstracting on that variable.

$$\dfrac{\dfrac{\dfrac{\dfrac{\text{gave}; \mathbf{give}; VP/PP_{to}/NP \quad \varphi; x; NP}{\text{gave} \circ \varphi; \mathbf{give}(x); VP/PP_{to}} \quad \dfrac{\vdots \quad \vdots}{\texttt{to} \circ \texttt{cleopatra}; \mathbf{cleop}; PP_{to}}}{\text{gave} \circ \varphi \circ \text{to} \circ \text{cleopatra}; \mathbf{give}(x)(\mathbf{cleop}); VP} \quad \text{antony}; \mathbf{ant}; NP}{\text{antony} \circ \text{gave} \circ \varphi \circ \text{to} \circ \text{cleopatra}; \mathbf{give}(x)(\mathbf{cleop})(\mathbf{ant}); S}}{\lambda\varphi.\text{antony} \circ \text{gave} \circ \varphi \circ \text{to} \circ \text{cleopatra}; \lambda x.\mathbf{give}(x)(\mathbf{cleop})(\mathbf{ant}); S \upharpoonright NP}$$

Figure 2: Variable introduction

The predicate in the final line of Figure 2 ascribes a property to some object; that object is in the set of things given to Cleopatra to Antony, and the proposition in the passage Noth quotes from Peirce is decomposable into λ-terms along these lines just as much as it is the composition of *Antony* with the denotation of the VP *gave a ring to Cleopatra*. In this sense, ↾ introduction is the logical warrant for Peirce's view that *Anthony gave a ring to Cleopatra* is "about" any (or all) of its "subjects", not just the NP which carries the grammatical function "subject".

But the empirical problem which led to ↾ introduction (under a different notation) in Oehrle (1994) was rather different, and took the form of the question, how can we capture the fact that quantified expressions such as *every student*, *some book*, *most journals* and so on have the same syntactic distribution as NPs, i.e., names and definite descriptions, while corresponding to radically different semantic objects? And, related to this question, is a second: how do quantifiers interact syntactically with the sentences they appear in such that they take scope over subportions of the semantic interpretation of those sentences? Various solutions have been proposed, e.g., the machinery introduced by Montague (1973), whereby all quantified expressions and names denote property sets, i.e., are functors on the properties denoted by the VPs that take them as syntactic subject, which require the use of meaning postulates; post-SpellOut movement operations ("Quantifier Raising", originating in May 1985) in the most recent incarnations of transformational grammar; Cooper's (1975, 1983) storage mechanism, adopted in Pollard & Sag (1994), and many others. In some cases the solutions involve formal devices that seem to be purpose-built for the description of quantifier's syntactic and semantic behavior, with little use outside the specific problem they were designed for, e.g., quantifier storage and retrieval; in others, there is is no connection to an actual model-theoretically accessible semantic denotation, as is the case with "Quantifier Raising" in transformational grammar and the Pollard & Sag (1994) proposal; and still others are problematic in both respects.

Oehrle's (1994) breakthrough, in contrast, is conceptually simple, of extremely broad application to problems of the syntax-semantics interface, and yields a directly interpretable expression in higher-order logic that is model-theoretically defined in a straightforward way. But this last point needs to be amplified: the basic approach is itself compatible with a wide range of explicit semantic frameworks, including proof-theoretic approaches that do not appeal to any model. Oehrle's key innovation was the application of a higher-order logic in the prosodic sector, with a corresponding type hierarchy, allowing the semantics and the prosody to operate independently of each other so that quantified expressions, and scopal operators generally, can in effect take the syntactic contexts in which they appear as their own arguments. The following simple example is representative of the setup generally. We have

(12) John gave someone that book.

We take the quantified expression *someone* to be a functor that intersects a property argument with the set of people, and returns a truth value of 0 if the intersection is ∅, and 1 otherwise. A proof along the lines of Figure 2 will directly yield the sign in (13):

(13) $\lambda\varphi$.john ∘ gave ∘ φ ∘ that ∘ book; $\lambda x.\mathbf{gave}(x)(\iota(\mathbf{book}))$; S↾NP

The semantics here is just what we need: the characteristic function of the set of entities who received some discoure-prominent (and in some sense pragmatically distal) member of the set of books from John. *Someone* intersects this set with that of people and, based on the model, returns a value of 0 or 1. But in that case, the pronunciation someone cannot itself be the prosody of *someone*, since in that case it would be an argument of the prosody in (13) despite *someone*'s semantics taking the latter's interpretation as its argument. Prosody and semantics would thus be at irreconcilable cross-purposes.

Oehrle's ingenious solution to this seeming contradiction takes the prosody of *someone* to be, not someone, but a function that applies the prosody of its S↾NP argument to someone. Since $\lambda\varphi$.john ∘ gave ∘ φ ∘ that ∘ book is a string-to-string function, *someone* is given a prosody which applies to such functions and positions them to take a string argument someone to the pronunciation of (12). The lexical entry for *someone* is then

(14) $\lambda\sigma.\sigma$(someone); $\lambda P.\exists(\mathbf{person})(P)$; S↾(S↾NP)

and we have the simple proof in Figure 3.

The quantified expression takes scope over the context in which it appears – its continuation, in Barker's terms (2002, 2004) (see also Barker & Shan 2015). If two quantified expressions are introduced into a single proof, the first one introduced into the proof will scope over the material included into the proof up to that point, and will then be part of the context which the second one scopes over when the latter is in turn added in the proof. A different proof, in which the two are introduced in the opposite order, will yield the opposite scoping. No special mechanism or operation is therefor required to obtain multiple scopings under the inference rules in (4) (see Kubota & Levine 2020: Section 2.3 for details).

Oehrle's solution to the parallelism of NP and quantifed expression distributions plays on the independent but linked relationship of prosody and semantics in type-logical grammar – a relationship made possible by the ↾ connective.

$$\dfrac{\dfrac{\begin{matrix}\vdots \\ \lambda\varphi.\ \mathtt{john} \circ \mathtt{gave} \circ \varphi \circ \mathtt{that} \circ \mathtt{book}; \\ \lambda x.\mathbf{gave}(x)(\iota(\mathbf{book}));\mathrm{S}{\upharpoonright}\mathrm{NP}\end{matrix} \qquad \begin{matrix}\vdots \\ \lambda\sigma.\sigma(\mathsf{someone}); \\ \lambda P.\exists(\mathbf{person})(P); \\ \mathrm{S}{\upharpoonright}(\mathrm{S}{\upharpoonright}\mathrm{NP})\end{matrix}}{\begin{matrix}\lambda\sigma\ [\sigma(\mathtt{someone})](\lambda\varphi.\ \mathtt{john} \circ \mathtt{gave} \circ \varphi \circ \mathtt{that} \circ \mathtt{book}); \\ \lambda P[\exists(\mathbf{person})(P)](\lambda x.\mathbf{gave}(x)(\iota(\mathbf{book}))); \mathrm{S}\end{matrix}}}{\dfrac{\begin{matrix}\lambda\varphi[\mathsf{john} \circ \mathsf{gave} \circ \varphi \circ \mathsf{that} \circ \mathsf{book}](\mathsf{someone}); \\ \exists(\mathbf{person})(\lambda x.\mathbf{gave}(x)(\iota(\mathbf{book}))); \\ \mathrm{S}\end{matrix}}{\begin{matrix}\mathsf{john} \circ \mathsf{gave} \circ \mathsf{someone} \circ \mathsf{that} \circ \mathsf{book}); \\ \exists(\mathbf{person})(\lambda x.\mathbf{gave}(x)(\iota(\mathbf{book}))); \\ \mathrm{S}\end{matrix}}\ \beta\text{-conversion}}\ \beta\text{-conversion}$$

Figure 3: Proof for *John gave someone that book.*

Quantified expressions parallel NPs precisely because they are in a sense parasitic on NP variables: they only appear in parts of the string where such variables can appear,b undergo abstraction and ultimately replacement by the string element in the prosody of quantifiers. At the same time, their syntax targets sentences which are "missing" NPs, in the sense that some argument position in the semantics is occupied by a λ-bound variable. These characteristics of $\upharpoonright$ play an essential role across a wide range of phenomena, one of which is considered in detail in Section 3 as a dramatic illustration of the way effects which require recourse to operations on phrase structure in other approaches can be reduced to mappings between valence values in type-logical grammar, with no need to posit syntactic configuration.

At this point, it's important that we take a step back from the technical details covered in this section in order to get a more global picture of the strategy embodied in an approach based on (4). The explicit correspondence between the implicational syntax and the operations of abstraction and function application in the semantics and prosody via independent type hierarchies with their own respective λ-calculi, guarantee a fully compositional derivation of signs, with the syntactic types guiding the composition on the basis of the familiar logic of modus ponens and hypothetical reasoning. The critical point here is that not only obviously local dependencies involving argument structure, but arbitrarily long-distance effects – in particular, the interpretation of quantifier scope – are reducible to the satisfaction of argument requirements; in effect, in the proof-theoretic architecture of type-logical frameworks, valence satisfaction is the source of all observed grammatical regularities, as well as constructional idiosyncrasies.[7] In the case of

[7]For a demonstration of how these eccentricities can be elegantly accounted for, see Kubota & Levine (2022).

scopal operators, such as generalized quantifiers – as well as symmetrical predicates such as *same*, *similar*, *different*, and various other varieties, the relationship between semantics and syntax is immediate and transparent: quantified expressions scope over the denotations of their syntactic arguments, in exactly the same way that modal auxiliaries and raising verbs scope over their VP arguments. In all cases, truth-conditional meaning is composed in accord with the valence of predicates and operators.

What about genuinely long-distance dependencies, of the sort exemplified by topicalization, wh-displacement, *tough* constructions and many others? These are standardly treated by machinery which "localizes" the dependency, but in neither derivational nor monostratal phrase-structure frameworks is the same mechanism employed for this localization as for garden-variety valence satisfaction. The point of the following analysis is to demonstrate the degree to which a proof-theoretic approach in which valence satisfaction, rather than syntactic configuration, yields the extraction dependency can capture the relevant phenomena in a simple and transparent fashion.

3 "Extraction" from ellipsis sites: What you don't see is what you don't get

3.1 The empirical problem

There is a sizable contemporary cross-linguistic literature on ellipsis, generally understood to refer to a varied range of phenomena in which semantic content from one part of a discourse context is part of the interpretation supplied by other(typically, but not necessarily, following) material, despite the absence of any overt phonology and syntax corresponding to that interpretation. We find, for example, patterns such as the following:

(15) a. John likes pizza, but Bill doesn't ∅ \
   'John likes pizza, but Bill doesn't like pizza.'
   (VP/Post-auxiliary ellipsis)

   b. John eats way more junk food than he does ∅ real food.
   'John eats way more junk food than he eats real food.'
   (Pseudogapping)

   c. John was arguing with someone, but I don't know who ∅ \
   'John was arguing with someone, but I don't know who John was arguing with.'
   (Sluicing)

d. Q: Who was John talking to?
A: ∅ Someone from his department.
'John was talking to someone from his department'
(Fragment answers)

There are a number of other subspecies of ellipsis, but those in (15) have had the lion's share of attention from theorists, most of whom appear to favor some version of the basic analytic line that originates in Kuno (1981) and has been most influentially developed in Merchant (2001) and subsequent work, whereby the interpretive glosses in (15) are, in essence, the syntactic sources of the examples themselves. (15a) on this approach arises from a series of processes that can be graphically summarized as something very much like (16):

(16) John likes pizza, but Bill doesn't [$_{\mathrm{VP}}$ ~~like pizza~~]

Pseudo-gapping, as in (15b), is the result of a movement to the left or right of the post-auxiliary "remnant" followed by the VP deletion process suggested in (17):

(17) John eats way more junk food than he does [$_{\mathrm{VP}}$ [$_{\mathrm{VP}}$ ~~eat~~ __] real food]

and so on. Most of the arguments in favor of this approach are necessarily indirect, based on patterns of acceptability judgments which seem to mirror judgments of corresponding non-ellipsed data; in Kubota & Levine (2020), a detailed examination of what appear to be the most persuasive of these arguments strongly suggests that they are in fact quite fragile on both empirical and methodological grounds. The central difficulty with such arguments is their pivotal assumption that the phenomena in ellipsis and corresponding non-ellipsed example which evoke parallel judgments of acceptability – e.g., island effects, restriction on anaphora, etc. – are themselves syntactic in nature. Building this assumption into any argument that parallel judgments of ellipsis and corresponding non-ellipsed data reflects the need to posit covert phrase structure which is deleted in the course of derivations thus appears to be a textbook instance of begging the question.

Defenders of the view that what you don't see in ellipsis was never there in the first place still have their work cut out for them, of course; it is necessary to construct plausibility arguments for the premises that (i) the putatively syntactic effects alluded to have non-syntactic origins and (ii) that the parallels between ellipsis and non-ellipsed examples can originate in the extragrammatical sources adduced in establishing (i). Examples such as the following are particularly challenging insofar as (i) is concerned;

(18) a. I know what John ate for lunch, but I don't know what Bill did.
b. I'm acutely aware of what I can do and what I can't. (Mahoney 2004: 735).
c. John is certain *he* would buy *this* kind of sports car, but I have no idea what kind *I* would.

Although examples of the sort displayed in (18) are not easy to discover in corpora, they can be found with a bit of persistence, though the third example is unattested (but has been checked with multiple informants, the great majority of whom found it altogether unproblematic with the right prosody (though the latter varied somewhat from speaker to speaker)). But there is one species of this class of *wh*-extractions, so called antecedent-contained deletion, which is quite common. Data parallel to (19) can readily be found in Google search results, for example.

(19) a. I hate feeling like everyone knows something I don't __.[8]
b. And perhaps they would nod with understanding at what a senior once told me: "Everyone knows something that I don't __. I keep asking until I find out what that is.".[9]
c. However, 4 months ago i said something which i shouldn't have.[10]

Dozens of such instances of the construction can be found in Google searches, and there is a very substantial literature on them. Versions of the sort shown in (18) are less well-studied, but there *has* been a certain amount of research devoted to them (see, for example, Schuyler (2002) and references there).

The problem for (i) is that whereas there is now a deep body of results constituting compelling evidence against the structural origins of island effects (for recent overviews of the relevant literature, see, e.g., Chaves & Putnam (2020), Kubota & Levine (2020), Liu et al. (2022)), most frameworks take filler-gap connectivity itself as irreducibly syntactic in nature. And while there are deep consequences that follow from rejecting movement operations as the source of extraction, this theoretical position does not, on its own, give us any particular help in explaining what the *wh*-word is doing in (18). In GPSG and its descendent HPSG, for example, a feature carrying information about the syntactic and semantic content of a *wh* constituent must be carried through the structure to the point where a category matching that content satisfies the valence requirements of a selecting

[8]https://twitter.com/therealkimj/status/1640857002896396288, 2024–03–14.
[9]https://www.ciomastermind.com/blog/the-arrogance-of-the-arrived, 2024–03–16.
[10]http://disq.us/p/1dhjjmu, 2024–03–16.

head. In the case of (18), the default analysis in these frameworks would license a connectivity linkage of this sort which would be "cashed out", as it were, by either an empty category corresponding to a valent of some transitive verb or, as in later work in HPSG assumed, in a reduction in the valence of such a verb (e.g., per the analyses of extraction patterns in van Noord & Bouma (1994) and Bouma et al. (2001)). And the entire "point" of VP ellipsis is that no such verb is present. Unsurprisingly, advocates of analyses based on covert-structure solutions to the problems posed by ellipsis seem to have been taken such examples as prima facie evidence for the presence of covert structure. Thus Johnson (2001) takes examples such as (18) to show that "the ellipsis site seems to have internal parts", while Elbourne agrees that "things seem especially difficult for [approaches to ellipsis] according to which there is nothing whatsoever in ellipsis sites" (Elbourne 2008: 216). So far as I am aware, there has to date been no account of the pattern exhibited in (18) in any work in the monostratal phrase-structure tradition that offers an explicit counteranalysis to the movement-and-deletion analysis assumed by transformationalists.

But such an alternative is readily available. It rests however on a particular approach to extraction connectivity and assumes a specific analysis of VP ellipsis, both of which differ considerably from standard positions shared by both transformational and monostratal frameworks. In the following section, I first outline a commonly assumed type-logical treatment of filler/gap linkage, and in the next section, recapitulate the treatment of VP ellipsis, and its generalization to pseudogapping, proposed in Kubota & Levine (2017). This background sets the stage for my account of (18).[11]

Muskens (2003) outlines a treatment of unbounded wh-dependencies, readily extendable to topicalization, which differs radically from previous analyses of extraction within both phrase-structure-based approaches and categorial grammar. In terms of wh-relatives, Muskens' proposal takes the form of the lexical sign in (20):

(20) $\lambda\sigma.\mathsf{which} \circ \sigma(\varepsilon)$; $\lambda P \lambda Q \lambda w.P(w) \wedge Q(w)$; $(\mathrm{N}\backslash\mathrm{N})\upharpoonright(\mathrm{S}\upharpoonright\mathrm{NP})$

Unpacking this operator a bit, we can see that its argument structure seeks a clause missing an NP, and its denotation is predicated of some entity, while the prosodic functor corresponding to the $\mathrm{S}\upharpoonright\mathrm{NP}$ argument applies to a string of length zero. To derive (21), then, we start with the subproof in Figure 4.

(21) the book which John lost yesterday

lost; $\lambda x \lambda y \mathbf{lose}(x)(y)$; $(NP\backslash S)/NP$ $\quad [\varphi_0; u; NP]^1$

lost ∘ φ_0; $\lambda y \mathbf{lose}(u)(y)$; $NP\backslash S$ $\quad$ yesterday; $\lambda P \lambda v.yst(P)(v)$; $(NP\backslash S)\backslash(NP\backslash S)$

`lost ∘` φ_0 `∘ yesterday;` $\lambda v.\mathbf{yst}(\mathbf{lose}(u))(v)$; $NP\backslash S$ $\quad$ `john;` $\mathbf{j}$; NP

john ∘ lost ∘ φ_0; $\mathbf{yst}(\mathbf{lose}(u))(\mathbf{j})$; S

①→ $\lambda\varphi_0$.john ∘ lost ∘ φ_0 ∘ yesterday; $\lambda u.\mathbf{yst}(\mathbf{lose}(u))(\mathbf{j})$; $S{\upharpoonright}NP$

Figure 4: Relative clause subproof 1

The operator in (20) takes arguments of this type and returns a function which picks up an N on the left, while β-converting a zero-length string into the position occupied by φ_0 in the last proof line in Figure 4, giving us *which John lost yesterday.*

`λσ. which ∘ σ(ε);` $\lambda P \lambda Q \lambda w.P(w) \wedge Q(w)$; $(N\backslash N){\upharpoonright}(S{\upharpoonright}NP)$ $\quad$ `λ`φ_0`. john ∘ lost ∘` φ_0 `∘ yesterday;` $\lambda u.\mathbf{yst}(\mathbf{lose}(u))(\mathbf{j})$; $S{\upharpoonright}NP$

`λσ [which ∘ σ(ε)](λ`φ_0`. john ∘ lost ∘` φ_0 `∘ yesterday);` $\lambda P[\lambda Q \lambda x.P(w) \wedge Q(w)](\lambda u.\mathbf{yst}(\mathbf{lose}(u))(\mathbf{j}))$; $N\backslash N$

which ∘ $\lambda\varphi_0$[john ∘ lost ∘ φ_0 ∘ yesterday](ε); $\lambda Q.[\lambda w.\mathbf{yst}(\mathbf{lose}(w))(\mathbf{j}) \wedge Q(w)]$; $N\backslash N$

which ∘ john ∘ lost ∘ ε ∘ yesterday; $\lambda Q[\lambda w.\mathbf{yst}(\mathbf{lose}(w))(\mathbf{j}) \wedge Q(w)]$; $N\backslash N$

Figure 5: Relative clause subproof 2

The final part of the proof supplies an N argument to the functional term in the last proof line in Figure 5.

book; $\mathbf{book}$; N $\quad$ which ∘ john ∘ lost ∘ ε ∘ yesterday; $\lambda Q[\lambda w.\mathbf{yst}(\mathbf{lose}(w))(\mathbf{j}) \wedge Q(w)]$; $N\backslash N$

book ∘ which ∘ john ∘ lost ∘ ε ∘ yesterday; $\lambda w.\mathbf{yst}(\mathbf{lose}(w))(\mathbf{j}) \wedge \mathbf{book}(w)]$; N

Figure 6: Relative clause subproof 3

[11] For a rather different, though ultimately related approach to a solution in a framework belonging to a distinct class of categorial grammar frameworks, see Jacobson (1992).

We thus obtain *book which John lost*, denoting the set of things which have the properties of being books and being objects that John lost.

The critical point for us is what happens at ① in Figure 4. Application of ↾ Introduction abstracts on the variable terms superscripted as 1 – an operation completely indifferent to the length of the string in which the variable φ_0 appears. Exactly the same step would take us from the expression above the proof line in Figure 7 to the sign below the line.

$$\frac{\texttt{mary}\circ\texttt{thinks}\circ\texttt{bill}\circ\texttt{remembers}\circ\texttt{ann}\circ\texttt{saying}\circ\texttt{john}\circ\texttt{lost}\circ\varphi;\ \mathbf{think}(\mathbf{remember}(\mathbf{saying}(\mathbf{yest}(\mathbf{lost}(u))(\mathbf{j}))(\mathbf{a})))(\mathbf{b})))(\mathbf{m});\ \mathrm{S}}{\lambda\varphi.\ \texttt{mary}\circ\texttt{thinks}\circ\texttt{bill}\circ\texttt{remembers}\circ\texttt{ann}\circ\texttt{saying}\circ\texttt{john}\circ\texttt{lost}\circ\varphi;\ \mathbf{think}(\mathbf{remember}(\mathbf{saying}(\mathbf{yest}(\mathbf{lost}(u))(\mathbf{j}))(\mathbf{a})))(\mathbf{b})))(\mathbf{m});\ \mathrm{S}}$$

Figure 7: Long-distance relative clause subproof

Essentially the same proof storyline in Figures 4–6 will give us *book which Mary thinks Bill remembers Ann saying John lost.* There is no local registration of the information linking the filler to the gap – nothing analagous to cyclic wh-movement, no SLASH feature shared between vertically adjacent nodes in a phrase-structure tree that gets realized at the tail end of the chain. Properly speaking, there isn't even anything that can be properly identified as a gap "site". We have a prosodic component of the sign with no marker corresponding to some missing substring, since the model theory for the prosodic calculus interprets $\texttt{a}\circ\texttt{b}\circ\varepsilon$ as $\texttt{a}\circ\texttt{b}$; nor is there any representation in the semantics or the syntactic type of something we would want to call a "gap". In a way, this treatment of extraction is an echo of the view in extraction in the earliest phase of transformational grammar, when wh-movement shifted a constituent to the left over an unconstrained variable. The appearance of Ross (1967) resulted in the almost universal rejection of this view, but the most recent research on the island effects that Ross first documented, as noted earlier, overwhelmingly supports a view of such effects which takes them to be epiphenomena of functional factors. Clearly, the nonlocal view of syntactic connectivity has an empirical claim on a second act.[12]

[12]This is not to say, of course, that Muskens' operator is completely unproblematic. For one thing, it has an obvious failure in its coverage, since obviously there's no way that (20) as given accounts for pied-piping. A second problem is that the linearity of the type logic shared by Muskens' λ-grammar and our own HTLG, inter alia, makes it difficult to derive multiple

3.1.1 VP ellipsis and pseudogapping

The correct explanation for data such as (18) obviously depends on an empirically sound analysis of VP ellipsis in the first place. The standard transformational approaches following Kuno (1981) face severe empirical challenges and serious conceptual problems, detailed in Kubota & Levine (2017). These problems are avoided in the proof-theoretic solution proposed there, whose central premise is that VP ellipsis itself is the expression of a kind of "zero derivation" whereby signs typed VP/VP are mapped to the type VP, whose denotation is the application of the modal/aspectual operator of the input sign to some salient property retrieved from the discourse context or, under certain conditions, inferred exophorically, per Miller & Pullum (2013). This approach is implemented via the operator in (22), where $ is a variable over sequences of arguments, following notation introduced in Steedman (2000):[13]

(22) VP ellipsis operator
$\lambda\varphi.\varphi$; $\lambda\mathcal{F}.\mathcal{F}(P')$; $((NP\backslash S)\$)\upharpoonright(((NP\backslash S)\$)\upharpoonright((NP\backslash S)\$)$
– where P' is a free variable whose value is resolved anaphorically

(23) Anaphora resolution condition on the VP ellipsis/pseudogapping operator:
1. if there is a syntactic constituent with category VP in the antecedent clause matching the syntactic category of the missing verb in the target clause, then the value of *P* is identified with the denotation of that constituent;
2. if there is no such syntactic constituent, then the value of *P* is anaphorically identified with some salient property in the discourse that is not inconsistent with the syntactic category VP.

An example of simple VP ellipsis, illustrating how the ellipsis operator in (22) works, is given in (24).

(24) Mary should call Ann, but Bill shouldn't.

At the grayed-in proof line in Figure 8, the free variable *P* is instantiated as the prominent contextually available property **call**(**a**).

extractions linked to a single filler. In Kubota & Levine (2020), we offer solutions for both problems, and are currently generalizing our proposal for pied-piping to take into account the interaction of the latter with a variety of coordination possibilities, an aspect of the pied-piping problem that does not appear to have been previously addressed. But for our purposes, the approach exemplified in (20) is completely serviceable.

[13] Because the prosodic term is a function, the main connective in the type description is ↾ rather than /.

```
                    call;              ann;
                    call;              a;
                    (NP\S)/NP          NP
               ①→  ─────────────────────────
should ;                 call∘ann;
λQλy.□Q(y);              call(a);
(NP\S)/(NP\S)            NP\S
──────────────────────────────────────── /E
        should∘call∘ann ;
        λy.□ call(a)(y);
        NP\S                                   mary; m; NP
──────────────────────────────────────────────────────────── \E
                 mary∘should∘call∘ann ;
                 □ call(a)(m);
                 S
```

```
            λφ. φ ;                        shouldn't ;
bill ;      λℱ.ℱ(call(a));                 λPλy.¬□P(y);
b;          (NP\S)↾(NP\S/NP\S)             (NP\S)/(NP\S)
NP         ──────────────────────────────────────────────── ↾E
                shouldn′t; λy.¬□call(y)(b); NP\S
───────────────────────────────────────────────────── \E
        bill ∘ shouldn,′ t; ¬□call (a)(b); S
```

Figure 8: VP ellipsis proof

More complex cases, e.g. those involving sloppy identity (*John thinks he deserves a promotion, and Bill does too*) and scopal operators (*John read every book before Bill did*) fall out altogether straightforwardly on this approach, as shown in Kubota & Levine (2017: 236–238). But for our purposes, what is relevant is the fact that the operator in (22) applies to a functional term taking a complete NP\S to a complete NP\S – which we can abbreviate as VP/VP – and returns a complete VP. Suppose now that we generalize the operator so that it applies to a functional term taking a *partial* VP to a partial VP, and returns a partial VP. This seems perhaps like a question completely orthogonal to the phenomena we're looking at, because auxiliaries are, in non-transformational frameworks generally, taken to apply to VPs and return VPs, period. But it is a strict theorem of our proof theory that every VP/VP type has a prosodically identical counterpart which applies to VP/NP objects and returns a VP/NP object – i.e., maps a transitive verb to a transitive verb. This is nothing more than a conversion into type logic of one consequence of the transitivity of implication in standard logics, and is simply demonstrated as in Figure 9 (where ○ is a variable over arbitrary operators).

$$\dfrac{\dfrac{\dfrac{\dfrac{[\varphi_2; R; \mathrm{VP/NP}]^2 \qquad [\varphi_1; u; \mathrm{NP}]^1}{\varphi_2 \circ \varphi_1; R(u); \mathrm{VP}}/\mathrm{E} \qquad a; \lambda T.\lambda z. \bigcirc T(z); \mathrm{VP/VP}}{a \circ \varphi_2 \circ \varphi_1; \lambda z. \bigcirc\ R(u)(z)); \mathrm{VP}}/\mathrm{E}}{a \circ \varphi_2; \lambda u \lambda z. \bigcirc\ R(u)(z); \mathrm{VP/NP}}/\mathrm{I}^1}{a; \lambda R \lambda u \lambda z. \bigcirc\ R(u)(z); \mathrm{(VP/NP)/(VP/NP)}}/\mathrm{I}^2$$

Figure 9: Type-logical Geach theorem proof

The point is that VP/VP ⊢ (VP/NP)/(VP/NP), and a completely parallel entailment can be proven between VP/VP and terms of type VP/NP/NP, VP/PP/NP, etc. In general, then, for any auxiliary, we have an entailment VP/VP ⊢ VP$/VP$. It follows that if we generalize the VP ellipsis operator to the type VP$↾(VP$/VP$), we derive an operator that yields a form of the auxiliary as a transitive verb, a ditransitive verb and so on. And such an operator enables us to extend the coverage of the VP ellipsis rule to the pseudogapping phenomenon illustrated in (15b) above; to evade the complexities of the comparative semantics, I use the somewhat less natural (though still typically acceptable) *but*-conjunction in (25):

(25) For some reason, John will read ESSAYS but he won't NOVELS.

Generalizing the VP ellipsis operator to the form in (26) would have the effect of taking *won't*, typed (VP/NP)↾((VP/NP)/(VP/NP)) (via application of the Geach entailment, with $ = NP) to an auxiliary typed VP/NP, i.e., a transitive verb. This revised operator can be stated as in (26):

(26) Generalized ellipsis operator
$\lambda\varphi.\varphi$; $\lambda\mathcal{F}.\mathcal{F}(P)$; VP$↾(VP$/VP$)
– where *P* is a free variable whose value is resolved anaphorically

(27) Anaphora resolution condition on the VP ellipsis/pseudogapping operator:
1. if there is a syntactic constituent with category VP$ in the antecedent clause matching the syntactic category of the missing verb in the target clause, then the value of *P* is identified with the denotation of that constituent;
2. if there is no such syntactic constituent, then the value of *P* is anaphorically identified with some salient property in the discourse that is not inconsistent with the syntactic category VP$.

We can now derive Figure 10 directly.

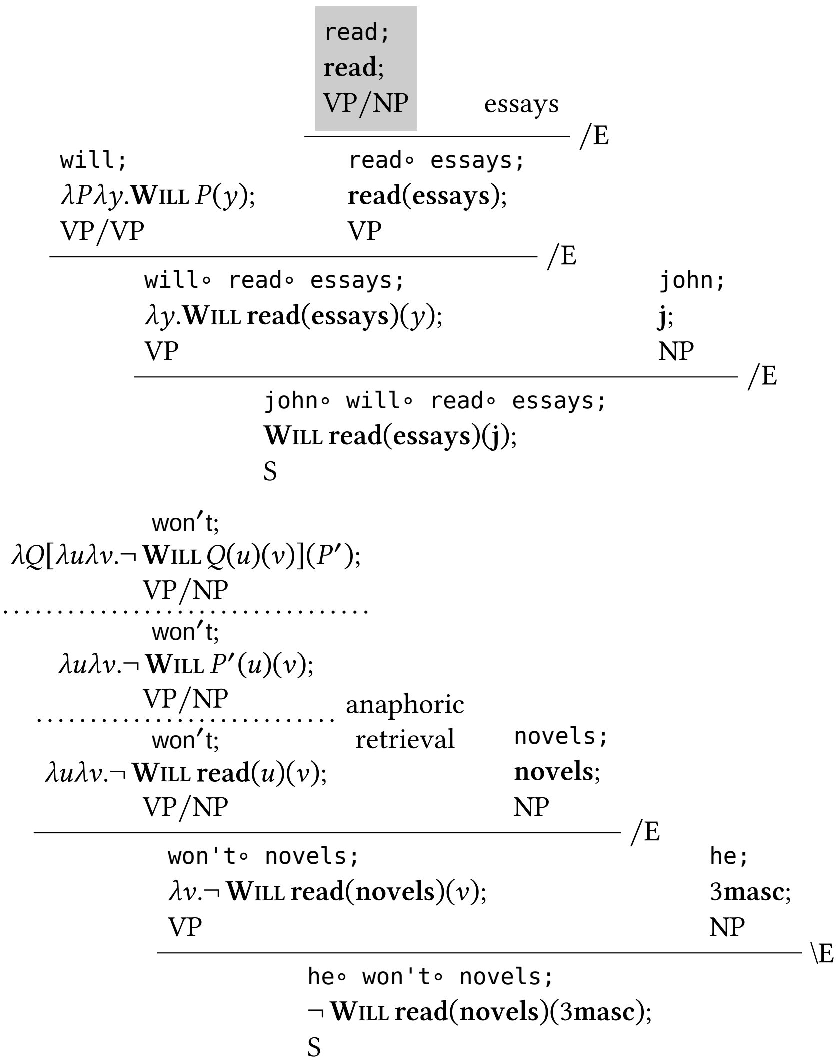

Figure 10: Pseudogapping proof

Intuitively, the application of the generalized operator in (26) is to repackage an auxiliary and a transitive verb as a somewhat longer and more complex transitive verb, so that rather than composing *read novels* as a VP and applying a standardly typed auxiliary to derive a VP, we in effect repackage *won't* and *read* as a transitive verb *won't*, whose type is the same as *read* itself, but which, after the retrieval of the corresponding predicate in the antecedent clause, applies the semantics of *won't* to the proposition derived by supplying this transitive verb with its arguments.

With our generalized account of ellipsis, we are now in a position to see how the proof-theoretic approached introduced in the preceding sections can license examples such as (18) without recourse to any actual material corresponding to the gap "site" in the antecedent clause ever being involved.

3.1.2 Pseudo-extraction via pseudogapping

In the analysis that follows, apparent extraction from an ellipsed VP arises as a result of Muskens-style extraction from one or another argument of the "transitive" auxiliary which is associated with the general ellipsis operator introduced in Section 3.1.1. That is, examples such as (18) involve not just a semantic object, as in purely semantic accounts of ellipsis (e.g., that given in Hardt 1993) analysis, but an actual syntactic extraction from an ordinary overt VP, as we show below. Treatments such as Hardt's, or that given in Dalrymple et al. (1991), have, as noted in the citation above from Elbourne (2008), a difficult time accounting in a simple way for cases such as (18); under the analysis which follows, in contrast, these constructions are predicted to conform to whatever conditions hold on extraction in general without any concomitant assumption of covert structure corresponding to an ellipsed VP.

What is distinctive about the filler-gap relationship, as vs. the standard picture of valence, is that while in both cases we have material that is missing other material required to compose a constituent, in the case of the former, the gap can be missing from anywhere within the partial constituent. That is, while Y/X is a sign that must compose on the left with a sign of type X, and $X\backslash Y$ is the same but seeking an X argument on its left to yield an object typed Y, the material missing from the string that is required for Y in $Y \upharpoonright X$ can, as noted in Section 2, appear anywhere. Thus, in *I wonder what John said to Mary*, the subconstituent *said* __ *to Mary* constitutes a VP with a medial NP gap, meeting the description $\mathrm{VP}\upharpoonright\mathrm{NP}$. In terms of sentences such as (18), what we want is a way to get *did* to have the type $\mathrm{VP}\upharpoonright\mathrm{NP}$, in which case we would, roughly speaking, apply a Muskens-style operator *what* to a clause composed from this $\mathrm{VP}\upharpoonright\mathrm{NP}$. As I show directly, given a sign *did* typed VP↾NP, we can use hypothetical reasoning to deduce S↾NP and then apply the *what* operator to obtain signs of expressions such as *what John did*. Furthermore, we predict on such an approach the well-formedness of e.g.,

(28) a. Do you think the British know something (about this) that we don't (at this point)? (Penn Treebank/Wall Street Journal corpus, cited in Bos & Spenader (2011), slightly modified)

b. Kollberg suspects Petrus, who Beck does __ as well (Kennedy 1997: 666)

(26) will not do the trick here, since it only gives us the possibility of elements missing on the right, not medially. What's needed, clearly, is some way to extend the generalized ellipsis rule still further. Fortunately, just as we were able to show that terms typed VP/VP can, by the Geach theorem proof given in Figure 9, be extended to the type (VP/NP)/(VP/NP), we can prove that for any term inhabiting VP/VP, there is a corresponding term with functional prosody having the schematic form (VP↾XP)↾(VP↾XP) for any type XP. The structure of the proof is essentially the same as that of Figure 9, but involving higher order terms.[14]

(29)

$$\dfrac{\dfrac{\dfrac{\varphi_1; \mathcal{O}; \mathrm{VP/VP} \qquad \dfrac{[\sigma_1; f; \mathrm{VP}\upharpoonright\mathrm{NP}]^1 \qquad [\varphi_2; x; \mathrm{NP}]^2}{\sigma_1(\varphi_2); f(x); \mathrm{VP}}}{\varphi_1 \circ \sigma_1(\varphi_2); \mathcal{O}(f(x)); \mathrm{VP}}}{\lambda\varphi_2.\varphi_1 \circ \sigma_1(\varphi_2); \lambda x.\mathcal{O}(f(x)); \mathrm{VP}\upharpoonright\mathrm{NP}}\upharpoonright\mathrm{I}^2}{\lambda\sigma_1\lambda\varphi_2.\varphi_1 \circ \sigma_1(\varphi_2); \lambda f\lambda x.\mathcal{O}(f(x)); (\mathrm{VP}\upharpoonright\mathrm{NP})\upharpoonright(\mathrm{VP}\upharpoonright\mathrm{NP})}\upharpoonright\mathrm{I}^1$$

With this result in hand, all that is needed to derive any given auxiliary as a VP seeking a gap-filling NP constituent *somewhere* is a further extension of the already-generalized ellipsis operator to such "vertically Geached" auxiliaries, mapping them to type VP↾XP, anaphorically supplying the meaning of the gapped VP. In (30), I give a "local" form of this extension of the ellipsis operator to internal gaps.

(30) $\lambda\rho\lambda\varphi_1.\rho(\lambda\varphi_0.\varphi_0)(\varphi_1); \lambda\mathcal{F}.\mathcal{F}(R'); (\mathrm{VP}\upharpoonright\mathrm{NP})\upharpoonright((\mathrm{VP}\upharpoonright\mathrm{NP})\upharpoonright(\mathrm{VP}\upharpoonright\mathrm{NP}))$
– where R' is the semantic term of a sign retrieved from the context whose type is VP↾NP

As before, we first specify how the antecedent clause of (31) makes available the predicate which is retrieved in the ellipsed clause, per Figure 11.

(31) I know what John ate for lunch, but I don't know what$_i$ Bill did ~~eat~~ $\_\__i$ ~~for lunch~~.

$$[\varphi_1; x; \mathrm{NP}]^1$$
$$\vdots \quad \vdots$$
$$\texttt{ate} \circ \varphi_1 \circ \texttt{for} \circ \texttt{lunch};\ \mathbf{ate}(x)(\mathbf{lunch}); \mathrm{VP}$$
① → $\overline{\lambda\varphi_1.\texttt{ate} \circ \varphi_1 \circ \texttt{for} \circ \texttt{lunch};\ \lambda x.\mathbf{ate}(x)(\mathbf{lunch});\ \mathrm{VP}\upharpoonright\mathrm{NP}}\ \upharpoonright\mathrm{I}^1$ $\quad \begin{bmatrix}\varphi_2\\ u;\\ \mathrm{NP}\end{bmatrix}^2$

$$\overline{\mathsf{ate} \circ \varphi_2 \circ \mathsf{for} \circ \mathsf{lunch};\ \mathbf{ate}(u)(\mathbf{lunch});\ \mathrm{VP}} \qquad \texttt{john};\ \mathbf{j};\ \mathrm{NP}$$
$$\overline{\mathsf{john} \circ \mathsf{ate} \circ \varphi_2 \circ \mathsf{for} \circ \mathsf{lunch};\ \mathbf{ate}(u)(\mathbf{lunch})(\mathbf{j});\ \mathrm{S}}$$
$$\overline{\lambda\varphi_2.\mathsf{john} \circ \mathsf{ate} \circ \varphi_2 \circ \mathsf{for} \circ \mathsf{lunch};\ \lambda u.\mathbf{ate}(u)(\mathbf{lunch})(\mathbf{j});\ \mathrm{S}\upharpoonright\mathrm{NP}}\ \upharpoonright\mathrm{I}^2 \qquad \lambda\sigma.\ \texttt{what} \circ \sigma(\varepsilon);\ \lambda P.\mathbf{what}(P);\ \mathrm{Q}\upharpoonright(\mathrm{S}\upharpoonright\mathrm{NP})$$
$$\overline{\mathsf{what} \circ \mathsf{john} \circ \mathsf{ate} \circ \varepsilon \circ \mathsf{for} \circ \mathsf{lunch};\ \mathbf{what}(\lambda u.\mathbf{ate}(u)(\mathbf{lunch})(\mathbf{j}));\ \mathrm{Q}}$$

Figure 11: Ellipsis pseudo-extraction antecedent proof

The grayed-in semantic term in Figure 11 is an available predicate with which the free variable R' obtained in the proof line ① can be anaphorically identified. The first part of the proof for *what Bill did* then takes the following form:

$$\vdots \quad \vdots$$
$$\lambda\sigma\ \lambda\varphi.\ \texttt{did} \circ \sigma(\varphi);\ \lambda f\lambda x\lambda y.f(x)(y);\ (\mathrm{VP}\upharpoonright\mathrm{NP})\upharpoonright(\mathrm{VP}\upharpoonright\mathrm{NP}) \qquad \lambda\rho\ \lambda\varphi.\ \rho(\lambda\varphi_0.\ \varphi_0)(\varphi);\ \lambda\mathcal{F}.\mathcal{F}(\lambda x.\mathbf{ate}(x)(\mathbf{lunch}));\ (\mathrm{VP}\upharpoonright\mathrm{NP})\upharpoonright((\mathrm{VP}\upharpoonright\mathrm{NP})\upharpoonright(\mathrm{VP}\upharpoonright\mathrm{NP}))$$
$$\overline{\lambda\varphi.\mathsf{did} \circ \varphi;\ \lambda x\lambda y.\mathbf{ate}(x)(\mathbf{lunch})(y);\ \mathrm{VP}\upharpoonright\mathrm{NP}} \qquad \begin{bmatrix}\varphi_3\\ v;\\ \mathrm{NP}\end{bmatrix}^3$$
$$\overline{\mathsf{did} \circ \varphi_3;\ \lambda y.\mathbf{ate}(v)(\mathbf{lunch})(y);\ \mathrm{VP}} \qquad \texttt{bill};\ \mathbf{b};\ \mathrm{NP}$$
$$\overline{\mathsf{bill} \circ \mathsf{did} \circ \varphi_3;\ \mathbf{ate}(v)(\mathbf{lunch})(\mathbf{b});\ \mathrm{S}}$$
$$\overline{\lambda\varphi_3.\mathsf{bill} \circ \mathsf{did} \circ \varphi_3;\ \lambda v.\mathbf{ate}(v)(\mathbf{lunch})(\mathbf{b});\ \mathrm{S}\upharpoonright\mathrm{NP}}\ \upharpoonright\mathrm{I}^3$$

Figure 12: Ellipsis pseudo-extraction 'gap' site

The term obtained at the last step of this proof, supplied as an argument to the extraction operator, yields an interpretation identical to the unellipsed embedded question *what Bill ate for lunch*. Note that the prosodic term derived in the last proof step, $\lambda\varphi_3.\mathsf{bill} \circ \mathsf{did} \circ \varphi_3$, is exactly what we would have obtained via the earlier version of the generalized ellipsis operator; the associated type would however been S/NP, and therefore ineligible to compose with *what*. Moreover, as noted above, only the vertically-slashed version of the ellipsis operator would allow us to derive a sentence with a non-peripheral "gap" as in (28). But the larger point is that long-distance dependencies into what appear to be ellipsis contexts are, on this analysis, based on what is in effect the extraction of a pseudogapping

[14] In (29), I gloss over certain important technical details in order to lay out most clearly the proof narrative.

remnant. For example, a proof along the lines of that began along the lines of (12) might have continued as in Figure 13.

$$\dfrac{\dfrac{\dfrac{\begin{array}{c}\vdots\quad\vdots\\ \lambda\sigma\ \lambda\varphi.\ \mathtt{did}\circ\sigma(\varphi);\\ \lambda f\lambda x\lambda y.f(x)(y);\\ (VP{\upharpoonright}NP){\upharpoonright}(VP{\upharpoonright}NP)\end{array}\quad \begin{array}{c}\lambda\rho\ \lambda\varphi.\ \rho(\lambda\varphi_0.\ \varphi_0)(\varphi);\\ \lambda\mathcal{F}.\mathcal{F}(\lambda x.\mathbf{ate}(x)(\mathbf{lunch}));\\ (VP{\upharpoonright}NP){\upharpoonright}\\ ((VP{\upharpoonright}NP){\upharpoonright}(VP{\upharpoonright}NP))\end{array}}{\lambda\varphi.\text{did}\circ\varphi;\ \lambda x\lambda y.\mathbf{ate}(x)(\mathbf{lunch})(y);\ VP{\upharpoonright}NP}\quad \begin{array}{c}\text{breakfast};\\ \mathbf{brkfst};\\ NP\end{array}}{\text{did}\circ\text{breakfast};\ \lambda y.\mathbf{ate}(x)(\mathbf{brkfst})(y);\ VP}\quad \begin{array}{c}\text{he};\\ 3masc;\\ S\end{array}}{\text{he}\circ\text{did}\circ\text{breakfast};\ \mathbf{ate}(\mathbf{brkfst})(3masc);\ S}$$

Figure 13: Non-extraction pseudogapping

This would then be an ordinary instance of pseudogapping as in *John ate lunch much faster than he did breakfast.* The upshot is that apparent extraction from ellipsis sites as in (18) is nothing other than the interaction of Muskens-style wh-operators with the object of a transitive auxiliary – a possibility that we would predict in advance on the analysis given above.

The reader might suppose that the possibility of this kind of extraction depends on some kind of parallel interpretation between the antecedent and the ellipsed clauses in (18), based on the extraction already visible in the former. But we also have examples where there is no extraction in the antecedent, such as (28) and (32):

(32) John is certain he would buy *this* kind of sports car, but I have no idea what kind *I* would.

To obtain such examples, we derive the antecedent by a derivation which includes the subproof in Figure 14.

From this point on, the proof for the ellipsed clause would proceed in exactly the same fashion as in the derivation of (31), with the free variable P' instantiated as the grayed-in term in (14).

The above (re)analysis of "extraction out of an elided VP" as extraction of a pseudogapping remnant gives us, in effect, a proof-of-concept argument for rejecting the assumption that covert structures in VP ellipsis necessarily exist in order that a "site of origin" be available for filler/gap linkages that appear to implicate material missing from deleted VPs.[15] There is, on the analysis presented

[15]While this approach has been challenged in Johnson (2001), on the grounds that apparent extraction from ellipsis sites is subject to different constraints from pseudogapping, counterexamples to his claims are already familiar from, inter alia, examples from corpora or naturally occurring data presented in Levin (1979). For detailed discussion of this point, see Kubota & Levine (2020: Section 8.4.2).

$$
\dfrac{
  \dfrac{
    \dfrac{
      \begin{array}{c}\mathsf{buy};\\ \mathbf{buy};\\ VP/NP\end{array}
      \quad
      \begin{array}{c}\varphi_1;\\ x;\\ NP\end{array}
    }{
      \begin{array}{c}\mathtt{buy}\circ\varphi_1;\\ \mathbf{buy}(x);\\ VP\end{array}
    }
  }{
    \begin{array}{l}\lambda\varphi_1.\ \mathtt{buy}\circ\varphi_1;\\ \lambda x.\mathbf{buy}(x);\\ VP{\restriction}NP\end{array}
  }
  \quad
  \begin{array}{l}\qquad\vdots\quad\vdots\\ \mathtt{this}\circ\ \mathtt{kind}\circ\ \mathtt{of}\circ\ \mathtt{sports}\circ\ \mathtt{car};\\ \iota(\mathbf{kind}(\mathbf{spcr}));\\ NP\end{array}
}{
  \begin{array}{l}\mathtt{buy}\circ\ \mathtt{this}\circ\ \mathtt{kind}\circ\ \mathtt{of}\circ\ \mathtt{sports}\circ\ \mathtt{car};\\ \mathbf{buy}(\iota(\mathbf{kind}(\mathbf{spcr})));\\ VP\end{array}
}
\quad
\begin{array}{l}\mathtt{would};\\ \lambda P\lambda y.\mathbf{W}\textsc{d}\,P(y);\\ VP/VP\end{array}
$$

$$
\overline{\begin{array}{l}\mathtt{would}\circ\ \mathtt{buy}\circ\ \mathtt{this}\circ\ \mathtt{kind}\circ\ \mathtt{of}\circ\ \mathtt{sports}\circ\ \mathtt{car};\\ \lambda y.\mathbf{W}\textsc{d}\,\mathbf{buy}(\iota(\mathbf{kind}(\mathbf{spcr})))(y);\\ VP\end{array}}
$$

Figure 14: Pseudo-extraction antecedent without parallel extraction

in this section, no extraction from a subsequently deleted (or phonologically suppressed) subpart of some structural arrangement of linguistic expressions, as in a phrase structure tree. Rather, an auxiliary is licensed whose type and semantics correspond to a VP missing an NP, and which composes by hypothetical reasoning to the type of a clause missing an NP. A wh-operator along the lines proposed by Muskens can then take this clause as an argument. The appearance in (18) of an extraction from a subsequently ellipsed constituent is, on this view, a illusion due to the string-identity of a VP ellipsis on the one hand and displacement of a pseudogapping remnant on the other.

4 Conclusion: Peirce's linguistics, logic, and mathematics and the sources of type logical grammar

It is important at this point to consider how the results reported above have been achieved. Fundamentally, treatment of syntactic categories as valence specifications means that grammatical rules and operations can map the combinatorial possibilities of signs to different possibilities without ever requiring those possibilities to be realized as actual structures e.g., the operators for auxiliary typeshifting given above. But just as basic to this kind of solution is the fact that in type-logical systems, the "categories" of phrase structure grammar are replaced by types which specify the argument requirements of their own arguments. The *what* operator discussed above can apply to a sign typed S↾NP, an object itself seeking an NP to yield a clause of arbitrary depth. Since on the analysis in Section 3.1.2 the auxiliary *did* in (18) is a VP missing an NP and thus, by hypothetical reasoning, *Bill did* is an S missing an NP, a wh-operator such as *what* can take the latter as an argument without there ever having been any material in its licensing corresponding to the transitive verb *eat* per (31). The interpretation of (18) involves the sign *eat* only in the antecedent; in the ellipsis clause, the predicate **eat** is understood in the meaning only as a result of anaphoric retrieval from the antecedent clause. The heavy lifting in this proposal is carried out entirely by valence-shifting operators and the treatment of extraction as just one more instance of a dependency mediated by valence satisfaction.

The possibilities of this kind of framework depend on a RESIDUATED logic, i.e., a logic in which the connectives, viewed as type-constructors, have the property that, in the notation of classical implication (but necessarily modulo the directionality of the type-constructor slashes), and with $\Longleftrightarrow$ denoting metalogical equivalence, is shown in (33):

(33) $(\psi \vdash \psi \supset \varrho) \Longleftrightarrow (\psi, \phi \vdash \varrho) \Longleftrightarrow (\phi \vdash \psi \supset \varrho)$

(For detailed discussion, see Restall 2018). Residuation is a property of the type-constructors /,\ introduced in Lambek (1958), for all practical purposes the founding document of contemporary type-logical formalisms, and so far as type-logic is concerned, can be understood in the following way: there is a natural relationship between the entailment/equivalence relations in (33), whereby if inhabiting a given type τ_1 entails inhabiting some other type τ_2, then $\tau_1 \vdash \tau_2$, i.e., $\vdash \tau_1 \to \tau_2$. Suppose that, given two types A, B, we can compose each member of A with each member of B to yield a term belonging to type C, i.e., $A \bullet B \vdash C$. Then necessarily every member of A belongs to the set of terms which form a member of C when they compose with a member of B on the right; if we call this set C/B then $A \vdash C/B$, and likewise for B. We thus have the relations

(34) $(A \bullet B \vdash C) \Longleftrightarrow (A \vdash C/B) \Longleftrightarrow (B \vdash A\backslash C)$

(34) is nothing more than the residuated implication relationship of standard logic displayed in sequent notation. But as discussed at length in Pratt (1992), Peirce himself developed a theory of binary relations that incorporated the key components of residuated relationships between terms, including a kind of proto-version of the left and right "division" relations that, per (34), are formally entailed by each of the arguments of the type composition operator • (and which are

in essence the upper adjoints of the monotone Galois connection which frames residuation in terms of partial orderings).[16] As Pratt notes, the upper and lower adjoint operators are effectively the functions corresponding to the composition and division connectives (which Peirce wrote with a semicolon and a horizontal-line fraction notation respectively).

It seems fair to say, then, that – to extend Peirce's original chemical metaphor only slightly – we can plausibly view Lambek's seminal work in his 1958 and 1961 papers as the reaction product of an imagined catalyst bonding Peirce's ideas about valence as the basis of linguistic combinatorics to his work on the algebra of relations. Any doubt about the correctness of such a view should be immediately dispelled by Lambek's own words; in one of his papers on pregroup grammars – an algebraic reformulation of type-logical grammar he developed in order to make transparent the logical foundation of his earlier systems as instances of (a fragment of) intuitionistic noncommutative linear logic – he comments of a very basic skeleton for the pregroup grammar formalism that the essential combinatorics "may even be implicit in the ideas of C.S. Peirce [i.e. Peirce (1897) – RDL]", noting that certain combinators in this "rudimentary" version may have been seen by Peirce as comparable to "the unsaturated bonds of an atom. *I believe pregroup grammars developed from this rudimentary setup.*" (Lambek 2007: 352; emphasis added).

The system exhibited in (4) combines Lambek's earliest formulation of a type-based logic for linguistic composition with the version of type-logic developed in Oehrle (1994); but note that Oehrle's system is presented as itself an outcome of enriching the associated type-logic of Lambek's (1958) paper with the structural rule of permutation; this of course then requires word order to be somehow separated from type combinatorics, and Oehrle's own deep insight was to allow the prosody to contain functional operators. It is not unreasonable to see Lambek's 1958 paper as the fountainhead for the two separate research traditions that have developed under the broad heading of type-logical grammar, and as I hope to have made clear, Peirce's work in both linguistics and the algebra of relations had already provided the materials for Lambek's profound synthesis, as Lambek himself stressed. It is to be hoped that future overviews of the history of type-logical systems along the lines of e.g. Moortgat (2014) will take due note of

[16]Specifically, assume that for any two types A, B, $A \bullet B \leqslant C$, i.e., every inhabitant of the concatenation of the types A, B is an inhabitant of C. Then with $f_* = \lambda\alpha.\alpha \bullet B$ and $f^* = \lambda\beta.\beta/B$, there is a Galois connection between f_* and f^* iff $f_*(A) \leqslant C \Leftrightarrow A \leqslant f^*(C)$, which, if we also define an upper adjoint $f^{**} = \lambda\gamma.A\backslash\gamma$, and take the entailement relation $X \vdash Y$ to define a partial ordering $X \leqslant Y$, gives us exactly the "triquivalence" in (34).

Peirce's right to ancestral status in the lineage of the Lambek calculus, and therefore of all contemporary versions of type-logical grammar. And it strikes me as extremely likely that Peirce would have been particularly glad had he known the degree to which his key linguistic principles – valence satisfaction as the driver of grammatical composition and language as an extension of logic – would be unified so precisely and rigorously in Lambek's brilliant fusion of developments in logic and mathematics that can be traced, to a large extent, back to Peirce himself.

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