Appendix A: Radial velocity analysis per system

In this section, we give the details of the RV analysis for the individual targets.

Appendix A.1: Strong candidate

GJ 3470

Located in the hot-Neptune desert, GJ 3470 b has been object of many studies trying to bring light to the rare population to which it belongs. As a result, we account with multiple high-precision RV measurements. The RV analysis requires a linear trend probably caused by an undetected stellar companion, for which we inferred a slope of $(-0.0022 \pm 0.0011)\,\mathrm{m\,s^{-1}\,d^{-1}}$. The secondary eclipse of this planet has been observed, allowing us to set normal priors on the c and d parameters that were derived in this work using the eclipse published in Benneke et al. (2019), and obtaining compatible results with those from Kosiarek et al. (2019) (see Table **??**). Even there is no clear correlation between the RV measurements and the activity indicators, we included a GP informed with the S_{HK} index as it shows a peak in its GLS at $P_{\mathrm{rot}} = 21.5 \pm 0.5$ d. The resulting α places this target as a SC with a 3-σ significance.

Appendix A.2: Weak candidates

GJ 486

This single-planet target has an α parameter different from 0 in 1-σ in both slightly eccentric ($e < 0.1$) and circular orbit scenarios. We do not include a GP to the analysis since, according with Caballero et al. (2022), none of the periodograms of the activity indicators (e.g., H_α, Ca II, or Na I) show any significant peak that could suggest the star is active.

GJ 3473

Our RV analysis for the confirmed planet GJ 3473 b resulted in an α parameter different from 0 within 1-σ for both tested models, the slightly eccentric ($\alpha = -0.30^{+0.23}_{-0.24}$) and the circular orbit ($\alpha = -0.28^{+0.21}_{-0.22}$). Based on the H_α emission line, this target is considered inactive (Jeffers et al. 2018).

HD 260655

Both planets in this system transit the star with short periods, lying in a close 2:1 MMR ($P_b = 2.8$ d, and $P_c = 5.7$ d). However, these rocky worlds do not show significant TTVs according with Luque et al. (2022). Based on the values obtained for the H_α emission and R'_{HK}, this star is thought to be inactive with a rotation period of around ~30 d. Our analysis found that, contrary to the inner planet, the outer one has an α parameter different from zero within 1-σ ($\alpha = -0.32^{+0.22}_{-0.25}$) indicating that the mass of its potential co-orbital companion could be higher than the one for the inner planet. This result is hold when considering circular orbits. As warned in Leleu et al. (2017) (and discussed in Sect. **??**), planets in 2:1 MMR are susceptible to mimic co-orbitals in the α-test method. Nonetheless, as no TTVs are detected and the RV time span is huge (24 years) in comparison with the orbital periods, a false positive caused by the planetary configuration is not expected. Detailed searches to constrain their presence is needed, such as photometrically inspect the Lagrangian regions in the search for dimmings (see Sect. **??**).

K2-18

With an orbital period of 32.9 d, K2-18 b is a Super-Earth in the habitable zone (HZ) of an M-dwarf. We use the RVs extracted with the recent line-by-line (LBL) method (Artigau et al. 2022) published in Radica et al. (2022). They identified 14 outliers in CARMENES and three in the HARPS datasets, resulting in a total time series of 147 measurements after discarding those data points. This system has been claimed to host an additional not-transiting planet in a 9.2 d orbit confidently detected with the LBL RVs after rejecting a particular night (December 25, 2016). We include a GP informed with the dLW, indicator showing a peak in the GLS at the estimated stellar rotational period (~ 39 d). The analysis suggests the presence of a co-orbital for K2-18 b with an $\alpha = 0.48^{+0.29}_{-0.28}$.

K2-141

K2-141 b is an ultra-short period planet orbiting at 0.28 d. There is an additional validated transiting planet with a period of 7.7 d that is not detected with the RV dataset. Therefore, we carried out the analysis only considering planet b. The RV dataset shows a considerably big scatter. Besides, all the GLS of the activity indicators present a peak at $P_{rot} = 14$ d, so we add a GP by using the FWHM as proxy. Our analysis was compatible with a configuration with an low-mass co-orbital within L_4 as $\alpha = 0.07 \pm 0.07$.

K2-199

The K2 mission detected two transiting planets orbiting around K2-199, at 3.2 and 7.4 d orbital periods. Activity tracers suggest the star is moderately active. Nonetheless, Akana Murphy et al. (2021) opted for a model not including a GP since the spot behaviour of the star changed between the campaigns and therefore they did not achieve the convergence of the hyperparameters. Not accounting for the activity either, we found that the outer planet has a similar α for both eccentric ($\alpha = -0.25^{+0.23}_{-0.24}$) and circular ($\alpha = -0.26^{+0.20}_{-0.21}$) models.

LHS 1140

Two transiting planets orbit this M-dwarf star with periods of 3.8 (LHS 1140 c) and 24.7 d (LHS 1140 b, lying within the HZ). LHS 1140 has already been target of co-orbital search through the α-test in Lillo-Box et al. (2020). There, it is discarded the presence of a co-orbital for LHS 1140 b more massive than 1 $M_\oplus$, but found a co-orbital candidate for LHS 1140 c. Here, we re-analyze the system using the same observations but with a different RV extraction (using the LBL method, Cadieux et al. 2024) that is expected to improve the uncertainty on the inferred α parameter, and with some changes in the analysis as mentioned in Sect. **??**. An additional external planet was proposed in Lillo-Box et al. (2020) based on the RV signal, but it was not supported by the new LBL dataset (Cadieux et al. 2024). Therefore, we consider LHS 1140 as a two-planets system. A GP is clearly needed in the model, with a rotational period of around 131 days appearing in the FWHM. Both transiting planets arise to be WC, with $\alpha_b = -0.11^{+0.07}_{-0.02}$, and $\alpha_c = -0.04 \pm 0.02$, assuming circular orbits.

LP 714-47

LP 714-47 b is a planet located in the hot-Neptune desert ($P = 4.1$ d, $R = 4.7\,R_\oplus$). The scientific interest of this target motivated its RV follow-up using multiple instruments, such as ESPRESSO and CARMENES. In the original publication, Dreizler et al. 2020 found that the rotational period of the star is present in the GLS of the RVs (~ 33 d). Though, according with the periodogram of the activity tracers presented in their work, that signal is not obvious in any of them. For this reason, and since such indicators are not available and our residuals do not show much scatter, we do not include a GP in our model.

LTT 3780

A pair of TESS-candidate planets were confirmed using spectroscopic data around LTT 3780 with orbital periods of 0.8 d and 12.3 d. The star is known to be rather inactive, as indicated by the $R'_{\rm HK}$ value of −5.59 (Cloutier et al. 2020a). Additionally, no modulation appears in the photometric data. Given that no strong activity impact is expected, we chose not to include a GP in this test.

Qatar-2

Qatar-2 b is a hot Jupiter in a 1.3 d orbit around a K-dwarf star. Its strong photometric variations reveal the active nature of the star with a rotational period of ~ 18.5 d, which corresponds with a gyrochronological young age of around 1.4 Gyr (Esposito et al. 2017; Močnik et al. 2017). From the RV dataset in hand, only HARPS-N data have activity tracers available (nine out of the 80 out of transit data points). Such few FWHM measurements are not enough to account for the activity contribution on the signal and thus we do not include a GP to our analysis. We obtain an α different from zero in 1-σ ($\alpha = -0.02 \pm 0.02$, in the circular case).

TOI-269

The sub-Neptune (2.8 $R_\oplus$, 8.8 $M_\odot$) orbiting this M2-dwarf star with a 3.7 d period has been reported to be highly eccentric (~ 0.43). This solution is obtained by Cointepas et al. (2021) when fitting together transits and RV datasets. Based on the mature stage of the system (~2-4 Gyr) and the short orbital period of the planet, it would be expected to have circularized the orbit. To explain the puzzling result, they invoked the inward migration of the planet through a planet-planet interaction as an hypothesis. As explained in Sect. **??**, the eccentricity is highly degenerated with α, and thus it was not surprising to obtain an α different from zero ($\alpha = 0.29^{+0.20}_{-0.18}$ for a circular orbit) in our analysis. Since the occultation of this planet has not been measured, the eccentricity is not constrained and the co-orbital scenario is also a plausible alternative to the uncommon high-eccentricity. As the authors explain in their work, the host star is though to be quiet and probably has a long rotation period (~ 69 d). As in the original work, we did include a linear trend to the analysis with a resulting slope significantly different from zero (0.021 ± 0.008 m s^{-1} d^{-1}).

TOI-544

The star hosts a small planet discovered by TESS in a short period of 1.5 days. The RV dataset is composed of HARPS and HARPS-N campaigns taken over nearly three years, which enabled the detection of a non-transiting Neptune-mass planet at a 50.1 day period. The S-index shows a very clear peak at 19 days, which we interpret as the rotational period. When we include a GP the scatter of the RV residuals are greatly reduced, yet it is still larger than in Osborne et al. (2024) since they use a two-dimensional GP. The result of the analysis is compatible with the existence of a co-orbital companion within the L_4 region ($\alpha = 0.57^{+0.19}_{-0.22}$).

TOI-776

Three signals are required in order to model the RVs of this system: two Keplerians to account for the two transiting planets, and a sinusoidal function describing the activity. The sinusoidal function has the periodicity of the rotation of the star (34.4 days), and an amplitude of 2.7 m s^{-1}. It was shown by Luque et al. (2021) that this model explains well the observations and it is not necessary to appeal to a GP. Both planets are mini Neptunes with orbital periods of $P_b = 8.3$ d and $P_c = 15.7$ d. The signal for the outer planet is inadequately sampled based on the criterion adopted at the beginning of this section (i.e., there is a phase gap greater than 15% for P_c), and hence, we only put attention on the α parameter inferred for the inner planet. The inferred aforementioned parameter is different from zero ($\alpha = -0.62^{+0.29}_{-0.24}$ for the circular case) and thus it could point to a mass imbalance towards the L_5 region. We warn that, even the period of planet b is well sampled according with our criterion, there exists a scarcity of data around the orbital phase $\phi = 0$.

TOI-836

Two transiting planets were detected using two TESS Sectors (11 and 38): a super-Earth (P_b = 3.8 d), and a mini-Neptune (P_c = 8.6 d). The K V star is very active being imperative the use of a GP to detect the planetary signals in the HARPS RV dataset. We use the FWHM, which informs on the stellar rotational period being around 22 days. Based on the discovery paper (Hawthorn et al. 2023), the existence of TTVs with an amplitude of ∼20 min for TOI-836 c may suggests the existence of an outer planet not yet detected either by RVs or transits. As discussed in Sect. **??**, TTVs can induce false positive co-orbitals since the T_0 varies. However, the TTVs amplitude is very low in comparison with the RV time-span so we do not expect a big impact on our analysis. The results locate both targets as interesting candidates with not-null α at a 2.4 and 2.8-σ level for the inner and outer planets respectively. Furthermore, the light curve inspection comes as a suprise, with an interesting dimming in the L_4 region of TOI-836 b that is compatible with the location and the radius inferred from its α value (see Sect. **??**).

TOI-1130

This system is composed of a K-dwarf orbited by an inner Neptune-sized planet (TOI-1130 b) and an outer hot Jupiter (TOI-1130 c). TESS light curves show that the time of mid-transit of both planets varies at each orbit, with an amplitude of 2.5 h for TOI-1130 b, and 0.25 h for TOI-1130 c. This is cause by the resonant configuration in 2:1 MMR (P_b = 4.1 d, P_c = 8.4 d). The HARPS RV time series present a linear trend with a slope of $\sim 0.5\,\mathrm{m\,s^{-1}\,d^{-1}}$ likely induced by an undetected outer companion. Both planets are clearly detected with these RVs, and the result of our analysis return not-null α values for both planets with very high significance ($\alpha_b = -0.44^{+0.12}_{-0.11}$, and $\alpha_c = -0.09 \pm 0.01$). However, there is a high probability that these results are affected by the known TTVs, even the time span of the RV dataset is higher than the TTVs period. For this reason, we do not consider them as SC but as WC (see Sect. **??**).

TOI-1235

This M0.5 dwarf is orbited by the transiting planet TOI-1235 b, which is located in the radius gap (∼1.7 $R_\oplus$, ∼6-7 $M_\oplus$) in a 3.44 days orbit (Bluhm et al. 2020; Cloutier et al. 2020b). The RV data set comprises CARMENES, HARPS, and HIRES measurements. Its GLS shows an additional signal around 21 days, which is also present in the dLW GLS. Therefore, we include a GP informed with this activity indicator and considering a normal prior for η_3 of $P_{\rm rot} = 20.93 \pm 0.6$ d. The result barely places the target as a WC, with a not-null α just at 1-σ level ($\alpha = 0.12^{+0.12}_{0.11}$).

TOI-1452

TOI-1452 b is a super-Earth in a 11.1-days orbital period first detected by its transit with TESS, and later confirmed with SPIRou and IRD RVs (Cadieux et al. 2022). For the analysis, we opt for not including the IRD data since they are few measurements in comparison with the SPIRou not including additional information in our case but increasing in one the number of free parameters. For most of the SPIRou nights around three to four measurements were taken. For this reason, we use the night-binned measurement to reduce the measurement uncertainty. As detailed in the discovery article, the host star does not show any obvious rotational period, so we consider it as a quiet star. As the orbital period is higher than 10 days, we do not test the circular model (see Sect. **??**). The inferred α suggest the presence of a co-orbital companion ($-0.51^{+0.33}_{-0.30}$).

TOI-1695

A wealth of SPIRou and HARPS-N measurements enabled confirming the planetary nature of the transiting super-Earth TOI-1695 b identified by TESS (P = 3.13 d, $\sim 2.0 R_\oplus$, $\sim 6 M_\oplus$, Kiefer et al. 2023, Cherubim et al. 2023). The photometric estimation of the rotational period of the star ($P_{\rm rot} \sim 48$ d) corresponds with an inactive early M-dwarf, which is consistent with the spectroscopic indices showing very weak indicators of activity. The circular model of our analysis is compatible with the presence of a co-orbital within 2.5-σ ($\alpha = -0.40^{+0.14}_{-0.16}$).

TOI-3757

TOI-3757 b is one of the few gas giants orbiting an M-dwarf (P = 3.4 d, around 12 $R_\oplus$, and $\sim 85 M_\oplus$, Kanodia et al. 2022). The available RV dataset is composed of HPF and NEID measurements. We do not include a GP since it does not show hints of activity. The slightly eccentric and the circular model provide compatible results, with an $\alpha_{\rm c} = -0.25 \pm 0.13$. We note that this target labeled as a WC is a good opportunity to keep its RV monitoring in order to improve the data set (higher precision and more measurements).

TOI-3884

The TESS photometry of this M4-dwarf shows the transit of a super-Neptune of 6.4 $R_\oplus$ in a 4.5 days orbit affected by star-spots in several orbits. An upper limit to its mass has been confirmed with a dataset of HPF (Libby-Roberts et al. 2023) and a couple of measurements from ESPRESSO (Almenara et al. 2022a). Interestingly, even the star is known to be active, a GP does not improve the RV analysis as tested by Libby-Roberts et al. (2023). In both our eccentric ($\alpha = -0.42^{+0.27}_{-0.28}$) and circular ($\alpha = -0.41^{+0.25}_{-0.27}$) models, this target is a WC with a 1-σ significance.

TOI-3984 A

The RV analysis for this M4-dwarf star is based on a dataset of 41 NEID and HPF measurements. The star is orbited by a transiting giant of 4.4 day period (0.14 M_J, 0.71 R_J), and is the primary of a multiple system with a wide binary. With the available data, this target results a WC at a 1.3-σ level ($\alpha = 0.34^{+0.27}_{-0.26}$).

WASP-43

The case of the hot Jupiter WASP-43 b ($\sim 2M_J$ with a period of 0.8 days) around a K7V star has been widely studied since its discovery (Hellier et al. 2011). Despite what would be expected, considering its location and mass, this planet does not experience orbital decay (Hoyer et al. 2016). For our analysis, we opted for using only the HARPS and ESPRESSO data available in the ESO Archive with Program IDs 0102.C-0820, 60.A-9128, 089.C-0151, 096.C-0331, and 0104.C-0849, since there are a high quantity of them and are of great quality. We note that the observations were mainly taken around the transit as a cause of the original scientific goal. Such observational strategy probably favours obtaining a more accurate measurement of the mid-transit time through RVs, which favours our methodology. Moreover, the occultation of the planet has been observed breaking the α-eccentricity degeneracy (Blecic et al. 2014). The host star is though to be very active based on the high value of the $\log(R'_{\rm HK})$, which could be enhanced by the close Jupiter and its youth (Esposito et al. 2017). However, the rotational period of the star (around 16 d according with Hellier et al. (2011)) is not present in the GLS of the FWHM, therefore we do not include a GP. The result of the analysis suggest the presence of a co-orbital up to 11 $M_\oplus$ within L_5 ($\alpha = -0.007 \pm 0.005$).

Appendix A.3: Inconclusive

GJ 143

Apart from the confirmed planet (GJ 143 b), this star hosts an additional inner transiting candidate at 7.8 d (GJ 143 c). However, the available RVs are not sensitive enough to detect that candidate (Dragomir et al. 2019). Consequently, we only include GJ 143 b in our model and a linear trend to account for a long-term. Among the GLS periodogram of the available activity indicators for the HARPS data subset, we find a peak at 37.1 d (close but not overlapping with the planet orbital period, $P_b = 35.6$ d) in the FWHM of the cross correlation function (CCF), which is compatible with the rotational period proposed in Dragomir et al. (2019). Therefore, we add a GP informed with the FWHM as activity proxy. The uncertainty in the α parameter is greatly increased when including the GP in contrast with the not-null α value (2-σ) inferred for the model without the GP. Therefore, although it falls within the group of INC targets, it is convenient to study this planet by alternative methods such as transits (see Sect. **??**).

GJ 3929

The RV dataset shows the presence of the confirmed transiting hot Earth-sized GJ 3929 b ($P_b = 2.6$ d), and a candidate not transiting sub-Neptune ($P_c = 15.0$ d) around the M3.5V star (Kemmer et al. 2022; Beard et al. 2022). This star is thought to be inactive based on different indicators (e.g., Hα, flat light curve). Even in the discovery article they include a GP, based on the quiet behavior and since the α-test do not show hints for the presence of a Trojan candidate, we opt for not including a GP.

HD 73583

This system is composed of two transiting mini-Neptunes discovered by TESS (TOI-560) which are close to a 3:1 MMR ($P_b = 6.4$ d, and $P_c = 18.9$ d). This is a clear case of a young ($\sim$ 500 Myr) active star affecting the RVs. As in Barragán et al. (2022), we used the $S_{\rm HK}$ time-series as activity indicator to feed the GP, which has a peak in the GLS periodogram at $P \sim 12$ d, as the periodicity found in the TESS light curves. The posterior distributions found are compatible with those from their work. We note that they use a more sophisticated method using a multidimensional GP by adding the derivate of the QP kernel as a component of the RV model, resulting in cleaner residuals of the model. None of the two planets resulted in an α parameter different from zero.

K2-25

Young planetary system (730 $\pm$ 50 Myr, Mann et al. 2016) hosting a Neptune-sized planet at an orbital period of 3.5 d. As expected due to its youth, our preliminary RV analysis show large residuals and thus we decide to test a GP based on the dLW as activity tracer. We set a normal prior for η_3 as the rotational period estimated by Stefansson et al. (2020), $P_{rot} = (1.878 \pm 0.005)$d. The result did not support the presence of a co-orbital. Nonetheless, we note that obtaining more precise RVs is currently possible for this target and it would likely improve this test. Thus, this analysis should be revised in the future.

L 168-9

This M1V dwarf hosts a transiting super-Earth discovered by TESS in a close orbit (1.4 days). Looking at the GLS of the activity indicators, it is not evident whether there is an effect of stellar activity in the RVs: HARPS indices show a forest of peaks near P_{rot} and its harmonics (∼20 days based on Astudillo-Defru et al. 2020), meanwhile for PFS, there are no peaks related. Thus, we opt for not including a GP. Our analysis resulted in an α compatible with no co-orbital mass.

LHS 1478

TESS detected a hot super-Earth in a 1.9 days orbital period that was later confirmed through CARMENES RVs (Soto et al. 2021). In our analysis, we decide to not include the IRD data since they are sparse and do not add significant information to the α-test. This M-dwarf star is inactive based on the H_α emission and therefore, we do not include any GP. We do not find hints for the presence of co-orbitals, but improvement in the analysis could be made by acquiring more RV measurements.

TOI-532

This M-dwarf hosts a super Neptune ($\sim 62\,M_\oplus$) orbiting with a 2.3 day period. The flat light curve detected by TESS suggests that this star is probably inactive (Kanodia et al. 2021). There is no suspect so far that the planet could harbor a co-orbital based on the RVs dataset. Although we note that this target have not been extensively monitored and the uncertainty in α is potentially narrowable.

TOI-969

The late K-dwarf star TOI-969 is transited by a close mini-Neptune in a 1.8 day period (TOI-969 b, located at the lower boundary of the hot-Neptune desert). The combination of 94 RV measurements with HARPS, PFS and CORALIE revealed the presence of a second companion in the system, the candidate TOI-969 c ($\sim 11\,M_J$). We do not include the CAROLIE dataset in our analysis since the associated uncertainties are too high to detect the transiting planet (tens of m/s). As in the original article (Lillo-Box et al. 2023), we model the RV long-term with a Keplerian of $P_c = 1700 \pm 290$ d, which is the signal of the outer candidate. To account for the activity, we include a GP informed with the $\log R'_{\rm HK}$ computed for HARPS. The GLS of such indicator has its strongest peak at ∼24 d, with aliases at 12 and 8 d. The same signal appears in the WASP photometry according with Lillo-Box et al. (2023), which confirms that it corresponds with the rotational period of the K-dwarf. The result does not favor the presence of a co-orbital companion ($\alpha = 0.03^{+0.16}_{-0.14}$).

TOI-1468

Two transiting low-mass planets were detected and confirmed around this M3 V star using TESS photometry, and a dataset of 81 CARMENES and MAROON-X RVs (Chaturvedi et al. 2022). The inner planet orbits the star with a 1.9 d period (TOI-1468 b), while the outer orbits every 15.5 d. The Hα index is a good proxy for the rotational period as its GLS shows a clear signal close to 41 d. The preferred model of our analysis (including the GP and with the inner planet in a circular orbit) is compatible with no co-orbitals ($\alpha_b = 0.12^{+0.17}_{-0.10}$, and $\alpha_c = 0.01^{+0.11}_{-0.21}$).

TOI-1470

The RV monitoring of the M1.5 V TOI-1470 with CARMENES confirmed the TESS transiting sub-Neptune TOI-1470 b (P_b = 2.5 d), as well as other transiting planet in a 18.1 d orbit first reported by González-Álvarez et al. (2023), TOI-1470 c. We use Hα as spectral activity indicator of the rotational period (~ 29 d) to perform a GP analysis. The results are compatible with no co-orbitals but with wide uncertainties ($\alpha_b = -0.18^{+0.14}_{-0.19}$, $\alpha_c = 0.44^{+0.39}_{-0.51}$).

TOI-1685

The CARMENES (Bluhm et al. 2021) and IRD (Hirano et al. 2021) RV time series allowed us to confirm the planetary nature of the transiting super-Earth TOI-1685 b, which orbits its M3 V star with an ultra-short period of just 0.7 d. Bluhm et al. (2021) reported a moderated evidence for the presence of a second not-transiting planet in the system (TOI-1685 c) with an orbital period of 9.0 d based on the CARMENES dataset. In our IRD + CARMENES combined analysis, we find an increase in the evidence for its presence, with a logarithm of the Bayes Factor of 10 when comparing the two-planets model with the single transiting planet. We also include a GP informed with the CARMENES dLW indicator, with a normal prior in the rotational period of (18.7 ± 0.7) d. The result is affected of high uncertainty in the inferred α with the available dataset: $\alpha = 0.18^{+0.27}_{-0.16}$.

TOI-1728

With an orbital period of 3.5 d, the super-Neptune TOI-1728 b transits an old and inactive M0-dwarf star (Kanodia et al. 2020). We include a linear trend to our RV model to account for a long-term with an inferred slope of $(-0.14 \pm 0.17)\,\mathrm{m\,s^{-1}\,d^{-1}}$. The results of the α-test (compatible with no trojans with $\alpha = -0.22^{+0.23}_{-0.29}$) are here limited due to the relatively low-precision of the uncertainties associated with its HPF RV measurements, which are in the order of tens of m/s.

TOI-1759

The sub-Neptune TOI-1759 b was first detected through three transits from different TESS sectors. Later, independent works using CARMENES (Espinoza et al. 2022) and SPIRou (Martioli et al. 2022) RVs confirmed the planet orbiting the M0-dwarf star with a 18.8 d period. Our analysis is only based on the CARMENES dataset since it has a better presicion compared with the SPIRou measurements, and we checked that the latter do not improve the inferred α. We introduce a GP informed with the dLW, which shows the rotational periodicity at $\sim$ 36 d and is consistent with the inferred from the SPIRou dataset. The inferred α is $-0.23^{+0.24}_{-0.27}$.

TOI-1801

TOI-1801 b was identified by TESS photometry and confirmed by CARMENES and HIRES RVs. It is a transiting young mini-Neptune orbiting with a 10.6 d period a M dwarf younger than 1 Gyr (Mallorquín et al. 2023). We include to the MCMC analysis a GP informed with the Hα time series (peak at $P_{rot} \sim 16$ d in its GLS). The high scatter in the RVs even after the GP correction results in high uncertainties for the inferred α ($-0.05^{+0.33}_{-0.25}$).

TOI-2018

The metal poor K-dwarf TOI-2018 is transited by a mini-Neptune with an orbital period of 7.4 d. We use the available HIRES measurements correcting them from the stellar activity informing a GP through the $S_{\rm HK}$ index ($P_{\rm rot} \sim 21$ d). A vague constraint can be made for the presence of co-orbitals given the high uncertainty of the result ($\alpha = 0.19^{+0.33}_{-0.25}$).

TOI-3785

The M2-dwarf star TOI-3785 hosts a low-density transiting Neptune of 4.7 d orbital period. The parent star is thought to be inactive (Powers et al. 2023) thus no GP is introduced even there is a high scatter in the RV measurements. Note that six of the RV data points have a bigger uncertainty since they were taken with a shorter exposure time. The quality of the data do not allow the α parameter to be constrained, resulting in a non-informative $\alpha = -0.33 \pm 0.33$.

Appendix A.4: Null detections

GJ 1214

This is a extensively studied planetary system in the literature. Only one planet has been detected (the sub-Neptune GJ 1214 b) and the star is known to be inactive. The recent detection of its secondary eclipse (Kempton et al. 2023) confirmed that the planet is almost in a circular orbit. Our analysis was very well constrained not requiring a GP and with informative priors on c, and d. For this reason and in light of the result, this target is perfectly compatible with an scenario with no Trojan, with a very low upper limit to the Trojan mass (0.8 $M_\oplus$, see Sect. **??**).

HAT-P-20

HAT-P-20 has been thoroughly studied since its discovery (Bakos et al. 2011). Multitude of transits and RV measurements have been gathered, including observations of the R-M (Esposito et al. 2017), the absence of significant TTVs have been confirmed (e.g., Sun et al. 2017; Lillo-Box et al. 2018b), and the secondary eclipse has been observed twice in two *Spitzer* bands (Deming et al. 2015) confirming a slightly eccentric orbit. As first noticed by Knutson et al. (2014), the RVs have a linear trend which is potentially caused by the visual companion of HAT-P-20. Thanks to these previous efforts, α is estimated with an small uncertainty being able to reject this object as a co-orbital candidate based on the α-test ($\alpha = 0.000 \pm 0.002$). The result is compatible with the one already presented in Lillo-Box et al. 2018b.

HIP 65 A

This star (TOI-129) hosts a hot-Jupiter of $\sim 3\,M_J$ in a 0.98 d orbital period. Its light curve reveals that the star is active, showing modulations with a periodicity of $\sim$ 13 d. As argued by Nielsen et al. (2020), the available RV dataset is unaffected by the activity since the impact is of the order of $\sim 10\,{\rm m\,s^{-1}}$, which is comparable to the RV uncertainty of the FEROS measurements, and 2-3 times smaller than those from CORALIE and CHIRON. Additionally, none of the available activity indicators show any relevant periodicity. The result places this target as a co-orbital candidate. Interestingly, the result differs when assuming a circular orbit, where the α drops to a value compatible with zero. This is a good example to show the importance of breaking the α-eccentricity degeneracy through the secondary eclipse. In Sect. **??** we predict the expected occultation depths for all the targets, being HIP 65 A one of the most promising ($\delta > 10^4$ ppm). Our solution where α differs from zero corresponds with a slightly eccentric orbit that would imply that the occultation would happen around 18 min before of the expected time, at an orbital phase of 0.487 ± 0.003. Nonetheless, since this planet has a grazing transit, this exercise might not be possible.

LTT 1445 A

LTT 1445 A is the primary star of a hierarchical triple system. It is orbited by the transiting planet LTT 1445 A b, which has an orbital period of 5.4 d. Winters et al. 2022 found the RV signal of a second planetary body in a closest orbit, LTT 1445 A c with 3.1 d. The result of the analysis reject this target as a candidate for both planets (α_b = -0.07 ± 0.08, and $\alpha_c = 0.10^{+14}_{-0.13}$, for the circular case).

TOI-244

The bright M2V dwarf hosts a transiting super-Earth in a 7.4 days orbital period with a characteristic low density. The spectral activity indicators (FWHM, H_α and S-index) clearly show the stellar rotational period in their periodograms at $P_{rot} \sim 56$ days, indicating that this is an active star. As in the discovery article (Castro-González et al. 2023), we include a GP to mitigate this effect from the RVs by using the FWHM as proxy. Since after subtracting the stellar contribution the HARPS data still showing a big scatter, we opt for only including the ESPRESSO dataset to our analysis. As a result of the wealth of high-precision RV time-series, we can constrain the presence of co-orbitals to be below 0.8 $M_\oplus$ ($\alpha = -0.01 \pm 0.12$).

TOI-1075

The Super-Earth TOI-1075 b orbits its M0-type parent star with an ultra-short period of just 0.6 d. The star does not show any sign of activity. The RV dataset requires a linear trend with an slope that we infer to be of 0.14 m s^{-1} d^{-1}, which might be attributed to an outer planetary-mass companion. The PSF RV time-series do not show any shift in the mid-transit time of TOI-1075 b ($\alpha = 0.07 \pm 0.13$).

TOI-3629

TESS discovered a Jupiter orbiting close to this M1-dwarf star with a 3.9 day period. The subsequent gathering of 28 RV measurements with NEID and HPF confirmed this candidate and enables rejecting the possibility of having a co-orbital object more massive than 28.2 $M_\oplus$. Even this target is classified according with Table **??** as a *Null Detection* ($\alpha = -0.01 \pm 0.13$), it would be desirable to decrease the Trojan upper limit by extending the sample and even increasing the precision of the RVs (< 10 m s^{-1}).

WASP-80

This interesting confirmed transiting planet, halfway between warm Neptunes and hot Jupiters (171 $M_\oplus$, P = 3 days), was already studied through the α-test in Lillo-Box et al. (2018a). As its occultation was already known in this previous article, the target has a low uncertainty with an $\alpha = 0.038^{+0.083}_{-0.085}$. Six years later, the RV sample has been increased from 47 to 73 measurements, mainly grouped around the transit since its atmosphere has been studied with HARPS. Thanks to that observational effort, we now get $\alpha = -0.03 \pm 0.04$, reducing the uncertainty in a factor two and therefore, greatly decreasing the upper limit of the Trojan mass (from 44.7 $M_\oplus$ to 8.7 $M_\oplus$ in L_4, and from 28.5 $M_\oplus$ to 19.4 $M_\oplus$ in L_5). This example shows that gathering numerous RVs with orbital phases close to 0 might favor the search for co-orbitals by this method as the mid-transit time could be better constrained.

Appendix A.5: Sparsely sampled

GJ 1252

GJ 1252 b stands out as a Super-Earth of particular interest for characterization due to its notably short orbital period ($P = 0.5$ d), low stellar activity, and the observation of a day-side eclipse. Even though, the RVs for GJ 1252 b have been poorly monitored, with a limited time span of less than 11 days. Consequently, continued tracking of this target is imperative to accumulate more comprehensive data and derive a conclusive α parameter.

TOI-824

The K4-dwarf star is orbited every 1.4 days by one of the few known hot Neptunes (with a radius of 2.9 $R_\oplus$, and a mass around 18.5 $M_\oplus$). The exotic of its nature and its elevated surface temperature makes it a very interesting target for atmospheric characterization. Therefore, its occultation has been measured with *Spitzer* which has constrained the planet to be in a circular orbit. This under-sampled target is perfect to gather additional data to better constrain the α parameter and thus its possibility of harboring a co-orbital companion.

Appendix A.6: Rejected case

GJ 3090

This star hosts a transiting mini-Neptune on a 2.9 d orbit. Almenara et al. (2022b) reported the RV signal of a candidate second planet with an orbital period of 12.7 d. Their RV analysis resulted in a solution where both planets are in very eccentric orbits ($e_b \sim 0.2$, and $e_c \sim 0.3$). Our priors do not allow for eccentric orbits for GJ 3090 b since Eq. **??** is only valid when e < 0.1. Despite the degeneracy between e and α, our solution do not favor the co-orbital scenario. The model found is quite noisy, which translates into a big uncertainty for the α parameter, probably caused by the presence of stellar activity. Indeed, the GLS periodogram of the RVs show some signal at the rotational period ($\sim$ 18 d) and its first harmonic ($\sim$ 9 d). As argued in Almenara et al. (2022b), the quasi-periodic kernel in the GP does not converge, but adding a cosine component (QPC kernel) it improves. We repeat the same procedure using the QPC informed with the H_α indicator. Nonetheless, we still found problems in the convergence since GJ 3090 b is restricted to not very eccentric orbits. This target is therefore outside the validity domain of the α-test. For this reason, we do not include this target in the sample for the discussion.

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