

Increasing the Lock-in Probability of Blind Symbol Time Offset Estimators in OFDM Systems

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Abstract—For orthogonal frequency division multiplexing (OFDM) receivers, the mutual orthogonality of the subcarriers is preserved only if the estimated symbol timing error lies in the lock-in region. Conventionally, a bias is added to the coarse symbol time offset (STO) estimate to move the symbol timing error into the lock-in region, thus increasing the lock-in probability. Upon review of the literature, no analysis has been provided on determining which bias values can improve the lock-in probability. This paper is the first to investigate adding a bias to blind coarse STO estimators, and analyze its performance in terms of the lock-in probability. It will be shown that the optimal value of the bias term is dependent on the channel, which is not ideal for blind estimators. Hence, an upper bound on the optimal value is derived that is valid for all channels, and then the bound is made tighter for realistic wireless communication channels. It is shown that these bounds are dependent solely on the length of the cyclic prefix, thus the coarse STO estimators remain blind.

Keywords—lock-in probability, OFDM, synchronization

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a popular multicarrier modulation method that has been adopted in numerous wireless networking and broadcasting standards such as IEEE 802.11a/g/p, LTE, and DVB-T/T2. Much research has focused on blindly estimating the time and frequency offsets for OFDM, thus preserving high bandwidth efficiency (see [1] for a comprehensive survey). The most widely adopted symbol time offset (STO) estimator is given by [2], however it requires a-priori knowledge of the signal-to-noise ratio (SNR) [1]. In [1], two comparable STO estimators are derived that are simpler in complexity: the low-SNR unconditional maximum likelihood (UML) and the conditional maximum likelihood (CML) estimator. The UML STO estimator (eq. (73) in [1]) agrees with the estimator from [2] for low SNR and the CML STO estimator (eq. (63) in [1]) agrees with the estimator from [2] for high SNR.

A shortcoming of the analysis in [1] is that the performance of the two STO estimators is evaluated solely based on the variance of the STO estimate. However, the mutual orthogonality of the OFDM subcarriers is preserved only if the

estimated timing error lies in the lock-in region. If lock-in occurs, demodulation can successfully be accomplished using the fast Fourier transform (FFT) and the resulting phase offset can easily be corrected. If lock-in does not occur, the mutual orthogonality of the subcarriers is destroyed by intersymbol interference (ISI) and the bit error rate performance degrades.

The contribution of this paper is the investigation of adding a bias to the blind UML and CML STO estimate, and analyzing its influence on the lock-in probability. It is shown that the optimal value for the bias depends on the channel, which is not ideal for blind estimators. Hence, an upper bound on the optimal value is derived that is valid for all channels. However, this bound turns out to be rather high since it is derived using a “worst case” channel model. Therefore, the bound is made tighter for realistic wireless channels. The layout of the paper is as follows. Section II briefly describes the signal model for OFDM and reviews lock-in probability. Section III derives bounds on the optimal value for the bias. Section IV presents simulation results to demonstrate the improved performance of the biased estimator in terms of lock-in probability. Section V presents a possible application of the derived bounds in the time synchronization procedure of OFDM receivers. The paper is concluded in Section VI.

II. SIGNAL MODEL

In OFDM systems, the channel is divided into subchannels (i.e., mutually orthogonal subcarriers) where each subcarrier experiences flat fading. The transmitted OFDM symbol $s(n)$ $n = 0, \dots, N+N_{cp}-1$ is produced by taking the N point inverse FFT of the modulated data symbols and pre-pending the last N_{cp} samples, where the length of the cyclic prefix is greater than or equal to the order of the channel (i.e., $N_{cp} \geq L$). The critically sampled (i.e., $N+N_{cp}$ samples per OFDM symbol) received OFDM signal is given by

$$r(k) = e^{j2\pi k/N} \sum_{l=0}^{L-1} h(l)s(k-l) + n(k) \quad k = 0, \dots, \theta, \dots, 2N + N_{cp} - 1 \quad (1)$$

where $\theta \in [0, N-1]$ is the integer STO, $\varepsilon \in (-0.5, 0.5]$ is the carrier frequency offset (CFO) normalized to $1/NT_s$, and n is

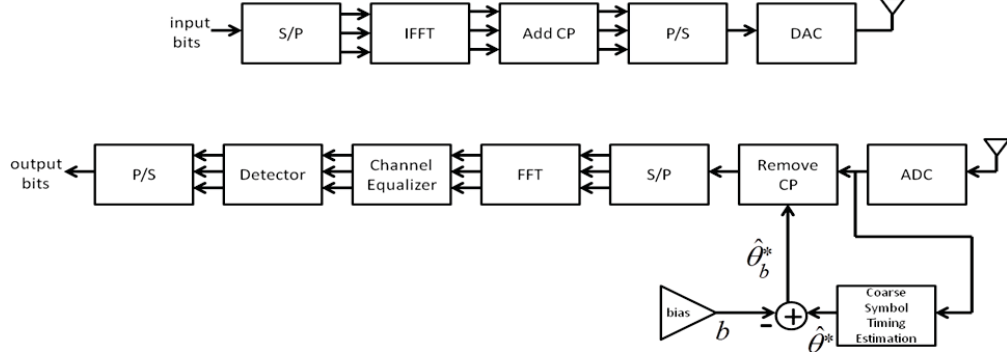


Fig. 1. Block diagram of OFDM system (transmitter - top, receiver - bottom)

additive white Gaussian noise (AWGN) with variance σ_n^2 . The STO θ is defined to be the first arrival path received (i.e., the first sample of the received OFDM symbol still including the cyclic prefix). Since θ is unknown, $2N+N_{cp}$ samples is the minimum number of samples that need to be collected at the receiver in order to estimate θ . However, only one complete OFDM symbol is actually contained in the $2N+N_{cp}$ samples.

As will be shown later, there are $L-1$ ISI samples and $N_{cp}-L+1$ ISI free samples in the cyclic prefix of the received OFDM symbol. Let $\hat{\theta}^*$ denote the STO estimate. Then the probability that the symbol timing error, defined to be $\hat{\theta}^* - \theta$, lies in the ISI free region of the received cyclic prefix (i.e., $P(L-1-N_{cp} \leq \hat{\theta}^* - \theta \leq 0)$) is called the lock-in probability.

As mentioned earlier, blind estimators (including the UML and CML estimators) have been designed to accurately estimate the STO, but not necessarily designed to lock-in with high probability. Many of the blind estimators presented in the literature evaluate a cost function for the N possible time offsets and then search for the minimum or maximum, which results in $\hat{\theta}^*$. The proposed approach in this paper subtracts a bias $b \geq 0$ from $\hat{\theta}^*$, thus yielding a biased estimate $\hat{\theta}_b^* = \hat{\theta}^* - b$, as shown in the bottom right portion of Fig. 1.

III. BOUNDS ON OPTIMAL BIAS

Both the UML and CML estimators exploit the correlation between the data portion of the OFDM signal and its corresponding cyclic prefix. From (1), the correlation between the received OFDM samples $r(k)$ and $r(k+N)$ is

$$E[r(k)r^*(k+N)] = \begin{cases} e^{-j2\pi\epsilon} \sigma_s^2 \sum_{l=0}^{k-\theta} h(l)h^*(l), & k \in I_1 \\ e^{-j2\pi\epsilon} \sigma_s^2 \sum_{l=0}^{L-1} h(l)h^*(l), & k \in I_2 \\ e^{-j2\pi\epsilon} \sigma_s^2 \sum_{l=k-N_{cp}+1}^{L-1} h(l)h^*(l), & k \in I_3 \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

where ϵ is the CFO, σ_s^2 is the signal power, $h(l)$ is the complex amplitude of the l -th multipath arrival, $I_1 = \{\theta, \theta+1, \dots, \theta+L-2\}$, $I_2 = \{\theta+L-1, \theta+L, \dots, \theta+N_{cp}-1\}$, and $I_3 = \{\theta+N_{cp}, \theta+N_{cp}+1, \dots, \theta+N_{cp}+L-2\}$. It can also be shown that the zero lag correlation is constant for all values of k , so it can be ignored in the analysis of the CML estimator.

The UML and CML estimators accumulate N_{cp} consecutive values of the correlation values, i.e.,

$$\sum_{k=\hat{\theta}}^{\hat{\theta}+N_{cp}-1} r(k)r^*(k+N) \quad (3)$$

for $\hat{\theta} = 0, \dots, N-1$. Since the estimators were derived under the assumption of a single path channel, the desired correlation values belong to I_2 and the undesired correlation values belong to I_1 and I_3 . I_3 moves the STO estimate outside of the lock-in region for multipath channels, since its correlation values cause the set of maximum accumulated correlation values Δ to increase in value (for clarity, the set Δ consists of STO estimates where each estimate has a corresponding accumulated correlation value; increase in value is defined as the STO estimates themselves increase, not the correlation values). Applying the expectation operator to (3) and substituting (2) yields the expected accumulated correlation values. On average, the $N_{cp}-L+2$ STO estimates that yield the largest accumulated correlation values are the most likely estimates produced by the UML and CML STO estimators. In order to ensure that these STO estimates fall within the lock-in region, a bias $b \geq 0$ is subtracted from $\hat{\theta}^*$, thus yielding a biased estimate $\hat{\theta}_b^* = \hat{\theta}^* - b$ and increasing the lock-in probability. The optimal bias is given by the value of b that moves these $N_{cp}-L+2$ STO estimates into the lock-in region (i.e., $L-1-N_{cp} \leq \hat{\theta}_b^* - \theta \leq 0$).

Note that this approach for determining the optimal bias is applicable to any channel, however the drawback is that the optimal bias is dependent on the channel (since the correlation between the received samples $r(k)$ and $r(k+N)$ depends on the channel coefficients), which is not ideal for blind estimators. Next, a bound for the optimal bias is determined that is independent of the channel.

The “worst case” channel model results in the largest expected correlation between the received samples $r(k)$ and $r(k+N)$ for $k \in I_3$, which causes the set of maximum accumulated correlation values Δ to increase in value the most. This is attained by a length N_{cp} channel where all the channel coefficients are zero, except the last one.

Proof The UML and CML estimators accumulate N_{cp} consecutive values of the correlation values as in (3). Applying the expectation operator to (3), the goal is to find a channel h that results in the largest symbol timing error, i.e.,

$$\begin{aligned} \max \quad & \hat{\theta}^* - \theta \\ \text{subject to} \quad & \hat{\theta}^* = \arg \max_{\hat{\theta} \in [0, N-1]} \sum_{k=\hat{\theta}}^{\hat{\theta}+N_{cp}-1} E[r(k)r^*(k+N)] \end{aligned} \quad (4)$$

Looking at (2), the correlation values in I_2 are the largest correlation values because they are proportional to the sum of the power of all the channel coefficients. However, the correlation values in I_3 equal the correlation values in I_2 only if

$$h(l) = \begin{cases} 0, & 0 \leq l \leq L-2 \\ 1, & l = L-1 \end{cases} \quad (5)$$

since all of the correlation values in I_3 include $h(L-1)$ in their summations. Hence, the correlation values are equal for $\theta+L-1 \leq k \leq \theta+N_{cp}+L-2$ and 0 otherwise. In order to maximize the cost function in (4), L is set to its maximum value (i.e., $L = N_{cp}$). Therefore, the channel given by (5) with $L = N_{cp}$ is the channel that results in the largest symbol timing error $\hat{\theta} - \theta = N_{cp}-1$. (Note that practically speaking, the channel would be some small value for $l=0, \dots, L-2$ and 1 for $l=L-1$.) *QED*

Using the results from the proof, the most likely estimate produced by the UML and CML estimators is $\hat{\theta}^* = \theta + N_{cp}-1$. Since the lock-in region holds for $L-1-N_{cp} \leq \hat{\theta}_b^* - \theta \leq 0$, which in this “worst case” translates to $-1 \leq \hat{\theta}_b^* - \theta \leq 0$, the optimal bias b is given by either $N_{cp}-1$ or N_{cp} (assuming the received signal is critically sampled [1]).

The “worst case” channel requires the largest bias to move the STO estimates within the lock-in region since it causes the set of maximum accumulated correlation values Δ to increase in value the most. Therefore, the optimal bias value of the “worst case” channel model provides an upper bound on the optimal bias value for any channel.

Theorem 1 For any channel, the optimal bias is less than or equal to the length of the cyclic prefix (i.e., $b_{opt} \leq N_{cp}$). This optimal bias results in a higher lock-in probability, on average, than the unbiased UML or CML estimator. \square

The upper bound provided by Theorem 1 is rather high, especially if one is interested in using the optimal bias for OFDM systems in realistic wireless communication channels. It has been observed in measurements of wireless channels that the power delay profile (PDP) can be approximated by a decaying exponential function [3], so that the power of each tap decreases with the delay. In an effort to make the optimal bias bound tighter for realistic channels, but still be

independent of the channel, the author proposes to upper bound the decaying exponential PDP by a uniform PDP of length $L = N_{cp}$. Since the uniform PDP channel causes the set of maximum accumulated correlation values Δ to increase in value more than or equal to any exponential PDP, its optimal bias provides an upper bound on the optimal bias value for realistic channels.

Proof Let the exponential PDP be given by

$$P_{h,\text{exp}}(l) = \begin{cases} e^{-al}, & 0 \leq l \leq L-1 \\ 0, & \text{otherwise} \end{cases} \quad (6)$$

where $a \geq 0$. Also, let $L = N_{cp}$ (if $L < N_{cp}$, the proof is trivial). There are two limiting cases: 1) $a \rightarrow \infty$ and 2) $a \rightarrow 0$. As $a \rightarrow \infty$, the channel approaches that of a single path channel and $\hat{\theta}^* = \theta$ is the most likely estimate produced by the UML and CML estimators. As $a \rightarrow 0$, the channel approaches that of the uniform PDP. Therefore, the uniform PDP channel of length $L = N_{cp}$ causes the set of maximum accumulated correlation values Δ to increase in value more than or equal to any exponential PDP. *QED*

In the analysis of the uniform PDP of length $L = N_{cp}$, the channel coefficients are independent and identically distributed (i.i.d.) where the real and imaginary components are each normally distributed with variance 0.5. Fig. 2 depicts (2) for an example uniform PDP. Note that the maximum accumulated correlation values occur for the STO estimates $\Delta = \{\theta, \dots, \theta+L-1\}$ because these estimates include the entire set of correlation values belonging to I_2 . Symmetrical argument can be used to find that the maximum of the accumulated correlation values occurs in the middle of Δ at

$$\hat{\theta}^* = \theta + \frac{L-1}{2} \quad (7)$$

Hence, the optimal bias is given by

$$b_{opt} = \frac{N_{cp}}{2} \quad (8)$$

This leads to Proposition 1.

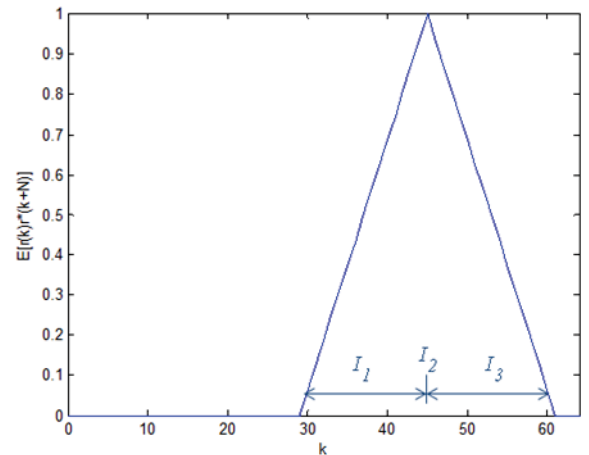


Fig. 2. Mean of correlation between the received samples $r(k)$ and $r(k+N)$ for uniform PDP of length $L = N_{cp} = 16$, $N = 64$, $\theta = 30$, $\varepsilon = 0$, and $\sigma_s^2 = 1$

Proposition 1 For any realistic channel, the optimal bias is less than or equal to half the length of the cyclic prefix (i.e., $b_{opt} \leq N_{cp}/2$). This optimal bias results in a higher lock-in probability, on average, than the unbiased UML or CML estimator. \square

Proposition 1 reduces the range of possible optimal biases by half when compared to Theorem 1. Furthermore, note that the bound in (8) only depends upon N_{cp} , so the coarse STO estimator remains blind.

IV. SIMULATION RESULTS

The lock-in probability performance of the proposed approach is demonstrated through Monte Carlo simulations using 10,000 realizations. In all of the simulations (except as noted), $N = 32$, $N_{cp} = 8$, $\theta = 16$, $\varepsilon = 0$, and BPSK modulation and uniform PDP Rayleigh fading channels are used where the channel coefficients are normalized to unit power and remain constant over the duration of a couple of OFDM symbols. For this channel model, the optimal bias is given by (8). Only $2N+N_{cp}$ samples are collected at the receiver (i.e., in the samples, there is one complete OFDM symbol and part of one or both adjacent symbols), since this type of constraint is relevant to real-time applications where a fast acquisition time is critical such as timing and navigation systems.

In these results, the simulations are intended to draw more general results rather than using a specific channel model. Factors such as channel order, length of the cyclic prefix, and number of symbols used by the estimator are investigated.

Fig. 3 looks at how the improvement in lock-in probability (i.e., the difference between the lock-in probabilities of the optimally biased estimator and unbiased estimator) for the UML and CML estimators varies with the channel order. The large improvements in lock-in probability for both estimators occur for multipath channels satisfying $N_{cp}-L > 1$. These large improvements in lock-in probability plateau for higher SNR because of the plateau in estimation variance of the unbiased estimator due to the finite cyclic prefix region [1].

Complementary to Fig. 3, Fig. 4 looks at the improvement in lock-in probability for the UML and CML estimators by

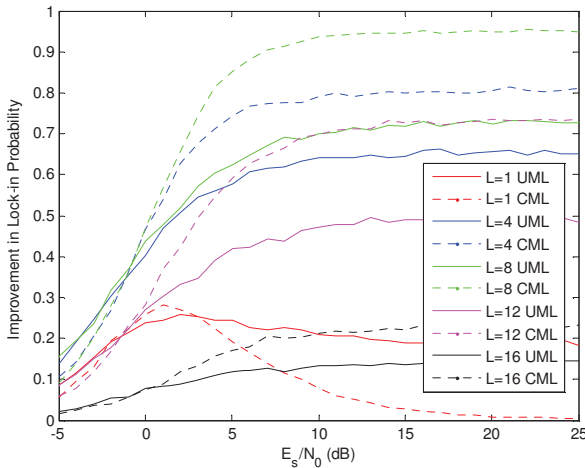


Fig. 3. Improvement in lock-in probabilities for the UML (solid) and CML (dashed) estimators by varying the channel order

holding the channel order fixed and varying the length of the cyclic prefix. The blue lines are for $L = 1$, the red lines are for $L = 4$, and the green lines are for $L = 8$. Note that the CML estimator's lock-in probability improves more than the corresponding UML estimator for $L > 1$. $L = 1$ is the channel order that sees the least improvement in lock-in probability with increasing N_{cp} . For higher order channels at high SNR, the improvement in lock-in probability increases with increasing N_{cp} , but eventually plateaus at various values of N_{cp} . This plateau is again caused by the aforementioned plateau in estimation variance of the unbiased estimator. On the contrary, the improvement in lock-in probability increases with increasing N_{cp} for higher order channels at low SNR. For example, the improvement in lock-in probability for the CML estimator ($L = 8$) plateaus at high SNR for $N_{cp} \geq 14$, but does not plateau for low SNR.

Fig. 5 shows the improvement in lock-in probability for the UML and CML estimators as the number of symbols used increases. The blue lines are for $L = 1$, the red lines are for $L = 4$

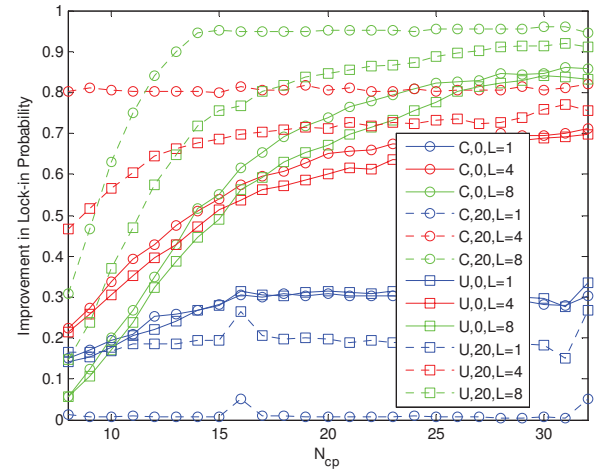


Fig. 4. Improvement in lock-in probabilities by varying the cyclic prefix length (in the legend: C stands for CML estimator, U stands for UML estimator, and the second entry is E_s/N_0 in dB)

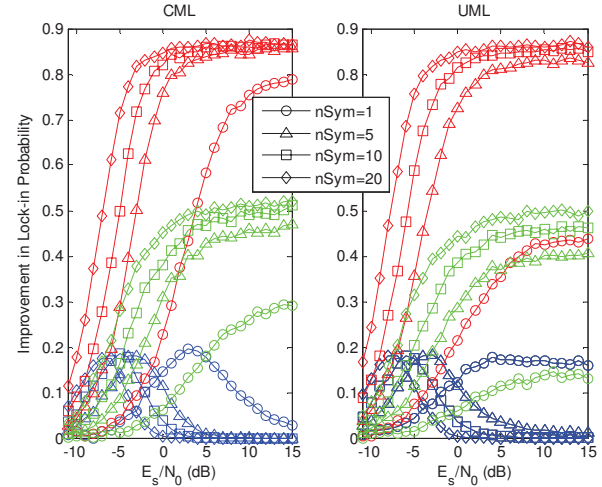


Fig. 5. Improvement in lock-in probabilities by varying the number of symbols used (blue lines - $L = 1$, red lines - $L = 4$, and green lines - $L = 8$)

4, and the green lines are for $L = 8$. Note that for $L > 1$, the curves shift to the left and up as the number of symbols used increases; the leftward shift shows that the optimal bias increases the lock-in probability for lower SNR as the number of symbols used increases, and the upward shift shows that the improvement in lock-in probability increases as the number of symbols used increases. However, the change in improvement in lock-in probability diminishes as the number of symbols used increases.

Finally, a realistic channel model is used to demonstrate Proposition 1. Differing from the previous results, $N = 64$, $N_{cp} = 16$, and the 3GPP Rural Area channel (Rax) [4] model is used, which is effectively a twelfth order channel for an OFDM signal bandwidth of 20 MHz (similar to the IEEE 802.11a signal standard). Fig. 6 shows the lock-in probability of the CML estimator for different bias values, ranging from 0 to N_{cp} . $b = 0$ is the unbiased CML estimator, and the grey lines are for $b > 8$, with the top grey line belonging to $b = 9$ and the bottom grey line belonging to $b = 16$. Proposition 1 is satisfied since $b_{opt} = 5 \leq N_{cp}/2$ results in the highest lock-in probability of greater than 98% for $\text{SNR} \geq 15\text{dB}$. Notice that, in this case, the bias values ranging from 1 to $N_{cp}/2$ all result in a higher lock-in probability, on average, than the unbiased CML estimator. Determining which bias value is optimal is discussed in the next section.

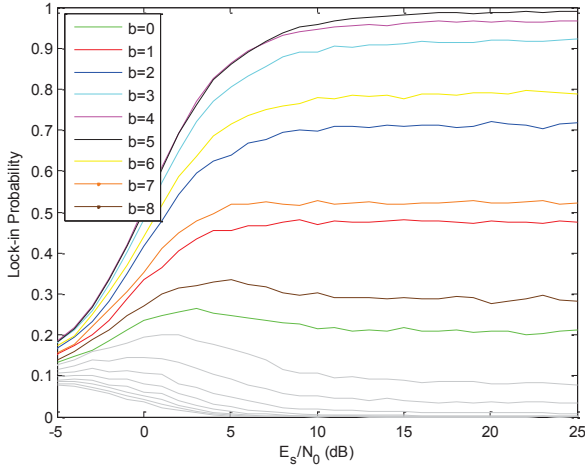


Fig. 6. Lock-in probability of biased CML estimator in Rax model

V. APPLICATION

The addition of a bias to the blind coarse STO estimate is a time domain technique to improve the synchronization of the OFDM receiver. However, it is only the first step in the time synchronization process. There are methods [5]-[8] to improve the synchronization after the N point FFT is applied (see Fig. 1), and they all require an initial coarse STO estimate. Utilizing Proposition 1, a parallel architecture is proposed where each of the $N_{cp}/2+1$ possible biases are used to control the FFT window (see Fig. 7). The reasoning behind the proposed architecture is that Proposition 1 provides a bound on the optimal bias value, but does not determine which bias value is optimal. Therefore, the $N_{cp}/2+1$ most likely sets of FFT windows are provided to the next step in the time synchronization process, so that it is likely that the mutual

orthogonality of the subcarriers is preserved in at least one of the sets. Note that the feedforward approach in [8] investigates $2N_{cp}+1$ biases, so its computational complexity can significantly be reduced using the proposed architecture.

Instead of applying $N_{cp}/2+1$ separate FFT operations (which has a computational complexity of $O[(N_{cp}/2+1)M\log_2 N]$), the sliding discrete Fourier transform (SDFT) can be used to reduce the computational complexity [9]-[10]. The SDFT algorithm works as follows: let $\tilde{x}(n-1) = \{x(0), x(1), \dots, x(N-1)\}$ and $\tilde{x}(n) = \{x(1), x(2), \dots, x(N)\}$ be two successive length N sequences. Denote the k th spectral component of the sequence $\tilde{x}(n-1)$ by $X_k(n-1)$ and the k th spectral component of the sequence $\tilde{x}(n)$ by $X_k(n)$. Then,

$$X_k(n) = e^{j2\pi k/N} (X_k(n-1) + x(n) - x(n-N)) \quad (9)$$

Since the biases produce $N_{cp}/2+1$ successive length N sequences, the SDFT can be used where each successive subcarrier only requires one complex multiplication and two additions. This can be extremely useful if the synchronization method only uses a small subset of the subcarriers. If all N subcarriers are needed by the synchronization methods, then the computational complexity using the SDFT is $O[M\log_2 N + (N_{cp}/2)N]$, which is lower in computational complexity than computing $N_{cp}/2+1$ FFTs and lower in memory requirements than using the sliding FFT [9].

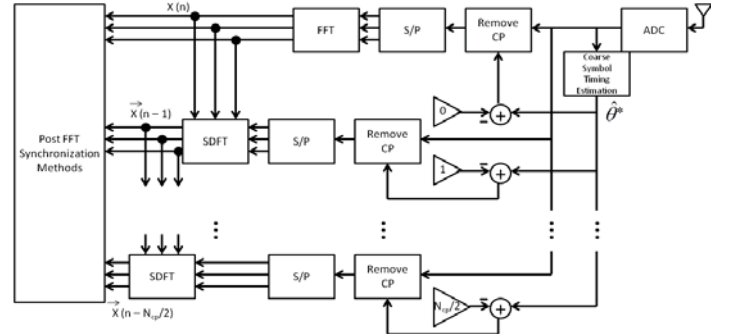


Fig. 7. Proposed time synchronization architecture

VI. CONCLUSION

This paper investigated adding a bias to blind coarse STO estimators, and analyzed its performance in terms of the lock-in probability. Bounds on the optimal value of the bias term were derived for the blind UML and CML STO estimators, and were found to be dependent solely on the length of the cyclic prefix thus keeping the coarse STO estimators blind. Using the proposed approach, it was shown through simulation that a higher lock-in probability can be achieved using only one complete OFDM symbol, thus resulting in low computational complexity. If more than one symbol is used, it was seen that the change in improvement in lock-in probability diminishes as the number of symbols used increases. Thus, there is a tradeoff between improvement in lock-in probability and computational complexity. Finally, it was discussed how the biased estimator and SDFT can be used in the time synchronization procedure of OFDM receivers.

REFERENCES

- [1] J.A. Lopez-Salcedo, E. Gutierrez, G. Seco-Granados, A.L. Swindlehurst, "Unified framework for the synchronization of flexible multicarrier communication signals," *IEEE Trans. Signal Process.*, vol.61, no.4, pp.828-842, Feb. 2013.
- [2] J.J. van de Beek, M. Sandell, P.O. Borjesson, "ML estimation of time and frequency offset in OFDM systems," *IEEE Trans. Signal Process.*, vol.45, no.7, pp.1800-1805, July 1997.
- [3] A. Molisch, "Channel models," in *Wireless Communications*, 1st ed. Chichester, England: Wiley, 2009, pp. 117-123.
- [4] 3GPP, "Technical Specification Group Radio Access Network; Deployment aspects," TR 25.943, Sept. 2012, V11.0.0.
- [5] Y. Mostofi, D. Cox, "A robust timing synchronization design in OFDM systems-part I: low-mobility cases," *IEEE Trans. Wireless Commun.*, vol.6, no.12, pp.4329-4339, Dec. 2007.
- [6] Y. Mostofi, D. Cox, "Robust timing synchronization design in OFDM systems-Part II: high-mobility cases," *IEEE Trans. Wireless Commun.*, vol.6, no.12, pp.4340-4348, Dec. 2007.
- [7] W. Chin, S. Chen, "A Blind Synchronizer for OFDM Systems Based on SINR Maximization in Multipath Fading Channels," *IEEE Trans. Veh. Technol.*, vol.58, no.2, pp.625-635, Feb. 2009.
- [8] A. Al-Dweik, S. Younis, A. Hazmi, C. Tsimenidis, B. Sharif, "Efficient OFDM Symbol Timing Estimator Using Power Difference Measurements," *IEEE Trans. Veh. Technol.*, vol.61, no.2, pp.509-520, Feb. 2012.
- [9] E. Jacobsen, R. Lyons, "The sliding DFT," *IEEE Signal Process. Magazine*, vol.20, no.2, pp.74,80, Mar. 2003.
- [10] E. Jacobsen, R. Lyons, "An update to the sliding DFT," *Signal Processing Magazine, IEEE*, vol.21, no.1, pp.110,111, Jan. 2004.