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Paper

THEORY OF NULL SYNTHESIS OF PLANAR ARRAYS

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Introduction. Most of the voluminous literature on antenna pattern synthesis is devoted to the synthesis of linear arrays. Studies of planar arrays, which span several decades, are quite scarce in design techniques. These may be grouped into three major categories: One class of design techniques [1] requires separable illumination and conventional rectangular or circular arrays. These techniques place at the designer's disposal all linear array synthesis methods. A second group of design techniques [2] uses numerical iterative procedures. They allow for optimization but necessarily require a great deal of computational capability. Yet another class of techniques [3] uses a linear transformation of linear arrays. In this case some linear array synthesis techniques may be utilized.

These synthesis methods for planar arrays usually have a small number of (one or two) degrees of freedom in pattern control; except for the numerical method, they do not take full advantage of all the available degrees of freedom. The null synthesis method discussed here is non-iterative and does not require separable illumination. It permits specification of arbitrary pattern nulls, has an increased number of degrees of freedom and uses a two-dimensional convolution process. The method also incorporates a basic set of array configurations referred to here as canonical configurations; consequently the method can retain the pattern symmetries of the canonical configurations.

Convolution Method of Synthesis. Consider a four element rhomboid array, R , consisting of isotropic sources, as shown in Figure 1(a). If two identical arrays R are convolved, the result of the two-dimensional convolution is the nine element rhomboid array, $RA_2 (= R * R)$, shown in Figure 1(b). The convolution process may be performed by replacing each element of the array R by an array R with its center at the element location. Convolution of RA_2 with R or equivalently convolution of R with itself twice, leads to the 16 element array RA_3 . In general, an N -fold convolution of R with itself will produce a $(N+1) \times (N+1)$ element array RA_N .

The convolution process can be extended to define the array and its excitation coefficients, using R_i to refer to the i th array. Then the rhombic array RA_N with its excitation coefficients given by the convolution of the arrays R_i , can be represented by

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$$R_N = R_1 * R_2 * \dots * R_N. \quad (1)$$

A consequence of the above relationship is that the array factor $f(\theta, \phi)$ of R_N is readily expressed in terms of the array factors of $f_{ei}(\theta, \phi)$ of R_i . Equation (2) results from the Fourier transform relationship between the array factor and the array illumination (excitation coefficients).

$$f(\theta, \phi) = \prod_{i=1}^N f_{ei}(\theta, \phi) \quad (2)$$

From equation (2), it is easily recognized that the nulls of f_{ei} are also the nulls of f . Thus, a null synthesis procedure for f is equivalent to the null synthesis of the element array factor f_{ei} . The number of zeros that may be specified for f_{ei} are three (since there are four excitation coefficients, three of which may be chosen independently) [4]. Consequently, the total number of zeros of f that may be prescribed is $3N$. This is also the number of degrees of freedom associated with the synthesis procedure. For quadrant symmetry of the array factor f , the degrees of freedom reduces to N .

Canonical Configurations. In the null synthesis method, for the rhomboid array discussed above, the nulls of a four element rhomboid array R have a fundamental role in the synthesis of larger rhomboid arrays. The basic array R is thus considered a canonical configuration for R_N . Most practical array geometries are special cases of the array R . Almost all of the commonly used array geometries may be synthesized from the following four canonical configurations: (1) A 4-element diamond array D (obtained from R by setting $\phi_1 = 0$ and $\phi_2 = \pi/2$); (2) a 2-element linear array L ; (3) a 7-element hexagonal array H ; and (4) a 3-element triangular array T . These canonical arrays exhibit basic quadrant, 180 degree, 60 degree and 120 degree pattern symmetries, respectively.

It is obvious that repeated convolution of a canonical array with itself leads to larger arrays of the same type. For example, a 16-element diamond array DA_3 is created by $(D * D * D)$. On the other hand, convolution between two different canonical arrays would lead to a different array boundary, although the lattice structure may be preserved. For instance, a 7-element hexagonal array HA_1 may be synthesized by convolution of D and L .

$$HA_1 = D * xL.$$

The prescript x with L indicates an x -oriented linear array. The array HA_1 differs from H in that HA_1 has quadrant symmetry whereas H has 60° symmetry. A more general example of such a convolution synthesis is shown in Figure 2. Here a 3-ring hexagonal array HA_3 is synthesized from DA_3 and xL_3 . xL_3 is, of course, obtained from two convolutions

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of χ_L . The array HA_3 can also be synthesized by $(H * H * H)$. Examples of these different types of symmetries using the null synthesis procedure are discussed in detail in a companion paper [5].

Inasmuch as the nulls of canonical arrays play an important role in the null synthesis procedure, they are investigated in detail. Hexagonal arrays H with uniform excitation (zero degrees of freedom) and with 60° symmetry (one degree of freedom) have been studied [6]. Figure 3 shows the locus of zeros for values of the excitation coefficient a for the canonical array H . Here the six elements on the hexagonal ring have excitation a with respect to the unity center element. Line OC_1 is along the x -axis and OD is at an angle of 30° to OC_1 . Because of symmetry it is necessary to show only $1/12$ of the hexagonal reciprocal lattice cell. For a uniformly excited array H , it can be shown that the sidelobe levels at C_1 and D are -16.90 dB and -10.88 dB, respectively, with respect to the mainlobe at O . Figure 4 shows the locus of the zeros of a diamond array with equilateral triangular lattice. Here the parameter a is the excitation of elements along the x -axis with respect to that of the elements along the y -axis. Once again because of quadrant symmetry only a portion of the reciprocal lattice cell need be shown.

Conclusion. The theory of a null synthesis method suitable for a variety of planar arrays is presented. Fundamental to this theory, which utilizes a two-dimensional convolution process, is the concept of canonical array configurations. The null locations of these canonical arrays with various symmetries are studied. The null synthesis method, which is non-iterative and does not require separable illumination, allows for arbitrary specification of pattern nulls and provides a greater number of degrees of freedom than other synthesis methods.

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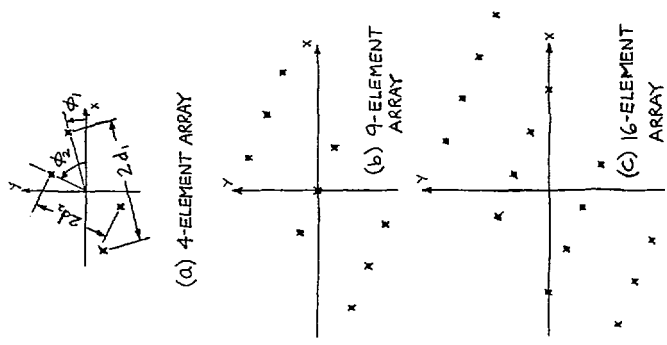


FIGURE 1. GEOMETRY OF RHOMBOID ARRAYS

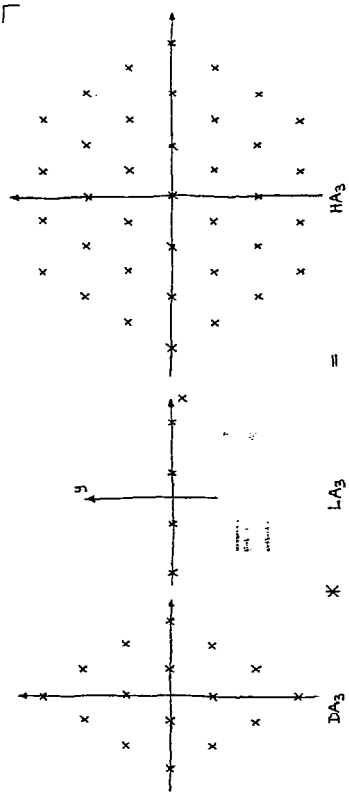


FIGURE 2. DEVELOPMENT OF HEXAGONAL ARRAY FROM DIAMOND AND LINEAR ARRAYS

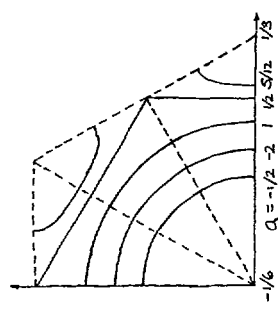


FIGURE 3. ZERO LOCUS OF CANONICAL HEXAGONAL ARRAY, H

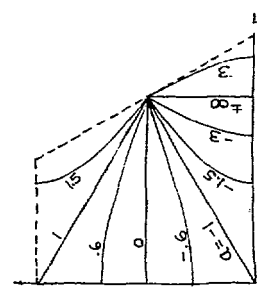


FIGURE 4. ZERO LOCUS OF CANONICAL DIAMOND ARRAY, D