

# Comparison of Timebase Interpolation Methods for Traceable, Wideband mm-Wave Communication Signals

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**Abstract** — An integral part of processing a signal from a calibrated, equivalent-time sampling oscilloscope is accurately determining sampling times and then interpolating the resulting irregularly-spaced time grid to create a uniform time spacing. In this paper, we present simulations and measurements to determine the effect of the interpolation choice on the fidelity of a modulated signal. We judge the effect by calculating the error-vector-magnitude of the resultant signal from each interpolation.

**Index Terms** — Digitally modulated signal, error vector magnitude, interpolation, microwave measurement, millimeter-wave wireless communication, oscilloscope, wireless system.

## I. INTRODUCTION

Calibration of a high-speed oscilloscope, traceable to primary standards, provides a traceability path for receivers and transmitters across the spectrum. The key to the traceability path is the correction of an equivalent-time sampling oscilloscope by a photodiode that has been characterized by an electro-optic sampling system (EOS) traceable to the primary volt, meter, ohm and second standards. A recorded signal is first processed to remove drift, systematic timebase distortion (TBD), and timing jitter [1]. The resulting trace no longer has a regular grid of time points. Therefore, in order to convert back to regular timebase-points for voltage correction and conversion to frequency space for mismatch correction, the corrected signal must be regularized in time by use of an appropriate interpolation method. In this paper, we examine optimization of this interpolation step with both simulated and measured precision, wideband, millimeter-wave (mm-Wave) signals using the error-vector-magnitude (EVM) as the figure-of-merit.

There are two reasons we chose to use mm-Wave signals for this study. First, there is intense interest in expanding the available wireless spectrum to these high frequencies [2]. Second, we have developed at NIST a method using predistortion for generating a precise, calibrated communication signal at these frequencies that allows a rigorous estimate of uncertainties [3].

The EVM is a useful metric for characterizing the quality of the modulated signal produced since it provides information on both the amplitude and phase of a signal [4]. Prior work [5]–[8], including our own [3] has allowed accurate uncertainty estimates for this metric. Thus, the EVM provides an ideal

measurement with which to compare the choice of an interpolation method in reconstructing a modulated signal.

In the remainder of this paper, we will first describe the signal generation process using predistortion to motivate the role of the interpolation. We then compare the effect of three different interpolation methods on the final signal EVM in both simulation and measurement.

## II. PREDISTORTION

While the method for creating a broadband, modulated signal with minimized EVM has been discussed previously [3], a brief summary will be presented here. This method will work for a modulated signal in any protocol. The signal we use in the present study for both measurement and simulation is a pseudo-random bit-sequence encoded with a 64-quadrature-amplitude-modulation (64-QAM) scheme. The signal is generated to produce 1 GSymbol/s and is 320 ns long (320 symbols in length). The signal is modulated at an intermediate frequency (IF) of 4 GHz and a root-raised-cosine filter is applied. We then create a periodic multisine [9], based on the Fourier components of this signal, to upload to an arbitrary waveform generator (AWG). The output of the AWG is fed to a frequency converter. We use a 10 GHz signal, multiplied by 4, to create a 40 GHz local oscillator to upconvert our IF to 44 GHz. This same 10 GHz signal is also used as the internal clock for the AWG. The AWG also creates a trigger for the equivalent-time sampling oscilloscope to measure the 44 GHz signal. Repeat measurements of the signal are carried out to improve the signal-to-noise and reduce measurement uncertainty.

At this point, the set of modulated signals measured are corrected for drift, timebase errors, and jitter, yielding signals with irregular time grids. Before the voltage can be corrected by the EOS calibrated photodiode, the timebase must be interpolated back to a regular grid. In the study presented here, we look at three interpolation methods typically used in our group. The first we call the least-squares method with midpoint knots. All acquired data sets are combined with simultaneous triggers. Then, the time points that lie between the midpoints of the desired time grid are collected and approximated by a linear least-squares fit. The desired time grid point is then calculated from this line. Each linear fit between the midpoints

(the knots) is independent from the next. The second method linearly interpolates the regular time grid from the nearest-neighbor measured time points. The linear interpolation is performed on each measured signal, then all of the evenly-time-spaced-signals are averaged. Finally, we use a shape-preserving interpolation based on piecewise cubic Hermite polynomials, termed PCHIP. Similar to the linear interpolation, the PCHIP interpolation is performed on each measured waveform before they are all averaged.

Once the measured signals are on an even time grid, averaged if necessary, and calibrated in voltage, the next step is to create a modulated signal that corrects for hardware non-idealities through predistortion. The resultant measured waveform is used to predistort the ideal desired signal. The predistorted signal is calculated element-wise according to

$$X_{predist} = X_{ideal} \left( \frac{X_{ideal}}{X_{meas}} \right), \quad (1)$$

where  $X_{predist}$  is the newly calculated predistorted signal,  $X_{ideal}$  is the ideal signal, and  $X_{meas}$  is the calibrated result of the interpolation step. The predistorted waveform is uploaded to the AWG, which now transmits a waveform closer to the ideal signal. A single predistortion step will account for linear distortions in the generation and upconversion, but this process can be repeated as many times as necessary to account for non-linear distortions. Once an adequate signal is produced, one additional interpolation step is necessary to process the final signal.

### III. SIMULATION

We examined the effect of interpolation choice using a simulated noisy signal. We used the random binary sequence that we then modulated with a 64-QAM protocol, as described above. The waveform was oversampled to be used with an AWG with an output of 20 GSamples/s. This waveform serves as an “ideal” reference for the calculation of EVM at the end of the simulation. The IF signal is filtered by a root-raised-cosine function, then digitally upconverted to 44 GHz.

At this point, sampling of the signal by the oscilloscope was emulated using time intervals randomly shifted from a nominal oscilloscope time step. We simulated oscilloscope time steps of 0.5 to 5 ps, replicating values used in our mmWave signal measurements. Representative results are shown in Fig. 1 for a simulation with a 2.5 ps oscilloscope time step. To model the results of the time base correction, the time points were shifted with a normally-distributed set of random offset values where the distribution width ranged from 0.5 to 2.5 ps. These values are shown as the  $\hat{y}$ -axis in Fig. 1. The resulting waveform provides the correct voltage at the estimated, but irregular sampling points. However, we know from measurements that the time base correction has a residual uncorrected jitter of 200 fs [1]. This uncorrected jitter is then added to the irregular time base by a normally distributed random value, with no change to the voltage value. Finally, we add voltage noise to the waveform with a normally distributed random number, where the distribution widths are between 0 and 1.25 mV, shown as the  $\hat{x}$ -axis in Fig. 1.

We simulated 25 noisy signals which were then interpolated with one of the three methods described. The root-raised-cosine filter was applied, the signal was demodulated, and the root-mean-square (RMS) EVM was calculated by comparing the resultant symbols with those originally encoded [10].

The EVM for each interpolation method, as a function of timebase offsets and voltage noise, is shown in Fig. 1. Since a nominal oscilloscope time increment of 2.5 ps is most routinely used in our lab, that is the one chosen. Table 1 shows the computation time to produce the values in each subplot of Fig. 1. We see that the least-squares method is the most computationally expensive. The other two methods are much faster and similar to each other.

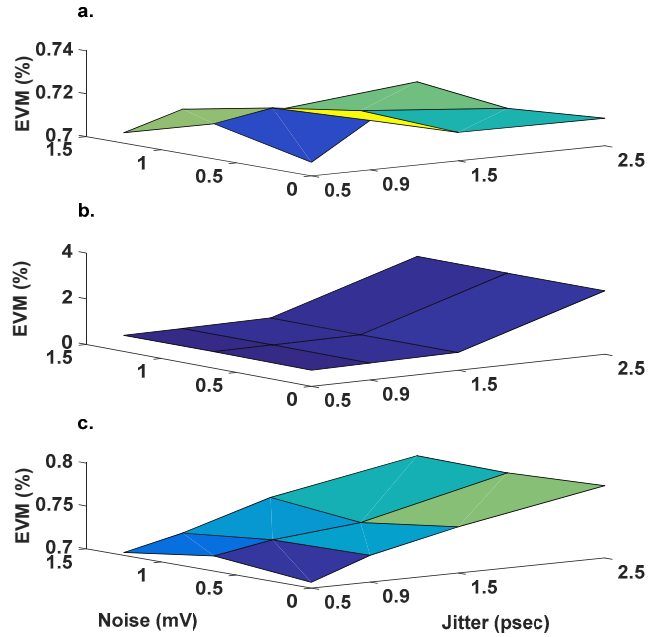


Fig. 1. (a) Surface plot showing the calculated EVM for an oscilloscope time base of 2.5 ps as a function of voltage noise and time base offsets for the least-squares interpolation. (b) Same plot for the linear, nearest-neighbor method. (c) For the PCHIP method.

Computation time at 2.5 ps time base			
	Least-Squares	PCHIP	Linear
CPU Time (s)	30.12	0.17	0.13

Table 1. Computation time for each interpolation on the 2.5 ps timebase data.

Although not plotted here, our simulations showed that for 0.5 ps oscilloscope time steps, the choice of interpolation method is immaterial since the fine grid produces equivalent EVM results for all values of offset and noise. As the time step increases, as seen in the 2.5 ps results of Fig. 1, both the PCHIP and the linear interpolation methods become sensitive to the increased time-step offsets introduced by the time-base correction. The least-squares method can handle the larger timing jitter and still provide a low EVM result. All interpolation methods are weakly sensitive to voltage noise. We conclude from this simulation study that, for smaller time

steps, we can use a computationally-cheaper interpolation method, such as PCHIP or linear interpolation. Yet, for longer time steps, or with an oscilloscope that requires large timing corrections, we can better use the more expensive least-squares method.

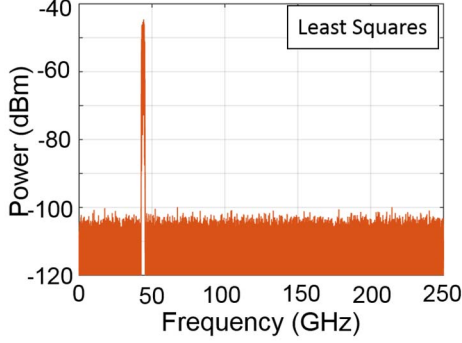


Fig. 2. Simulation of a 44 GHz, 64-QAM signal after combining 25 oscilloscope time records and fitting points between midpoint knots with a linear least-squares fit. The signal before interpolation was simulated with a 2 ps time increment, 1.5 ps noise added to the time points and 1.25 mV noise added to the voltage samples.

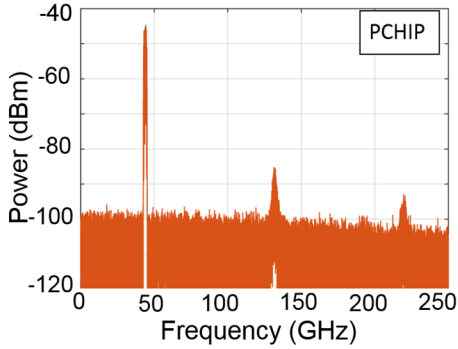


Fig. 3. Simulation of a 44 GHz, 64-QAM signal after averaging 25 time records that were individually interpolated onto a 2 ps time grid with a cubic interpolation. The signal before interpolation was simulated with a 2 ps timebase, 1.5 ps noise added to the time points and 1.25 mV noise added to the voltage samples.

A salient feature of each interpolation method is the effect on the frequency spectrum noise floor. The resultant 44 GHz signal, after the least-squares linear fit with midpoint knots, is shown in Fig. 2. The noise floor is shown to be spectrally flat. The 44 GHz simulated signal with PCHIP interpolation is shown in Fig 3, where there is now a higher noise floor near 44 GHz and a downward ramp toward higher frequencies. Also, the cubic interpolation imparts harmonic distortion to the signal. The same ramp without the distortion products was seen with the linear interpolation.

#### IV. MEASUREMENT

We also examined the effect of the interpolation choice on measurement. A difference in the effect of interpolation on measurement versus simulation is that the measurements require a minimum of two interpolation steps, rather than one,

assuming only linear corrections. In the simulations, an ideal signal with voltage and timing imperfections induced by the oscilloscope was created in software. Experimentally, in order to obtain this ideal signal from an AWG, we must first predistort the ideal signal. Therefore, we must measure the distorted signal from the AWG, correct the measurement, then interpolate it to obtain the predistortion. This predistorted signal is then measured, corrected and interpolated to see the effect on EVM. The 40 GSamples/s interleaved AWG we used at 20 GSamples/s single channel was linear in its distortion and did not require correction for the interleave imbalance. Therefore, we achieved low EVM values with only one predistortion step.

0.5 ps time step		2 <sup>nd</sup> Interpolation for Calculation	
1 <sup>st</sup> Interp for Predistortion	Least Squares 2 <sup>nd</sup> EVM(%)	PCHIP 2 <sup>nd</sup> EVM(%)	Linear 2 <sup>nd</sup> EVM(%)
<b>Least Squares w/ sm</b>	<b>0.602</b>	0.603	0.603
<b>PCHIP w/ sm</b>	0.600	<b>0.605</b>	0.602
<b>Linear w/sm</b>	0.615	0.615	<b>0.616</b>
2.5 ps time step			
1 <sup>st</sup> Interp for Predistortion	Least Squares 2 <sup>nd</sup> EVM(%)	PCHIP 2 <sup>nd</sup> EVM(%)	Linear 2 <sup>nd</sup> EVM(%)
<b>Least Squares w/ sm</b>	<b>0.598</b>	0.689	0.772
<b>PCHIP w/ sm</b>	0.731	<b>0.814</b>	0.883
<b>Linear w/sm</b>	0.583	0.682	<b>0.763</b>
5 ps time step			
1 <sup>st</sup> Interp for Predistortion	Least Squares 2 <sup>nd</sup> EVM(%)	PCHIP 2 <sup>nd</sup> EVM(%)	Linear 2 <sup>nd</sup> EVM(%)
<b>Least Squares w/ sm</b>	<b>3.04</b>	3.580	3.872
<b>PCHIP w/ sm</b>	2.785	<b>3.563</b>	3.862
<b>Linear w/sm</b>	2.8	3.705	<b>3.978</b>

Table 2. Resultant EVMs for measurements of the same 64-QAM signal as was used for simulation. The signal was predistorted as described earlier for the interpolation method shown on the left, but with the addition of a smoothing function. The final interpolations for EVM calculation are shown across the top of each table. Results are shown for oscilloscope time steps of 0.5, 2.5, and 5 ps.

Using two interpolations, we could not separate the effects of each individual interpolation. Therefore, we predistorted the waveform using each interpolation method described and measured 25 traces of the final 44 GHz signal from each. All three sets of data were calibrated, and then the three interpolation methods were again used on each set to interpolate the measured waveform and determine the EVM. Therefore, 9 combinations of interpolations were possible. The EVM results for oscilloscope time bases of 0.5, 2.5, and 5 ps time steps are shown in Table 2. The interpolation used for the predistortion is listed on the left. For the predistortion, a smoothing function was also applied to the correction so as to alleviate large noise induced offsets. The second interpolation leading to the final EVM calculation is listed across the top of the table. For simplicity, and to observe the trends in the results, we can look at only the diagonals of Table 2, shown in bolder font. The diagonals show the measurements where the same interpolation method was used for both the predistortion

determination and the final processing. The measured EVM values were higher than those simulated, but this is not surprising given nonlinear distortion introduced by the upconversion and other real-world effects, as compared to the simulated signal. The trends for each time step matched the simulation. The measurements had no dependence on the final interpolation used for 0.5 ps time steps due to the fine grid this provides. The relative dependence increased with increasing time steps. The least-square fit of combined data was the most robust method, followed by PCHIP and linear interpolations before averaging. This corroborated with what was observed in the simulations.

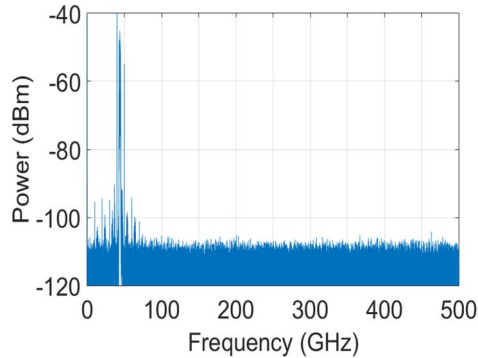


Fig. 5. 44 GHz measured signal after predistortion and interpolating with linear least squares of combined data.

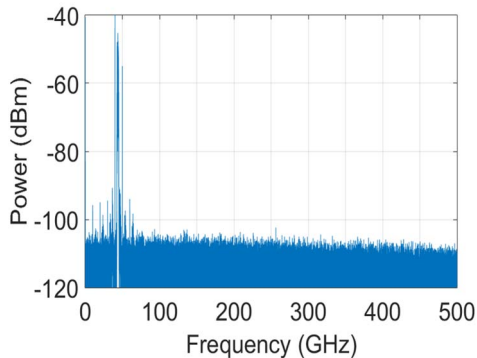


Fig. 6. 44 GHz measured signal after precorrection and interpolating with PCHIP method and averaging.

Trying to elucidate the trend in the measurements, we look at the effect of interpolation on the noise floor. In Fig. 4, the least-squares interpolation results in a flat noise power level. In Fig. 5, the noise floor treated with the PCHIP method is higher near the signal, but again has a downward ramp toward higher frequencies. The harmonic distortions seen in the simulation are not present.

## V. CONCLUSION

We have simulated and measured a 44 GHz, 2 GHz wide 64-QAM modulated signal to assess the effect of interpolation on our precision modulated-signal source. The simulations and measurements provided identical trends for the choice of interpolation.

At fine time grids for the recorded signal, the choice of interpolation is not critical so that the computationally-least expensive method can be chosen. For larger time steps, or more jitter, simulations show that the least squares linear fit of the combined data is the preferred method.

The simulations were corroborated by measurements. However, the time base corrections seen experimentally for our system, show that the interpolation choice has a relatively small effect on EVM for our typical 2.5 ps time step. Although the EVM values, as shown in Table 1, are small, the uncertainty analysis in [3] yielded an average EVM of approximately 1.5%. Thus, although we can minimize EVM of our precision source, the effect is not significant. Other quality metrics such as signal-to-noise-ratio, intermodulation distortion, or harmonic distortion may possibly be more sensitive to the choice of interpolation.

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