

A Comparison of Spectral Models Used in Developing Non-Kolmogorov Wave Structure Functions

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Abstract— We compare the Wave Structure Function (WSF) resulting from various models for refractive index fluctuation spectra in non-Kolmogorov turbulence. Closed form, analytical expressions for the WSF based on the generalized Kolmogorov, von Kármán, analytical, and exponential models are presented and compared. We find that the generalized von Kármán model is essentially identical to the WSF resulting from numerical integration of the Analytic spectrum at separations larger than the inner scale. In addition, we demonstrate that certain approximations to the Analytic model introduce significant inaccuracies at separations as small as a hundredth of the outer scale size. Asymptotic expressions for the WSF and phase variance in non-Kolmogorov turbulence with a finite outer scale are also presented.

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1. INTRODUCTION

Recently, there has been significant interest in using analytical models in order to understand the effects of non-Kolmogorov turbulence on various aspects of laser beam propagation and imaging [1],[2],[3],[4]. Generally, we obtain this understanding by modifying the spectral model of index of refraction fluctuations so that the power-law varies from the Kolmogorov value of $11/3$. Analytical models for the behavior of the received optical field are then readily available using the same integral expressions used to derive their Kolmogorov counterparts. Analytical models are useful in describing a variety of properties of laser beam propagation and imaging through atmospheric turbulence. The Wave Structure Function (WSF), in particular, determines spatial coherence and is used in the design of adaptive optics systems as well as beam parameters such as variance in angle of arrival. As we will demonstrate, the functional form of WSF can vary considerably depending on the form of the spectral model used and, therefore, warrants some examination.

The specific form and features of atmospheric spectral mod-

els vary depending upon the desired level of fidelity and intended application. Finite inner and outer scales are the common features and found in the Tatarskii and von Kármán spectra respectively [5], [6]. The modified von Kármán spectrum includes both features and is referred to here as the Analytic spectrum. Still, other models [7], [8] aim to account for physical processes in the transition between the inertial and dissipation regions. An ideal spectral model would include all of these features and, certainly, similar criteria apply in our studies of non-Kolmogorov turbulence.

However, these high fidelity spectral models can often result in intractable integrals, long, complicated expressions, or require the use of limiting series approximations. These complications are likely unavoidable in the search for closed form analytical models describing fluctuations in log-amplitude and irradiance. On the other hand, analytical models for phase fluctuation, like the WSF, are dominated by large scale fluctuations and inner-scale corrections can generally be ignored. Still, much of the work thus far [4], [9],[10],[11],[12] makes use of the either the analytical von Kármán or generalized forms of the Andrews analytical spectrum [8]. It is likely that more compact expressions are available for phase dependent beam parameters.

In this paper, we compare the plane Wave Structure Function (WSF) resulting from different generalized functional forms for the turbulence energy spectrum in non-Kolmogorov turbulence. In section 2, we revisit the fact that the inner scale in the analytical von Kármán spectrum results in an intractable integral. Closed form analytical expressions therefore, require the using limiting approximations. In section 3, we show that these approximations violate requirements of positivity on the structure function at large separations. Instead we show that the von Kármán spectrum is equivalent to its analytical counterpart for separations larger than the inner scale size. Finally, in section 4 we demonstrate that this equivalency holds over the range typical range of power-laws, $3 < \alpha < 4$. The generalized von Kármán spectrum is also used to produce generalized expressions for the variance in Angle of Arrival and phase variance. A summary of our conclusions is provided in section 5.

2. GENERALIZED WSF FORMULATION OF COMMON SPECTRAL MODELS

Following the development originating with Tatarskii [13], a generalized power law medium can be defined by the energy spectrum of index of refraction fluctuation and has the form [14]

$$\Phi^{NK}(\kappa) = A(\alpha)\beta\kappa^{-\alpha} \quad (1)$$

Noting here that β is a substitute for the familiar refractive index structure parameter, C_n^2 , and has units $m^{3-\alpha}$. $A(\alpha) = \frac{1}{4\pi^2}\Gamma(\alpha - 1)\cos\left(\frac{\alpha\pi}{2}\right)$ is a constant that ensures consistency

between the structure function definition for refractive index fluctuations and the spectral model in Eq.1. Also, owing to the use of the Markov approximation [15], α is limited in the range of $3 < \alpha < 4$ and we point out that the upper and lower bounds on α are explicitly not included in the valid range. The reader will note that when $\alpha = 11/3$, $A(\alpha)$ evaluates to 0.033 providing the familiar result $\Phi(\kappa) = 0.033 C_n^2 \kappa^{-11/3}$.

Assuming amplitude fluctuations are negligible [16], the WSF for an unbounded plane wave can be defined in terms of separation $\rho = |r_1 - r_2|$,

$$D_{pl}(\rho) = 8\pi^2 k^2 L \int_0^\infty \kappa \Phi_n(\kappa) (1 - J_0(\kappa\rho)) d\kappa \quad (2)$$

Substituting Eq.1 into Eq.2 and completing the integration results in [14][17]

$$D^{NK}(\rho) = 8\pi^2 k^2 L A(\alpha) \beta \left(2^{-\alpha} \alpha \rho^{\alpha-2} \frac{\Gamma\left[-\frac{\alpha}{2}\right]}{\Gamma\left[\frac{\alpha}{2}\right]} \right) \quad (3)$$

which evaluates to $D(\rho) = 2.91 k^2 C_n^2 L \rho^{5/3}$ when $\alpha = 11/3$ (Kolmogorov turbulence).

A noted weakness of the form of Eq.1 is the lack of a finite inner and outer scales of turbulence. Experimental evidence suggests that a finite outer scale leads to saturation in phase variance at large separations [6]. The generalized von Kármán spectral model,

$$\Phi^{gVK}(\kappa) = A(\alpha) \beta (\kappa^2 + \kappa_0^2)^{-\alpha/2} \quad (4)$$

accounts for this saturation by including an outer scale, L_0 , at wavenumber $\kappa_0 = 2\pi/L_0$. The resulting WSF can be expressed in terms of a K-type Bessel function, K_ν ,

$$D^{gVK}(\rho) = \frac{\kappa_0^{2-\alpha}}{\alpha-2} - \frac{2^{-\alpha/2} \alpha \kappa_0^{1-\alpha/2} \rho^{\alpha/2-1} K_{1-\alpha/2}(\kappa_0 \rho)}{\Gamma\left[1 + \frac{\alpha}{2}\right]} \quad (5)$$

In Eq.5 the leading factor of $8\pi^2 k^2 A(\alpha) \beta L$ has been omitted to make the expression more compact.

Finally, both inner and outer scale size effects can be included in what we refer to as the generalized Analytic spectrum [17]

$$\Phi^{gA} = A(\alpha) \beta \frac{\exp(-\kappa^2/\kappa_m^2)}{(\kappa^2 + \kappa_0^2)^{\alpha/2}} \quad (6)$$

Where $\kappa_m = c(\alpha)/l_0$ and $c(\alpha) = \left[\frac{2}{3}\pi\Gamma\left[\frac{5-\alpha}{2}\right] A(\alpha)\right]^{\frac{1}{\alpha-5}}$ [1] and l_0 is the inner scale size. The WSF integral including Eq. 6 does not have a closed form solution; it may, however, be evaluated numerically. Alternatively, the integral may be approximated by expanding the Bessel function in a power series [1], [5, p.194]. After several subsequent approximations the generalized form of the WSF may be expressed in terms

of a generalized hypergeometric function, ${}_1F_1$ [1],

$$\tilde{D}_{gA}(\rho) = \frac{1}{2} \left\{ \kappa_0^{4-\alpha} \frac{\rho^2}{4} \frac{\Gamma\left[\frac{\alpha}{2} - 2\right]}{\Gamma\left[\frac{\alpha}{2}\right]} + \Gamma\left[1 - \frac{\alpha}{2}\right] \kappa_m^{2-\alpha} \times \left[1 - {}_1F_1\left(1 - \frac{\alpha}{2}; 1; -\frac{\kappa_m^2 \rho^2}{4}\right)\right] \right\} \quad (7)$$

Again, the preceding constant terms have again been omitted. For all practical values of l_0 and ρ of interest of $\frac{1}{4}\kappa_m^2 \rho^2$ is large. Consequently, the hypergeometric function must be approximated. In [1], Toselli uses the approximation

$${}_1F_1(a; b; -z) \simeq \frac{\Gamma(c)}{\Gamma(c-a)} z^{-a} \quad (8)$$

while the approximation

$${}_1F_1(a; b; z) \simeq \left(1 - \frac{\alpha}{2}\right) z \times \left\{1 + z \left(\frac{2}{(\alpha-2)\Gamma(\frac{\alpha}{2})}\right)^{\frac{2}{\alpha-4}}\right\}^{\frac{\alpha}{2}-2} \quad (9)$$

is preferred by other authors [5], [4], [18]

Variations on the exponential model [5, Eq.23, p.69] [6, p.45] have also found application recently [19], [9], [10] possibly because they result in more tractable expressions of analytical quantities. In generalized form the exponential model is

$$\Phi^{GX}(\kappa) = A(\alpha) \beta \kappa^{-\alpha} (1 - \exp(-\kappa^2/\kappa_0^2)) \quad (10)$$

which has the generalized WSF

$$D^{GX}(\rho) = \frac{2^{-\alpha-1} \kappa_0^{-\alpha}}{\rho^2 \Gamma\left[\frac{\alpha}{2}\right]} \left(2^\alpha \pi \kappa_0^2 \rho^2 \csc\left(\frac{\pi\alpha}{2}\right) \times \left\{ L_{\frac{\alpha}{2}-1}\left(-\frac{1}{4}\kappa_0^2 \rho^2\right) - 1 \right\} - 4\Gamma\left[1 - \frac{\alpha}{2}\right] (\kappa_0 \rho)^\alpha \right) \quad (11)$$

in terms of Laguerre polynomial L_ν . Convenient as it may be, the asymptotic phase variance predicted at separations of $\rho \sim L_0$ differ significantly from the von Kármán model and do not necessarily agree with experimental data [6]. As we will see, this discrepancy results in very different behavior even at modest separations.

3. COMPARISON OF GENERALIZED WSF MODELS TO NUMERICAL EVALUATION OF ANALYTICAL MODEL

Normalized forms of the various WSFs are plotted together in Fig.1 on a log-log scale for $\alpha = 11/3$ with separation ρ , normalized to the outer scale value, L_0 . When required the inner scale is set to $l_0 = 0.001 L_0$. In Fig.1, the traditional Kolmogorov spectral model appears a straight line with the expected 5/3 slope. The von Kármán and Analytic spectrum, evaluated numerically, overlap exactly over most of the range differing only in the dissipation region where the von Kármán spectrum is equivalent to the Kolmogorov model. At larger

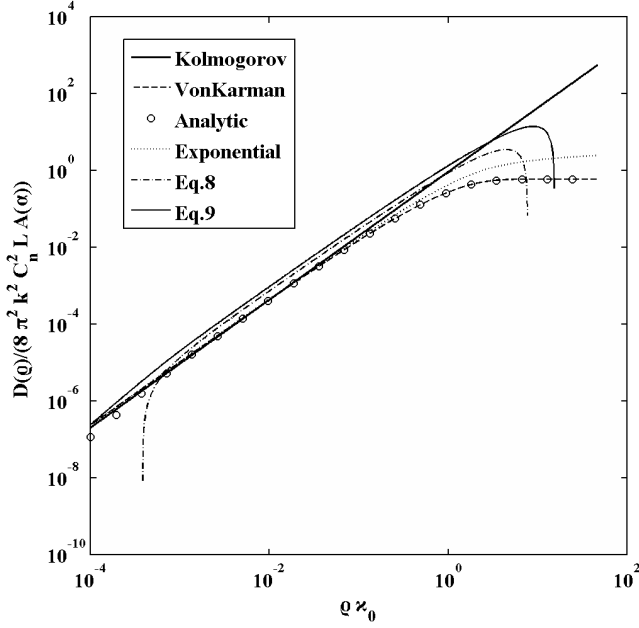


Figure 1. Comparison of the Normalized WSF resulting from various spectral models in terms of normalized separation variable ρ/ρ_0 . In all cases $\alpha = 11/3$

separations, the Analytic, exponential, and von Kármán WSF all deviate from the Kolmogorov spectrum significantly starting at about $\rho/\rho_0 = 0.1$. We point out that only the Analytic and von Kármán models demonstrate strong saturation behavior; the slope of the exponential model decreases but does not saturate over the range evaluated. Surprisingly, the approximations based on Eq.7 deviate significantly from the numerical evaluation of the WSF using the Analytic spectrum over the entire range examined.

As another point of comparison, spatial coherence, defined as $\exp\{-0.5D(\rho)\}$, is plotted in Fig.2 for each of the models in Fig.1. In Fig.2 $\alpha = 11/3$, $\lambda = 1.064 \mu\text{m}$, $\beta = C_n^2 = 10^{-17} \text{ m}^{-2/3}$, $L = 10 \text{ km}$, $L_0 = 5 \text{ m}$, and $l_0 = 5$, with corresponding log-amplitude variance of $\sigma_\chi^2 = 0.02$. As in Fig.1 the analytical WSF and the von Kármán WSF overlap over the entire range. Also, in Fig.2 the WSF as specified by the approximations to Eq.7 result in a negative values around $\rho \sim 0.75 \text{ m}$ and $\rho \sim 1 \text{ m}$ for the approximations given by Eq.8 and Eq.9 respectively. The inset plot shows the squared error between the approximations and the numerical analytical spectrum WSF at separations normalized to L_0 . Both figures suggest a limit on the validity of $\rho \sim 0.1 - 0.2 L_0$. However, the error in the approximations varies with L_0 and integrated turbulence strength. For example, when $\alpha = 11/3$, $\beta = C_n^2 = 3 \times 10^{-15} \text{ m}^{-2/3}$, $\lambda = 1.064 \mu\text{m}$, $L = 3 \text{ km}$, $L_0 = 3 \text{ m}$, and $l_0 = 3 \text{ mm}$ (i.e. daytime horizontal path conditions) the squared error has a value of 1 at $\rho/L_0 \sim 0.01$ and 0.1 for the approximations in Eq.8 and Eq.9 respectively. Similar limits are observed when $\alpha = 11/3$, $\beta = C_n^2 = 10^{-17} \text{ m}^{-2/3}$, $\lambda = 1.064 \mu\text{m}$, $L = 25 \text{ km}$, $L_0 = 20 \text{ m}$, and $l_0 = 10 \text{ mm}$ (i.e. high elevation horizontal conditions).

As illustrated in Fig.1 and 2 the choice of spectral model can have a significant impact on the behavior of the WSF. What's more it is important to note these differences when examining the effect of a change in power-law. Consider, for example,

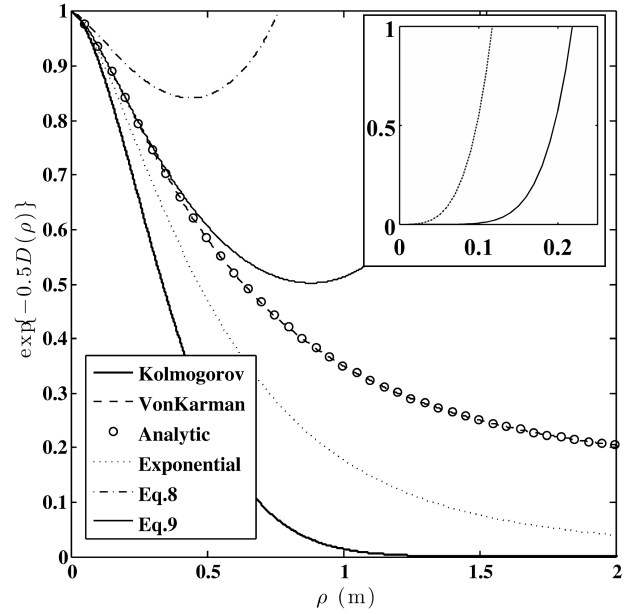


Figure 2. Spatial coherence as $\exp\{-\frac{1}{2}D(\rho)\}$ for $\alpha = 11/3$, $C_n^2 = 10^{-17} \text{ m}^{-2/3}$, $\lambda = 1.064 \mu\text{m}$, $L = 10 \text{ km}$, $L_0 = 5 \text{ m}$, $l_0 = 5 \text{ mm}$. (Inset) Squared error between approximations in Eq. 8, Eq. 9 and the WSF evaluated numerically using Analytic spectrum as a function of ρ/L_0 .

the variance of the angle of arrival, $\langle\beta_a^2\rangle$, defined in terms of the WSF, D , and aperture diameter, W_G [1], [9] as,

$$\langle\beta_a^2\rangle = \frac{D(W_G)}{(2kW_G)^2} \quad (12)$$

In Fig.3 $\langle\beta_a^2\rangle$ is evaluated for four aperture sizes as a function of α for the WSF models discussed in the previous section. The parameters in Fig.3 are the same as those used in Fig.2. In the figure we see excellent agreement between the Analytic and von Kármán models for all aperture sizes. The approximation in Eq.9 also matches the analytical model when aperture sizes are modest ($D < \sim 0.1 \text{ m}$). As aperture size increases so does the discrepancy between Eq.9 and the analytical model. Considering Fig.3(d) the approximation in Eq.9 is clearly not valid for large apertures. Also, outside the region near $\alpha \sim 3$ neither the approximation in Eq.8 or the exponential model match the other models. We expect this discrepancy in the case of the exponential model and observe it predicts consistently larger variance in the angle of arrival. On the other hand, the discrepancy associated with Eq.8 is inconsistent: predicting larger values for small apertures and negative, non-physical, values when the aperture is large.

4. EFFECT OF A CHANGE IN POWER-LAW ON ACCURACY OF VON KÁRMÁN MODEL

We conjecture that the higher fidelity of the generalized spectral model including an inner scale term has motivated its adoption in works related to non-Kolmogorov turbulence. After all, it makes sense that more complete models would increase the accuracy of quantities derived from these models. However, as we demonstrated in the previous section, using simplifying approximations can lead inaccuracies when improperly applied. Still, closed-form analytical expressions

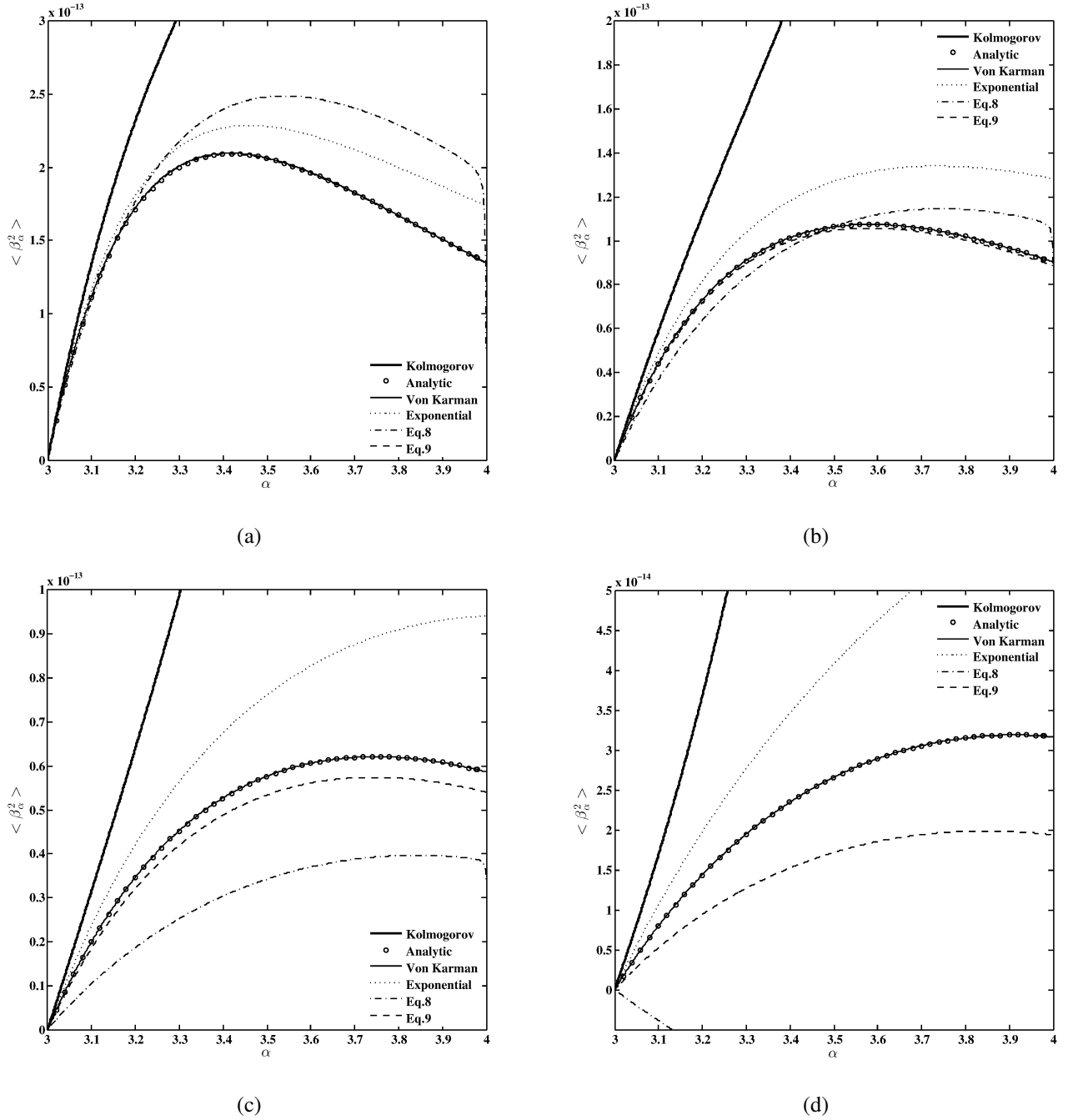


Figure 3. Aperture averaged variance in Angle of Arrival, $\langle \beta_\alpha^2 \rangle$, as a function of power-law exponent, α for the WSF models discussed in Sec.2 for (a) 0.1 m aperture, (b) 0.25 m aperture, (c) 0.5 m aperture, and (d) a 1 m aperture. In each sub figure $C_n^2 = 10^{-17} \text{m}^{-2/3}$, $\lambda = 1.064 \mu \text{m}$, $L = 10 \text{ km}$, $L_0 = 5 \text{ m}$, and $l_0 = 5 \text{ mm}$.

are no doubt useful. For that reason, we suggest using the generalized von Kármán for analytical modeling of optical propagation and imaging parameters that are dominated by phase effects.

Still, the generalized von Kármán model will not apply in every circumstance. For instance, in Kolmogorov turbulence ($\alpha = 11/3$) we would expect the model to be inaccurate at scale sizes on the order of the inner scale, l_0 . However, this is not necessarily case at other power laws. To that

point, the generalized von Kármán WSF at different values of α is plotted in Fig.4 for $\alpha = 3.1, 3.3, 3.66(11/3), 3.8, 3.9$ together with the analytical WSF evaluated numerically at the same values. In Fig.4, the WSF at different power-law exponents, α , appear in the same range only because the displayed values are normalized by the consistency factor, $A(\alpha)$. As pointed out by elsewhere, [15] for example, there is no defined equivalency between refractive index structure parameter, β in non-Kolmogorov turbulence. Nonetheless, the von Kármán model matches the Analytic WSF over the

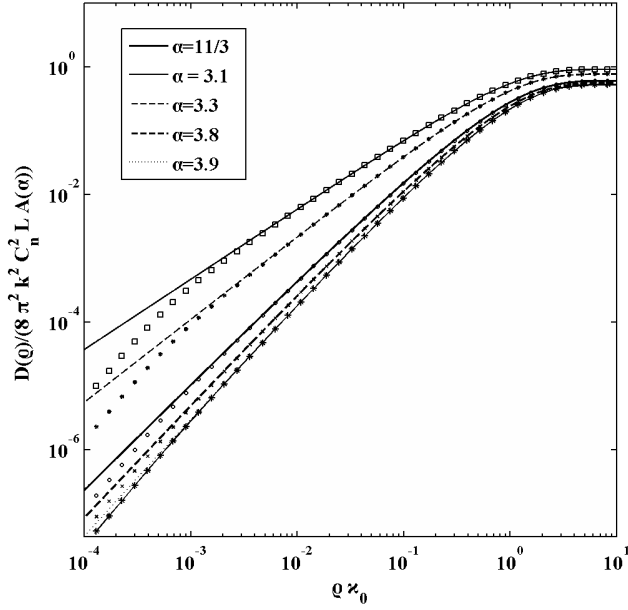


Figure 4. Inner-scale and inertial range comparison of the WSF using the generalized von Kármán models (solid lines) and the analytical model (markers) for various values of power-law exponent; in the figure $l_0 = 0.001L_0$.

range $\rho > l_0$ for all values of $3 < \alpha < 4$. As expected, deviations occur below this range and are more severe when $\alpha < 11/3$. Fig.4 also makes it clear that a decreasing power law exponent increases the saturation value, from Eq. 3, we find,

$$D_{sat}(\rho \gg L_0) = 2\beta k^2 L \kappa_0^{2-\alpha} \cos\left(\frac{\pi\alpha}{2}\right) \Gamma[\alpha - 2] \quad (13)$$

with the additional implication that the phase variance in the generalized von Kármán model saturates to $\langle \phi_{NK}^2 \rangle = \frac{1}{2} D_{sat} = \beta k^2 L \kappa_0^{2-\alpha} \cos\left(\frac{\pi\alpha}{2}\right) \Gamma[\alpha - 2]$.

5. CONCLUSION

In conclusion, we suggest the generalized von Kármán spectrum be used in developing analytical models for phase fluctuations in non-Kolmogorov turbulence. Fig.1,2, 3, and 4 clearly show equivalency between the generalized analytical, Eq. 6 and von Kármán spectrums, Eq. 4 for $\rho > l_0$ over $3 < \alpha < 4$. Equally important here are findings regarding approximation used to Eq. 7. Specifically, when Eq. 8 is used to approximate the hypergeometric function, Eq. 7 valid only when, $\rho < 0.01L_0$. Similarly, the range of validity is $\rho < 0.1L_0$, when Eq. 9 is used. Moreover, as demonstrated in section 4, the use the generalized Analytic and Andrews models for phase quantities results in cumbersome expressions. More compact and useful expressions result from instead incorporating the von Kármán spectrum. To that point we have presented generalized expression for saturated phase variance in non-Kolmogorov turbulence based on the generalized von Kármán model.

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