

## ERROR TOLERANCE, GAIN AND BEAMWIDTH OF A LOW SIDELobe PHASED ARRAY

James K. Hsiao  
Electromagnetics Branch  
Radar Division  
Naval Research Laboratory  
Washington, D.C. 20375

### Introduction

A radar system with a low sidelobe antenna is less vulnerable to jamming. Theoretically, one can design as low a sidelobe level as one wishes. However, in practice, the achievable sidelobe level is limited by the array errors. These errors are characterized by the element amplitude and phase errors. The relation between sidelobe level and array errors is expressed as a probabilistic function. For a given probability, the amount of error tolerance is a function of the deviation of the actual sidelobe level from the designed sidelobe level. It hence is possible to design an array which may have a higher tolerance when the designed sidelobe level is much lower than that of the desired sidelobe level. This paper describes how such an approach can be implemented, what amount of sidelobe degradation can be achieved for a given array error and how an optimal condition can be achieved. Furthermore, during system design phase, often only the array gain or main beamwidth is given and one is expected to design an array with acceptable sidelobe level and also to estimate the required error limits. In this paper, this relation shall also be discussed.

### Sidelobe Level Distribution and Array Errors

It is well known that the probability density function of an array sidelobe level is Rician. Its variance, which relates the array error and illumination has the following form:

$$\sigma^2 = 1/2(1 + \sigma_\delta^2 + |\phi(1)|^2) \sum_n A_n^2, \quad (1)$$

where  $\sigma_\delta^2$  is the variance of the amplitude error of each array element and  $\phi(1)$  is the characteristic function of the phase error. The  $A_n$ 's are the array element illuminations. We normalize the array side-lobe level and the array variance  $\sigma^2$  with respect to the designed (or ideal) sidelobe level. The cumulative probability that the normalized sidelobe level  $S$  is less than  $S_L$  is then

$$P(S \leq S_L) = \int_0^{S_L} \frac{S}{\sigma^2} \exp \left[ \frac{-S^2 + 1}{2\sigma^2} \right] I_0 \left( \frac{S}{\sigma^2} \right) ds, \quad (2)$$

U.S. Government work not protected by U.S. copyright.

where  $I_0$  is the modified Bessel function of zero order and  $\sigma^2$  is the normalized variance. In deriving the above relation, it has been assumed that the phase error is very small. Under this condition, it can be shown that

$$\Phi(1) \approx 1 - \sigma_\phi^2, \quad (3)$$

where  $\sigma_\phi^2$  is the variance of the phase error. Eq. (1) thus becomes

$$\sigma^2 \approx \sum_n A_n^2 (\sigma_\phi^2 + \sigma_\delta^2). \quad (4)$$

The variance consists of two parts. The first part is related to the array illumination  $A_n$ 's and the second part is the sum of the variances of the phase and amplitude errors. Equation (2) relates the probability of the deviation of the actual sidelobe from the designed sidelobe level as a function of  $\sigma^2$ . We shall utilize this equation for our array design problems. The first problem encountered very often is to determine the required array errors for a given deterioration of the sidelobe level from the designed value.

A set of constant probability contours of eq. (2) is shown in Fig. (1). In this figure the deviation of the actual sidelobe level from the designed value (in dB scale) is plotted as the abscissa while the normalized array variance is plotted as ordinate (also in dB scale). These curves are very useful in determining the required  $\sigma$  level to achieve a desired sidelobe degradation. For example, in order to achieve a sidelobe degradation of no more than 3 dB from the designed level with a probability of 90 percent the required  $\sigma^2$  value must be 11 dB below the designed sidelobe level. If the designed sidelobe level and the illumination function  $A_n$ 's are known, it is straightforward to compute the required array errors. In general, it is difficult to achieve a smaller array error. One may hence be forced to design an array with much lower sidelobe level than that which is required. The corresponding array error budget may be greatly eased. Curves of constant probability shown in Fig. (2) can be used to optimize such a design. The amount in dB that the sidelobe level should be designed below the desired sidelobe level is shown in abscissa while the corresponding amount in dB by which should be below the desired sidelobe level is used as ordinate. For example, we assume that an array is required to maintain a sidelobe of -50 dB with a probability of .9. If one designs the array illumination to achieve a -53 dB sidelobe level, the corresponding  $\sigma^2$  is -64 dB (-64 = -50-14) while if one designs a -55 dB array, the required  $\sigma^2$  becomes -60.8 dB. The slopes of the curve are very steep at the lower sidelobe reduction value and the slope becomes smaller as the sidelobe reduction value increases. It is thus evident that in the lower value region a small decrease in the designed sidelobe level yields a larger amount of increase of  $\sigma^2$  value. Therefore, an optimal value should be at the bend of these curves. Since  $\sigma^2$  consists of both array error and illumination function, the effect

of the illumination function must be included. One way to handle this problem is to compute the actual weight each time a new designed sidelobe level is determined.

#### Array Illumination Function and Gain

In order to compute the array errors, the array illumination coefficients  $A_n$  must be known. However, in many cases, a first order estimate of the array errors is needed and the exact configuration of the array is not yet known. There is no way to compute the illumination function. In this case, if the required array directive gain or beamwidth is known, it can be used for this estimate. If the array illumination is normalized in such a way that

$$\sum_n A_n = 1$$

then the radiated field voltage in the direction of the main beam is unity and

$$\sum_n A_n^2 = \sum_n A_n^2 / (\sum_n A_n)^2. \quad (5)$$

According to Elliott [1] the directive gain of an array with at least half wavelength spacing is

$$D = (\sum_n A_n)^2 / \sum_n A_n^2. \quad (6)$$

Therefore, if the required directive gain is known, one may use it in place of the  $\sum_n A_n^2$  for a first order estimate of the array error budget.

The directivity gain is defined as the quotient of peak intensity to the total radiated power. The latter is the sum of the main beam power and the sidelobe power. Hansen, [2] stated that for a modest length antenna, the main beam power predominates. Thus, in this case the change of sidelobe level does not appreciably change the gain and  $\sum_n A_n^2$  value. For a first order approximation the effect of the change of  $\sum_n A_n^2$  may be ignored when different sidelobe designs are used.

Elliott showed that in most cases, the beamwidth and gain of a linear array can be approximated by the relation

$$D = 101.5 / \text{beamwidth in degrees.}$$

Hence, if the required beamwidth is given one may use this relation to estimate the array errors.

## Planar Arrays

The above results can be applied directly to a planar array if element errors are independent. The gain of the planar array is the product of the gains of two linear arrays if the illumination of each element can be separated into row and column components. Under this condition, the gain and beamwidth can be used for estimating array errors as in the linear array case.

## References

1. R.C., Hansen Editor, "Microwave Scanning Antennas", Vol. 2, Chapter 1, Academic Press, 1966
2. R.C. Hansen, "Gain Limitation of Large Antennas", IRE Trans. on Antennas and Propagation, (8) September 1960.

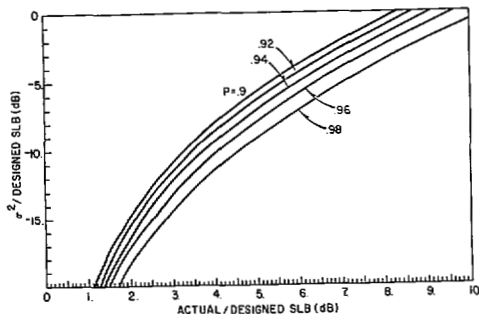


Fig. 1 - Constant Probability Contour of  $e^2$  & actual sidelobe - both normalized with respect to the designed sidelobe level

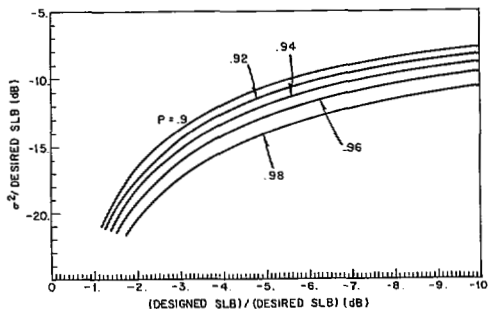


Fig. 2 - Constant Probability Contour of  $e^2$  & designed sidelobe - both normalized with respect to desired sidelobe level