Forecasting Models on Fuzzy Time Series Within Stock Market

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***Abstract*—This article firstly presents an analysis and survey regarding the traditional evaluation and forecasting model on fuzzy time series.It is pointed out that the maximum Subordination degree method and Subordination degree-Weighted average method is not suitable to attribute space usually, and a new evaluation model is proposed. The empirical study show that the new evaluation model is better able to evaluate and forecast the fuzzy time series within stock market**.

Key words*—*Fuzzy time series; Model；Attribute space; degree of confidence; stock market.

Ⅰ. INTYODUCTION

With the fast growth of our country's economy, it has become a hot topic within financial mathematics to analyze finance-derived products. To take an example of stock market, the investors are always willing to have a good grip of the trends of stock quotations so as to obtain as much repay as possible. Therefore, evaluation and forecasting on stock market has become the subject that many scholars are interested in and study. At present, there are various methods of evaluation and forecasting on stock market. For example, the paper [1] made the time series fuzzy, then evaluated and forecasted Taiwan weighted stock index, which provided us with a new thinking way. This study explored further on analyzing and forecasting fuzzy time series of stock market based on the results of the paper [1].

Ⅱ. EVALUATION AND FORECASTING BY USING THE MAXIMUM SUBORDINATION DEGREE-WEIGHED AVERAGE METHOD

Many scholars used the maximum subordination degree method and subordination degree-weighted average method to evaluate and forecast. They usually adopted the following 5 steps:

Step1: Construct fuzzy sets of universe of discourse, that is, confirm the intervals containing time series;

Step 2: Make the materials fuzzy, divide the intervals into many subintervals (a_i, b_i), and define the fuzzy sets F_i, ($i = 1,2,\cdots,m$), corresponding to the subintervals in the universe of discourse;

Step 3: Compute the relative matrixes, and confirm fuzzy subordination degree;

Step 4: Construct the appropriate pattern of fuzzy time series;

Step 5: Evaluate and forecast on the basis of the computed pattern of the fuzzy time series.

Following the 5 steps, the paper [1] made an evidence-based analysis of weighted stock index of Taiwan in April, 1999. Firstly, it abstained the set of universe of discourse $(-210,220)$ after making everyday weighted stock opening index a difference. Secondly it made the set fuzzy, that is, it divided $(-210,220)$ into 5 intervals:

$$I_1 = (-210,-124), I_2 = (-124,-38), I_3 = (-38,48),$$
$$I_4 = (48,134), I_5 = (134,220),$$

and defined 5 fuzzy sets F_1="Collapse", F_2="Drop", F_3="Steadiness", F_4="Up tick", F_5="Spike". The elements of every fuzzy set were composed of I_1, I_2,

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I_3, I_4， I_5 and their respective degree of subordination. Lastly, it computed the corresponding fuzzy matrixes and used the maximum subordination degree method and subordination degree-weighted average method to evaluate.

The paper [1] provided for constructed fuzzy sets I_1， I_2， I_3, I_4， I_5 with an evaluation space from "Collapse" to "Spike", and used the maximum subordination degree method and subordination degree-weighted average method to evaluate, which was an attribute space and was the evaluation and forecasting of ordered subdivision class. Well then, was it suitable to the real circumstances?

The study made a simple analysis of the two methods (it still used Taiwan weighted stock index of on April 6, 1999).

1) The maximum Subordination degree method.

The value was –18 after making the relative data a difference. After establishing the fuzzy relationship and the fuzzy matrixes, it obtained the degree of subordination of the output pattern of the day {1.00,1.00,1.00,0.68,0.68}. Because the intervals corresponding to max {1.00,1.00,1.00,0.68,0.68}=1 were (－210, －124), (－124, －38), (－38, 48), and the midpoint values were respectively －162, －81 and 5, the output value was $-81 \in I_2$ (see table 3.3 of the paper [1]). Therefore, relative to the last trading day, weighted stock index on April 4 should have been predicted "Drop" and the forecasting value should have been 7182－81=7101（7182 was the real value of the last day）, from the subordination degree of the output pattern of the day, we can see that the outcome evaluated as "Drop"(including "Collapse") made up only 46% and the rest outcome made up 54%. It didn't consist with the real circumstances.

2) Subordination degree-Weighed average method.

Firstly, the degree of subordination of the output pattern {1.00,1.00,1.00,0.68,0.68} was standardized and the standardized degree was {0.23，0.23，0.23，0.16，0.16}; then, we got the outcome $-14 \in I_3$ by using the formulation

$$S_1 \times (-167) + S_2 \times (-81) + S_3 \times 5 + S_4 \times 91 + S_5 \times 177$$

（ S_i was the standardized degree of subordination）. Therefore, weighted stock price index on April 6 should have been predicted "Steadiness" and the forecasting value should have been 7168, which accorded with the real circumstances of the day on the whole.

3) We made further analysis of the two methods.

When using the maximum Subordination degree method to evaluate, the output value were just the endpoint values of the intervals on April 15, 20, 23, 27, 30. The reason was that the intervals were all （－38,48），（48,134）corresponding to

15: max{0.68,0.68,1.00,1.00,0.68}=1；

20: max{0.79,0.79,1.00,1.00,0.79}=1；

23: max{0.92,0.92,1.00,1.00,0.92}=1；

27: max{0.94,0.94,1.00,1.00,0.94}=1；

30: max {0.50, 0.50, 1.00, 1.00, 0.50} =1.

and the midpoint values of the intervals were respectively 5 and 19. So, the same output value 48([5+91]÷2) was obtained, which was just the left or right endpoint value. We still need a further consideration of this method.

When using subordination degree-Weighed average method, the maximum of every component of standardized degree of subordination was 0.29 ,and the minimum was 0.13. The maximum and minimum of the output value was respectively 46.93 and –36.43, they were all in I_3 . Therefore, the forecasting value of everyday in April should have been all "Steadiness"，which didn't square with the real circumstances of the month.

We can see that there were some disadvantages for using the maximum subordination degree method and subordination degree-weighted average method to evaluate and forecast as far as the ordered subdivision class of attribute space was concerned. And the paper [2][3] also pointed out that degree of confidence recognition principle would be better in this instance.

III. DGREE OF CONFIDENCE RECOGNITION PRINCIPLE AND ITS APPLICATION

Definition 1 [2] :Suppose *X* is the whole studying objects, then it is called an attribute space. *F* is an attribute of the elements of *X*, and then *F* is called an attribute space. One situation of *F* is called an attribute set.

Definition 2 [4]: Suppose $\mathscr{B}$ is a set composed of attribute sets, and F is an attribute space. If the attribute sets in $\mathscr{B}$ is closed to complementary set and countable union operation, that is, for all $A \in \mathscr{B}$, such that $\overline{A} \in \mathscr{B}$, for all $A_i \in \mathscr{B}$, such that $\bigcup A_i \in \mathscr{B}$, then $\mathscr{B}$ is called σ--algebra, and $(F, \mathscr{B})$ is called an attribute measurable space.

Definition 3[5]: Suppose $(F, \mathscr{B})$ is an attribute measurable space, and then μ_x is called an attribute measure on $(F, \mathscr{B})$. If μ_x satisfies:

1) $\mu_x(A) \geq 0, \forall A \in \mathscr{B}$;

2) $\mu_x(F)=1$;

3) if $A_i \in \mathscr{B}, A_i \bigcap A_j = \Phi(i \neq j)$, then

$$\mu_x\left[\bigcup_{i=1}^{\infty} A_i\right] = \sum_{i=1}^{\infty} \mu_x(A_i),$$

then $(F, \mathscr{B}, \mu_x)$ is called an attribute measure space.

Definition 4[2]:If $(c_1, c_2, \cdots c_k)$ is a subdivision of attribute space F, and $c_1 > c_2 > \cdots > c_k$ or $c_1 < c_2 < \cdots < c_k$, $c_1, c_2, \cdots, c_k$ is called an ordered subdivision class.

Degree of confidence principle: Suppose $(c_1, c_2, \cdots c_K)$ is an ordered subdivision class of the attribute space F, λ is confidence degree. If when $c_1 > c_2 > \cdots > c_k$,

$$k_0 = \min_{k}\left\{k : \sum_{i=1}^{k} \mu_x(c_i) \geq \lambda, 1 \leq k \leq K\right\};$$

when $c_1 < c_2 < \cdots < c_k$,

$$k_0 = n - \min_{k}\left\{k : \sum_{i=1}^{k} \mu_x(c_i) \geq \lambda, 0 \leq k \leq n-1\right\},$$

then x is belong to c_{k_0} class.

The steps of using confidence degree recognition principle are as follows:

1) Confirm the subordination degree of output pattern by maximum subordination degree method;

2) Make λ=0.6（or 0.7), and confirm k_0 by

$$k_0 = \min_{k}\left\{k : \sum_{i=1}^{k} \mu_x(c_i) \geq \lambda, 1 \leq k \leq K\right\};$$

3) Elicit the evaluation outcome by k_0 (For example, if $k_0 = 3$,then the evaluation outcome is “Steadiness”) , and its forecasting value is the sum of the midpoint value between classes corresponding to k_0 and the real value of the last day.

Some scholars used this method in futures price forecasting and city transportation layout, and they achieved visible results.

We can get the forecasting outcome of Taiwan weighted stock index of in April 1999 (the real value of the before day of April 2 was 7049) by degree of confidence recognition principle（TableⅠ）.

TABLE Ⅰ STATISTICS

day	The real value	Degree of subordination of pattern output	k_0	The forecasting value
02	7233	0.50,0.77,1.00,0.74,0.77	3	7024 (Steadiness)
03	7182	0.50,1.00,1.00,0.50,0.50	3	7238(Steadiness)
06	7164	1.00,1.00,1.00,0.68,0.68	3	7187(Steadiness)
07	7136	1.00,1.00,1.00,0.87,0.87	3	7169(Steadiness)
08	7273	1.00,1.00,1.00,0.81,0.81	3	7141(Steadiness)
09	7266	0.50,0.78,1.00,0.73,0.78	3	7278(Steadiness)
12	7242	1.00,1.00,1.00,0.93,0.93	3	7271(Steadiness)
13	7338	1.00,1.00,1.00,0.84,0.84	3	7247(Steadiness)
14	7399	0.50,0.53,1.00,0.98,0.53	4	7427(Up tick)
15	7498	0.68,0.68,1.00,1.00,0.68	4	7490(Up tick)
16	7467	0.50,0.55,1.00,0.95,0.55	4	7589(Up tick)
17	7582	1.00,1.00,1.00,0.79,0.79	3	7472(Steadiness)
19	7623	0.50,0.64,1.00,0.86,0.64	4	7673(Up tick)
20	7628	0.79,0.79,1.00,1.00,0.79	4	7714(Up tick)
21	7474	1.00,1.00,1.00,1.00,1.00	3	7633(Steadiness)
22	7495	0.58,0.58,1.00,0.93,0.50	3	7479(Steadiness)
23	7613	0.92,0.92,1.00,1.00,0.92	4	7586(Up tick)
26	7629	0.50,0.66,1.00,0.85,0.66	4	7704(Up tick)
27	7550	0.94,0.94,1.00,1.00,0.94	3	7634(Steadiness)
28	7497	1.00,1.00,1.00,0.51,0.51	3	7555(Steadiness)
29	7290	1.00,1.00,1.00,0.66,0.66	3	7502(Steadiness)
30	7371	0.50,0.50,1.00,1.00,0.50	4	7382(Up tick)

On the basis of the accuracy ratio

$$p = \frac{1}{N}\sum_{i=1}^{N} I_{RL_i}(FL_t)$$

(FL_t denotes forecasting language variable value, RL_i denotes real language variable value, N is the sample length, I is an index function, namely, $I_t = \begin{cases} 1, & x = t, \\ 0, & x \neq t. \end{cases}$) provided by the paper [1]，we can figure out the accuracy ratio is 41%. Though the accuracy ratio is not perfect, the method can avoid the disadvantages the maximum subordination degree method and subordination degree-- weighed average method faced.

Ⅳ. CONCLUTIONS

When choosing evaluation and forecasting method, people will take many factors into consideration, such as deviation of forecasting results, concision and accuracy ratio of forecasting models and so on. We can draw the following conclusions from the above discussion:

1) The maximum Subordination degree method and Subordination degree-Weighted average method is not suitable to attribute space. The theoretical analysis and empirical study show that the confidence recognition method is better to evaluation and forecast the fuzzy time series time within stock market.

2) The different MSE of weighted stock price index of Taiwan in April,1999 can be predicted by using these forecasting method. The maximum subordination degree method: MSE=11079; subordination degree weighed average method: MSE=8499; One variable ARIMA: MSE=45146; confidence degree recognition method: MSE=9392; moving average method (2 MA): MSE=27084.

We can see that the MSE of the maximum subordination m degree method was larger than that of subordination degree-weighted average method and that of degree of confidence recognition method, and its forecasting accuracy ratio was not higher than that of the other two. In general, we believed that degree of confidence recognition method was better than the other two.

Moving average method has the largest MSE among these three methods, but the accuracy ratios were all 41%; moving average method was the simplest method. Therefore, moving average method was a considerable choice for forecasting weighted stock price index.

Of course, so far there still has not any universal method of evaluation and forecasting on time series. However, how to choose a method in reality depends on the integrated analysis and investment of specific circumstances.

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