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Challenges for Computational Electromagnetics in the Time Domain [1]

J.S. Shang,
Flight Dynamic Directorate, Wright Laboratory
Wright-Patterson Air Force Base, Ohio 45433

I. Introduction

The time-dependent Maxwell equations constitute the hyperbolic partial differential equation system [1,2]. In order to complete the description of the differential system, initial value and boundary condition are required. For Maxwell equations, only the source of the excited field and a few physical boundary conditions at the media interfaces are pertinent [1]:

$$\frac{\partial B}{\partial t} + \nabla \times E = 0 \tag{1}$$

$$\frac{\partial D}{\partial t} - \nabla \times H = -J \tag{2}$$

$$\nabla \cdot B = 0\,, \qquad \nabla \cdot D = \rho \tag{3}$$

$$\begin{aligned} n \times (E_1 - E_2) &= 0 \\ n \times (H_1 - H_2) &= J_s \\ n \cdot (D_1 - D_2) &= \rho_s \\ n \cdot (B_1 - B_2) &= 0 \end{aligned} \tag{4}$$

where the subscripts 1 and 2 refer to media on two sides of the interface. J_s and ρ_s are the surface current and charge densities of a perfect electrical conductor respectively.

This first-order divergent-curl equations system is not necessarily analytical and is difficult to solve by conventional numerical methods. Pioneering efforts by Yee and others have attained impressive achievements [3,4]. Recently, numerical techniques in computational electromagnetics (CEM) have been further enriched by the computational fluid dynamics (CFD) community [5,6]. Limited by the numerical resolution of present algorithms, accurate high frequency CEM simulations are still beyond the reach of conventional computing systems.

The computational accuracy requirement for diffraction and refraction dominant electromagnetic phenomena is well known [4,5]. For any numerical simulation of physics, fundamental efforts should always be concentrated on the requirements of numerical resolution, initial/boundary condition implementation, and accurate geometrical description. A major source of error is introduced into the solution by physically incorrect and inappropriate implementation of initial and boundary conditions. The placement of the farfield boundary and type of initial or boundary conditions also play an important role. These concerns are easily appreciated in the light of the fact that the governing equations are identical in CEM, only different initial or boundary conditions generate different solutions.

Numerical accuracy is also controlled by the algorithm and computing systems used. Error induced by the discretization consists of round-off and truncation error. The round-off error is contributed by the computing system. Since this error behavior is random, it is the most difficult to evaluate. The truncation error for time-dependent calculations appears as dissipation and dispersion. In multiple dimensional computations, the anisotropic error also emerges. All known numerical algorithms have a wavenumber range in which the error is small. As an illustration, the semi-discrete dispersive error of several well-known and a bidiagonal compact-differencing scheme are presented in Figure 1. The numerical results are obtained by solving the one-dimensional model wave equation. It is not surprising that the high resolution algorithm development is a pacing item in CEM.

For a wide range of CEM applications, the Maxwell equations can be cast on a general curvilinear frame of reference to accommodate different electrical configurations [6,7]. The system of equations on general curvilinear coordinates is derived by a coordinate transformation from the Cartesian frame to achieve the strong conservation form [6,7]:

$$\xi = \xi(x, y, z), \eta = \eta(x, y, z), \zeta = \zeta(x, y, z). \tag{5}$$

$$\frac{\partial U}{\partial t} + \frac{\partial F_\xi}{\partial \xi} + \frac{\partial F_\eta}{\partial \eta} + \frac{\partial F_\zeta}{\partial \zeta} = -J \tag{6}$$

where the dependent variables are now defined as, $U = U\,(B_x V, B_y V, B_z V, D_x V, D_y V, D_z V)$, and the V is the local cell volume is also the inverse Jacobian of coordinate transformation.

In order to provide a versatile numerical tool for CEM, the multiple-block structure technique becomes necessary to describe the field around complex electric configurations. The adopted grid topology, grid generation procedures, and the minimum grid spacing criteria will control the simulation accuracy. The aforementioned requirements are collectively described as pre-processing needs. On the other hand, the data reduction, data display, and validation efforts are considered to be post-processing endeavors. All these issues demand a systematic approach to establish a common ground for applications.

In the last decade, through remarkable progress in micro chip and interconnect data link technology, a host of multiple address, message passing computers have became available for data processing. These scalable multi-processors or multi-computers, in theory, are capable of providing essentially unlimited computing resources for scientific simulations. A synergism of new numerical procedures and scalable parallel computing capability will open up a new frontier in electromagnetics research [8]. This new opportunity also offers challenges for CEM research in high resolution numerical algorithm development, consistent and well-posed initial values and boundary conditions implementation, pre- and post data processing, and the ever important validation data base development.

II. Areas of Research Emphasis

In numerical simulation, the well-posedness requirement of initial or boundary conditions and the stability of a numerical approximation are also ultimately linked to the eigenvalues of governing equation. The solution of the hyperbolic differential system has a distinctive domain of dependence in which all data must meet the compatibility condition [2]. The well-posed boundary conditions at the farfield and the media interface is paramount for numerical stability and accuracy. Since all CEM computations in the time domain must be conducted on a truncated computational domain, this constraint requires a numerical farfield condition at the truncated boundary to mimic the behavior of an unbounded field. A highly developed technique to suppress the reflected wave from the artificial numerical boundary is to introduce an absorbing layer. Arecent contribution to this approach is due to Berenger [9]. Another approach is to treat the farfield by the characteristic-based formulation. Both approaches use split Maxwell equations, a comparative study should be performed to gain insight on the effectiveness and well-posedness features of these techniques.

A systematic evaluation of boundary implementation on media interface is urgently needed. Particularly for PEC scattering simulations, the unknown surface current and charge density are bypassed by extrapolations [6-8]. Although these approximated numerical boundary conditions are compatible with the basic attribute of the hyperbolic partial differential system, they also induce error. The error is easily identified by comparing with values derived from the asymptotic formulation in the optical limit by Kay [10]. Figure 2 depicts the comparison of computed electrical surface charge densities with asymptotes. The discrepancy exists over the entire wavenumber range examined. An exact boundary formulation on the PEC scatterer by including the conservation law for electric charge and current densities will be invaluable for high resolution procedure development.

The final area of CEM research emphasis is the accuracy enhancement of computations by high resolution schemes and spectral methods. Substantial progress is being made in the compact difference method, optimized algorithm research [11]. All these numerical techniques are devised to increase the numerical resolution of simulations over a wider range of the frequency spectrum. Several high resolution schemes for both the finite-difference and the finite-volume formulations are being developed. A detailed delineation of these numerical procedures will accelerate their maturation for applications.

In summary, recent progress in solving the three-dimensional Maxwell equations in the time domain has opened a new frontier in electromagnetics. The progress in micro chip and interconnect network technology has led to a host of high performance distributive memory computer systems. The synergism of efficient and high resolution numerical algorithms for solving the Maxwell equations in the time domain with high performance multicomputers will propel the relatively new interdisciplinary simulation technique to practical and productive applications.

III. References

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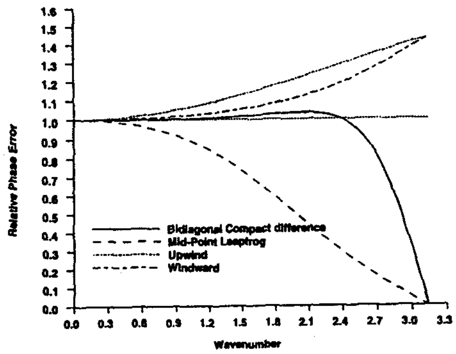

Figure 1. Dispersive Error of Various schemes for Solving Simple Model Wave Equation

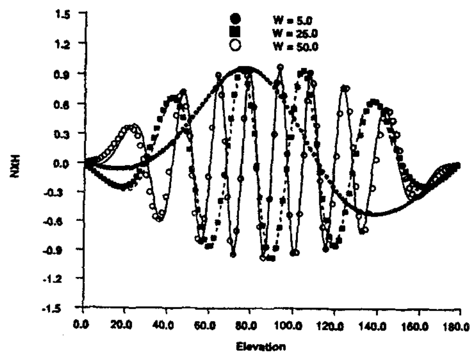

Figure 2. Comparison of Electrical Surface Charge Density on a PEC Sphere