Origins of Sparse Aperture Imaging[1]

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Abstract—Sparse aperture imaging has its roots in the work on optical interferometry by Fizeau and Michelson over one-hundred years ago, and the development of radio astronomy nearly fifty years later. In their quest for higher angular resolution at meter wavelengths radio astronomers were forced to seek alternatives to filled aperture telescopes. Radio interferometers, as these instruments are called, measure the complex visibility, which is the Fourier transform of the source brightness distribution. The earliest instruments measured only the amplitude of the visibility, and it was several years before phase measurements became routine. Both the amplitude and phase of the visibility are needed to produce images of a complex source by Fourier inversion. It wasn't until the 1970s and 1980s that the technology was available to allow modern optical stellar interferometers to be built. This paper traces the history of sparse aperture imaging from early radio measurements to the current generation of ground-based optical interferometers, and discusses their general principles of operation.

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1. INTRODUCTION

Sparse aperture imaging has its origins in astronomy and the quest for higher angular resolution. In 1867 Fizeau realized it should be possible to measure the diameters of the stars by observing the interference fringes produced by two small apertures place in front of the objective of a telescope. By 1890 Michelson had a fully developed theory of stellar interferometry, and applied the technique to the measurement of the diameters of the satellites of Jupiter. The first diameter measurement of a single star, Betelgeuse, was done in 1921.

The development of radio astronomy in the late 1940s and early 1950s included the first application of interferometry techniques at meter wavelengths. This was necessary in order to get enough resolution to study active regions on the surface of the sun.

During this period radio astronomers realized that it was possible to generate images from the interferometric data by Fourier inversion, and these techniques were perfected and refined over the next forty years. It wasn't until the late 1970s and early 1980s that sparse aperture techniques were developed at optical wavelengths using separate telescopes. These techniques have been extended and improved in the current generation of sparse aperture instruments.

2. EARLY STELLAR INTERFEROMETRY

Fizeau and Stéphan

Hippolyte Fizeau writing in 1868 in *Comptes Rendus* mentions, almost in passing, that it should be possible to measure the diameters of the stars using interference techniques similar to those used to observe Young's fringes [1]. The idea is to place mask over the objective of a standard astronomical telescope [2]. The mask will have two small holes aligned along a diameter, with a separation somewhat less than the full diameter of the objective. The arrangement is sketched in Figure 1 for a refracting telescope. Light from two objects, O_1 and O_2, separated by an angle θ on the sky, and at an effective wavelength λ_0, passes through apertures S_1 and S_2. The apertures will each give an interference pattern at P in the focal plane with the same fringe spacing, the resultant intensity distribution at P is formed by adding the intensities of the two patterns. Born and Wolf [2, p. 302] show that the two patterns are displaced by Δm fringe orders, where

$$\Delta m = \frac{|MS_2 - NS_2|}{\lambda_0} = \frac{\theta d}{\lambda_0}. \tag{1}$$

Here θ is assumed to be small and the index of refraction is about 1. The resultant fringes will be most distinct when Δm takes on integral values (0,1,2,3,...), whereas for half-integral values $(\frac{1}{2},\frac{3}{2},\frac{5}{2},...)$, the intensity maxima from O_1 coincide with the minima from O_2; and the fringes disappear if the intensities from the two objects are equal. As the separation of the apertures is increased from zero the fringes will show a periodic variation of distinctness. At the first minimum of distinctness

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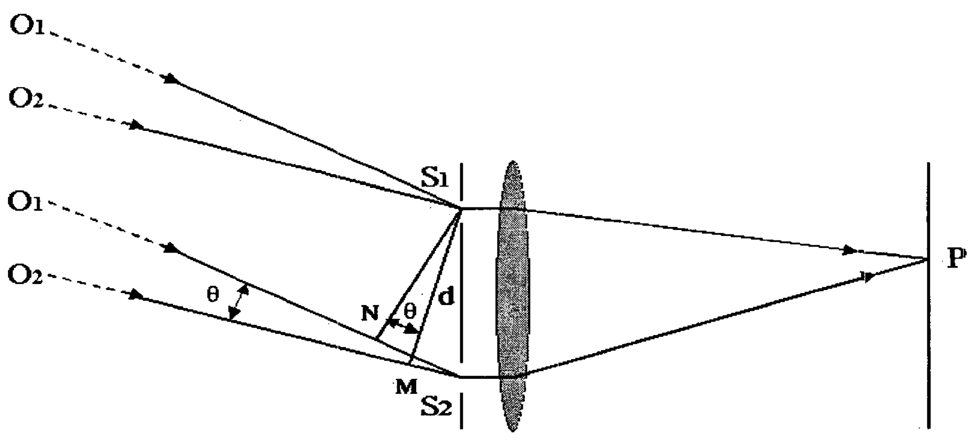


Figure 1 - Refracting telescope objective with a two-aperture diaphragm.

$$d = \frac{\lambda_0}{2\theta}. \qquad (2)$$

Thus, we can determine θ by noting the value of d the first time the fringes disappear.

About five years after Fizeau's original suggestion, Edouard Stéphan, the director of the Marseilles Observatory, reported the successful detection of stellar fringes using Fizeau's technique. In fact, Stéphan found fringes on all the stars he observed, and concluded that the fixed stars must have diameters significantly smaller than 0.158 arc seconds, the limit of his observations [3,4].

Michelson

By 1890, Albert Michelson had a fully developed mathematical theory of stellar interferometry [5]. It seems likely that Michelson developed his theory without any knowledge of the work by Fizeau and Stéphan, although he may have met Fizeau during the year he spent in Europe starting in the fall of 1880 [6]. In the 1890 paper Michelson introduces the concept of fringe visibility, define as

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}, \qquad (3)$$

Where I_{max} is the intensity at the center of a bright fringe and I_{min} is the intensity at the center of a dark fringe. Michelson then proceeds to derive expressions for V for various objects: a uniform disk, and a disk with a non-uniform illumination, and a double star. This remarkable paper contains all the elements of the modern theory of stellar interferometry. Within 18 months, Michelson had demonstrated his technique by measuring the diameters of the four brightest moons of Jupiter [7].

In general, the output of a two-aperture interferometer can be written as (Born and Wolf [2], Traub [35, Capt.3]),

$$I_{\mathrm{int}}(\theta) = 2I_{app}(\theta)[1 + V\cos(2\pi\theta B/\lambda)], \qquad (4)$$

where $I_{\mathrm{int}}(\theta)$, is the output intensity of the interferometer as a function of position in the focal plane, θ, B is the aperture separation and λ is the wavelength. $I_{app}(\theta)$ is the aperture function, which is

$$I_{app}(\theta) = \left[\frac{2J_1(\pi\theta D/2\lambda)}{\pi\theta D/2\lambda}\right]^2 D^2, \qquad (5)$$

for an circular aperture of diameter D. V is the visibility and is a function of the object observed. For a point source, V=1, and for a stellar disk with a uniform brightness distribution,

$$V_{UD} = \left[\frac{2J_1(\pi B\theta_{UD}/\lambda)}{\pi B\theta_{UD}/\lambda}\right]. \qquad (6)$$

Thus, what is seen is the diffraction pattern produced by the sub-apertures crossed with interference bands.

Nearly 30 years passed before the diameter of a single star was successfully measured. To achieve this, Michelson and Pease mounted small mirrors to a 20-foot iron beam set across the aperture to the 100-inch telescope, at the time the largest telescope in the world [8]. With this arrangement the diameter of Betelgeuse was determined to be 0.047 arc seconds, more than 3 times smaller than Stéphan's upper limit. Pease subsequently built a 50-foot system in an attempt to resolve a large number of nearby stars, but this instrument never worked successfully [9].

3. THE BEGINNINGS OF RADIO INTERFEROMETRY

The development of radio interferometry began at the close of World War II when scientists and engineers who were involved in radio and radar research returned to peacetime activities. In the event, the idea to use interference techniques to increase the resolution of radio observations developed simultaneously in England, at the Cavendish Laboratory of Cambridge University, and in Australia, at the Division of Radiophysics of the Commonwealth Scientific and Industrial Research Organization [10].

Radio Studies of the Sun

Radio radiation from the Sun had been discovered by a number of groups working with radar in the early 1940s, and by 1946 the groups in Australia and England were using interferometers in an effort to show that the enhanced radiation from the Sun came from sun spots. In the first paper from the Cambridge group, the authors note that it is not possible to build a telescope large enough to achieve the resolution they need. "An alternative method was therefore used, analogous to Michelson's method for determining stellar diameters. Two aerial systems were used with a horizontal separation of several wavelengths, and their combined output fed to the receiving equipment [11]." At the same time, the group in Sidney realized that an interferometer measures the Fourier transform of the source brightness distribution. The important conclusion is that, "It is possible in principle to determine the actual form of the distribution in a complex case by Fourier synthesis using information derived from a large number of components [12]."

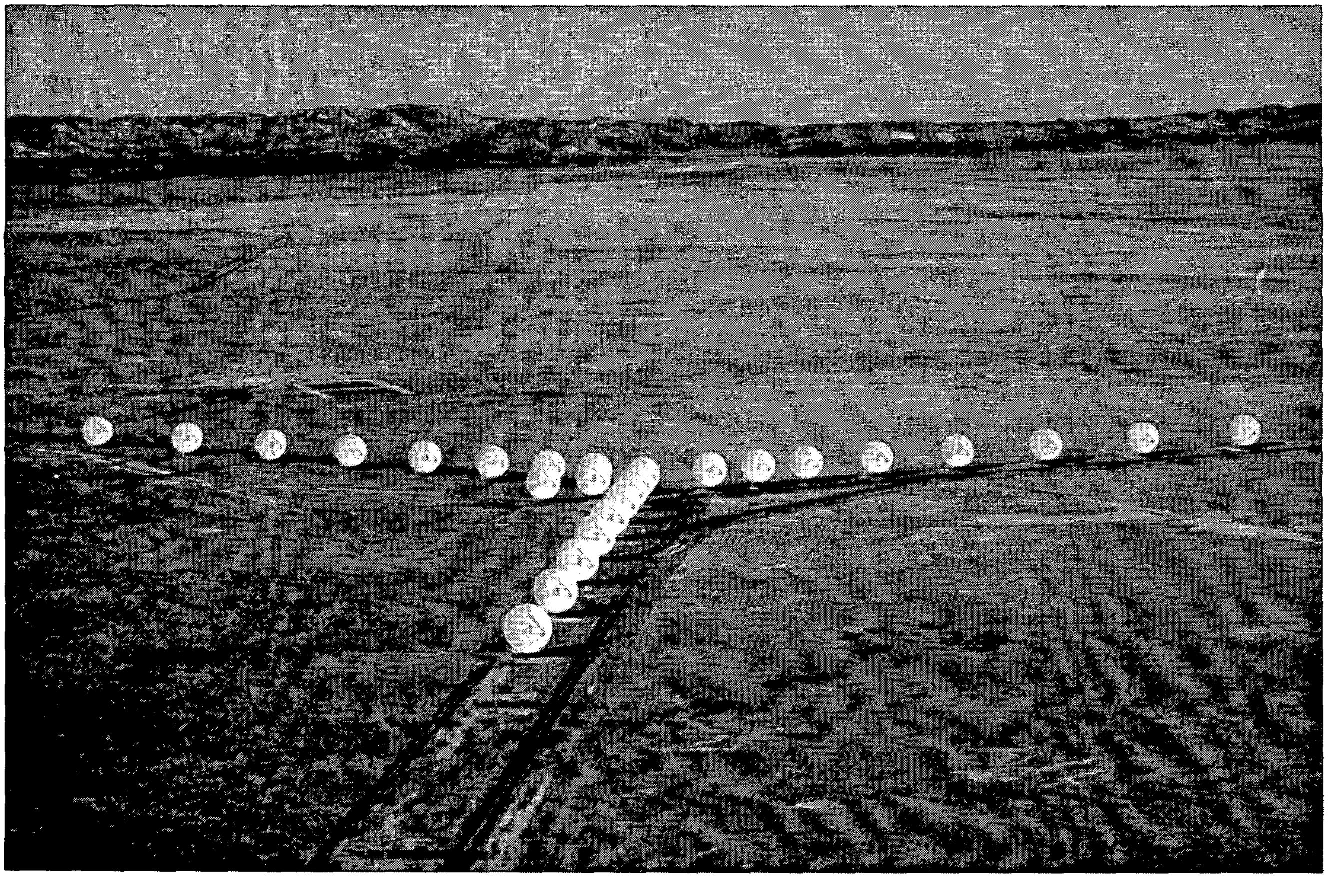


Figure 2 - The Very Large Array Radio Interferometer.

In fact, this relationship was already know in optics as the van Cittert-Zernike theorem developed by van Cittert in 1934 [13] and generalized by Zernike in 1938 [14]. Consider a two-element interferometer with a aperture separation vector $\boldsymbol{B}$. If the source has an intensity distribution on the sky of $I(\alpha)$, where α, is the two-dimensional sky coordinate, then, the quantity known as the *complex degree of coherence* μ is given by

$$\mu(\mathbf{B}) = \int I(\alpha) e^{-ik\mathbf{B}\bullet\alpha} / I(\alpha) d\alpha \quad . \ (7)$$

where $k = 2\pi/\lambda$, and the integral is over the field of view of the single apertures. Now it turns out that the visibility is just the modulus of μ,

$$V = |\mu|. \qquad (8)$$

I and μ form a Fourier transform pair, so that the inverse relation

$$I(\alpha) / \int I(\alpha) d\alpha) = \int \mu(\mathbf{B}) e^{+ik\mathbf{B}\bullet\alpha} d\mathbf{B} \qquad (9)$$

gives the source distribution derived from a set of visibility measurements, where the intergral on the right is over all baselines.

Aperture Synthesis

The realization that an interferometer measures the Fourier transform of the source brightness distribution, and that, for a given observing wavelength, the Fourier component measured is proportional to the separation of the interferometer elements lead to the development of moveable element systems. By measuring at a number of different separations it became possible to synthesize a large aperture [13]. The final important step toward modern aperture synthesis was taken by Martin Ryle in 1962. This technique uses the rotation of the earth to vary the length of antenna baselines projected on the sky [14]. The new instrument used parabolic dish antennas set on equatorial mounts and arranged along an east-west baseline. Thus, the antennas can track radio sources across the sky as the earth rotates making measurements on continuously varying projected baselines.

All modern aperture synthesis arrays use earth rotation synthesis. The Very Large Array (VLA), shown in Figure 2, is one of the largest such instruments, consisting of 27 antennas arranged along three arms. The individual antennas have diameters of 25 meters and can be moved to different stations along the arms giving baseline lengths up to 36 kilometers.

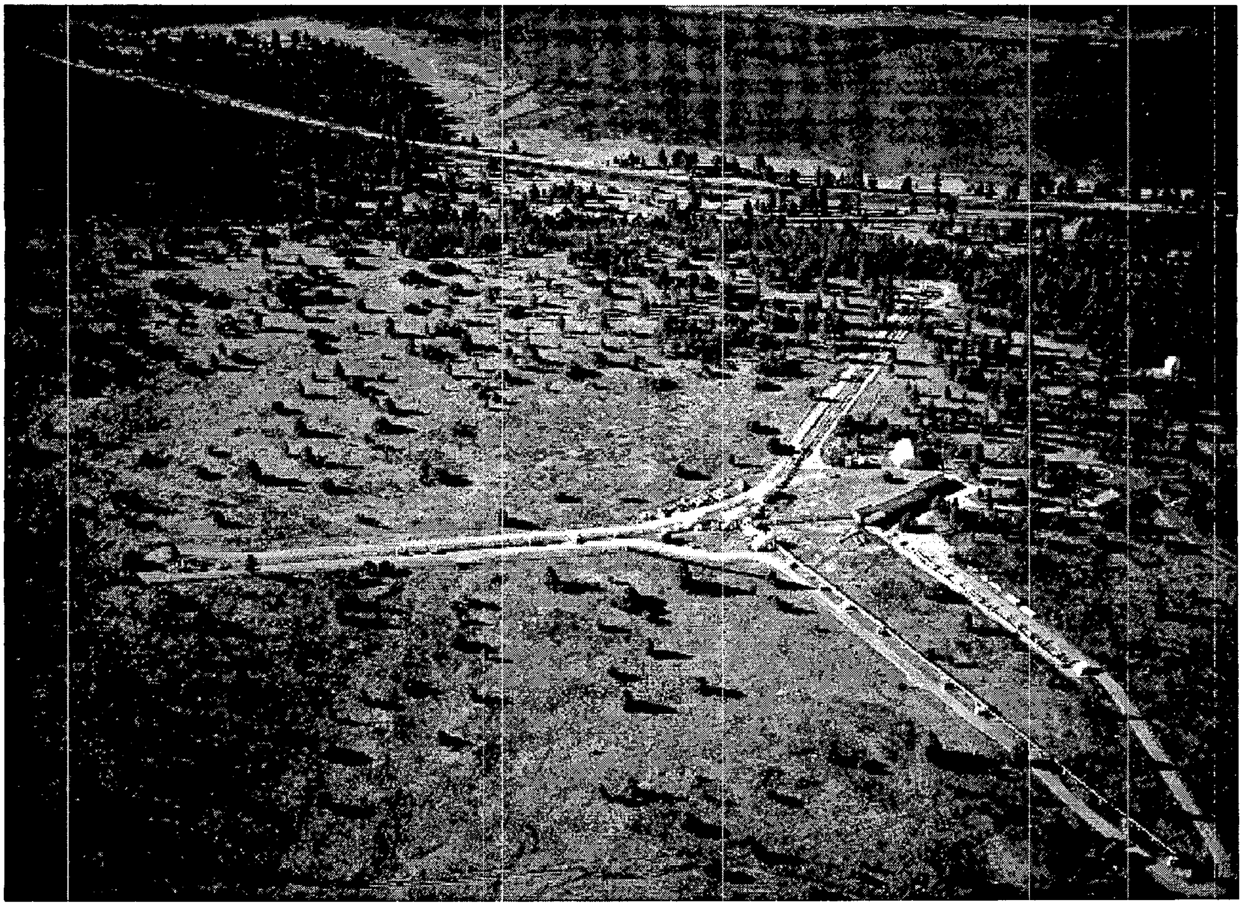


Figure 3 - The Navy Prototype Optical Interferometer

4. EARLY MODERN OPTICAL INTERFEROMETERS

The era of modern optical interferometry began in the 1950s with the development of the intensity interferometer by Brown and Twiss [15]. This instrument uses the fact that there is a correlation in the arrival times of photons in coherent beams of light. The degree of correlation is a function of the separation of the two telescopes that collect the light and the angular diameter of the source. Brown and Twiss eventually measured the diameters of 32 stars, but were prevented from doing more by the fundamentally low sensitivity of the instrument [16].

Antoine Labeyrie built the first separate-element optical interferometer in 1974 [17]. It consists of two small telescopes whose light is combined on an optical table in a separate enclosure. For many years this instrument has made observations of stellar diameters and binary star orbits. Originally designed to work at visual wavelengths, this instrument was later fitted with an infrared beam combination table as well [18].

5. STEPS TOWARD OPTICAL SYNTHESIS IMAGING

The visibility is the Fourier transform of the source brightness distribution and is, therefore, inherently a complex quantity which can be describe by an amplitude and phase. All early interferometers we have discussed so far have assumed that the source is circularly symmetric, so that the Fourier transform is real, and we can derive the diameter of a source by simply measuring the variation of the visibility amplitude with baseline length. For more complex source structure it is necessary to measure the phase of the fringe visibility as well.

Closure Phase

By the middle of the 1950s radio interferometers were able to measure the fringe phase, and a discovery by Jennison in 1958 at the University of Manchester proved crucial to the subsequent development of optical synthesis imaging [19]. With three or more antennas it is possible to construct the sum of the baseline phases around a closed triangle, and this sum contains phase information due to the source structure on the three baselines, but instrumental errors and phase fluctuations due to atmospheric turbulence cancel [20]. This sum of three phases around a closed triangle is called the *closure phase*. The use of the closure phase has been crucial to the technique of Very Long Baseline Interferometry (VLBI), where data from separate radio telescopes, sometimes located on different continents, are combined to form images of astronomical sources. VLBI has been a highly successful tool, producing images with resolutions higher than any other technique in astronomy.

The first closure phase measurements at optical wavelengths

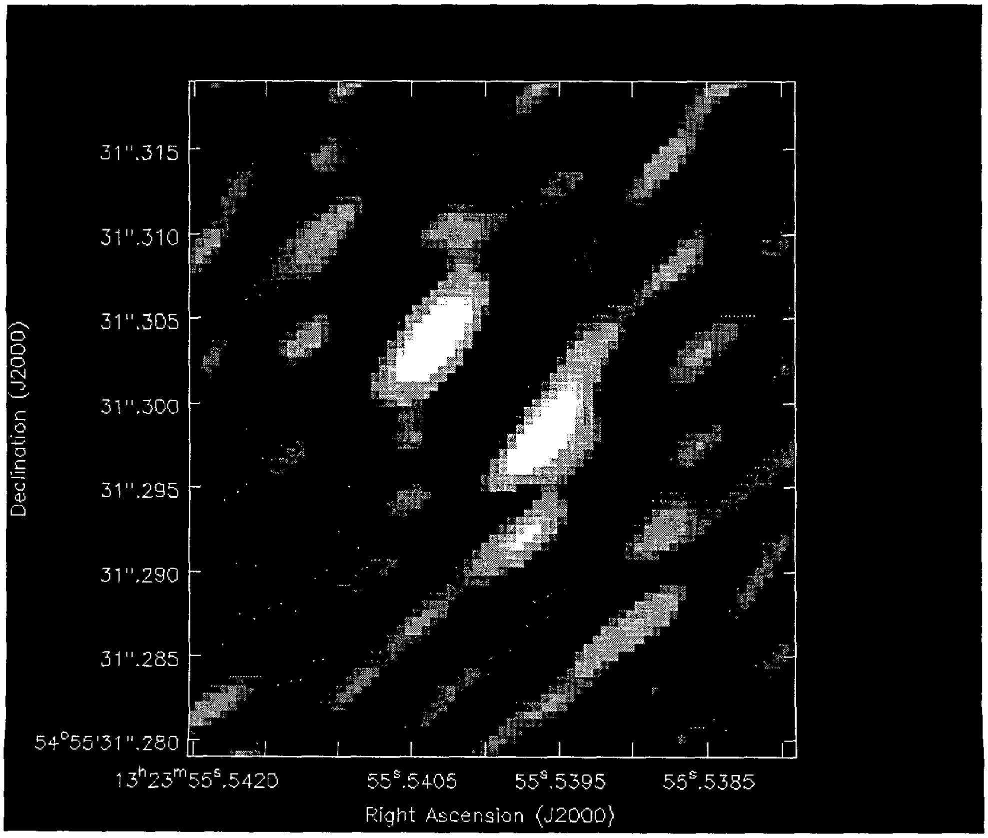


Figure 4 - Optical aperture synthesis image of the double star Mizar A.

were made by Baldwin and co-workers at the University of Cambridge in 1986 [21]. In this experiment an aperture mask was constructed with three or more holes arranged in a non-redundant pattern. The first optical aperture synthesis images using these techniques where made of the binary star ϕ And by the Cambridge group in 1987 [22]. As mentioned, this work used a single telescope with an aperture mask to generate the fringes. The next step is to combine the light from separated telescopes and track the fringes in real time.

Active Fringe Tracking

The measurement of the fringe phase in real time represents a major challenge at optical wavelengths. The earth's atmosphere has a coherence length of about 10 centimeters and the fluctuations have a time scale of about 10 milliseconds. The coherence length is the size of a patch of the incoming wave-front over which the r.m.s. phase fluctuations are less than 1 radian. In order to track the fringe phase, the telescope apertures must be of order or smaller than the size of the coherence length. Shao and Staelin constructed the first separate element stellar interferometer able to measure fringe phase and track atmospheric phase variations in real time [23]. This instrument was the first in a series of fringe tracking interferometers built by this group during the 1980s culminating in the highly successful Mark III stellar interferometer on Mount Wilson, where Michelson had measured the first stellar diameter seventy years earlier [24].

Optical Aperture Synthesis

By the early 1990s work was underway on two optical aperture synthesis stellar interferometers. In England, the Cambridge group began construction of the Cambridge Optical Aperture Synthesis Telescope (COAST) [25], while in the U. S. the Naval Research Laboratory and the Naval Observatory built the Navy Prototype Optical Interferometer (NPOI) [26]. The NPOI, shown in Figure 3, is a direct descendant of the Mark III interferometer and uses active fringe tracking. COAST made the first measurement of closure phase images at optical wavelengths [27], followed soon after by the NPOI [28].

These instruments are true aperture synthesis imaging systems. Figure 4 is an example of a double star image made with the NPOI using three telescopes. The two bright objects at the center of the figure are the components of the binary star; the reddish elongated features show the noise level and artifacts resulting from only having one closure phase available.

6. FURTHER READING

A very valuable collection of papers relating the history of stellar interferometry has been compiled by Lawson and published as part of the SPIE Milestone Series [31]. There have been several SPIE conferences dedicated to the subject [32,33], and there is a recent review article [34] and Summer school proceedings [35].

REFERENCES

[1] H. Fizeau, "Prix Bordin: Rapport sur le concours de l'année 1867," *C. R. Acad. Sci.* **66**, 932, 1868.

[2] M. Born and E. Wolf, *Principles of Optics, 7th (expanded) Edition*, Cambridge: University Press,, 1999.

[3] E. Stéphan, "Extrait d'une lettre de M. Stéphan à M. Fizeau," *C. R. Acad. Sci.* **76**, 1008-1010, 1873.

[4] E. Stéphan, "Sur l'extrême petitesse du diamètre apparent des étoiles fixes," *C. R. Acad. Sci.* **78**, 1008-1012, 1874.

[5] A. A. Michelson, "On the Application of Interference Methods to Astronomical Measurements," *Philosophical Magazine*, **30**, 1-21, 1890.

[6] P. R. Lawson, "Notes on the History of Stellar Interferometry," in *Principles of Long Baseline Stellar Interferometry*, JPL Publication 00-009 07/00, 2000.

[7] A. A. Michelson, "Measurement of Jupiter's Satellites by Interference," *Nature*, **45**, 160-161, 1891.

[8] A. A. Michelson and F. G. Pease, "Measurement of the Diameter of • Orionis with the Interferometer," *Astrophysical Journal*, **53**, 249-259, 1921.

[9] F. G. Pease, "Interferometer Methods in Astronomy," *Naturwissenschaften*, **10**, 84-96, 1931.

[10] J. S. Hey, *The Evolution of Radio Astronomy*, New York: Science History Publications, 1973.

[11] M. Ryle and D. D. Vonberg, "Solar Radiation at 175 Mc./s.," *Nature*, **158**, 339-340, 1946.

[12] L. L. McCready, J. L. Pawsey and R. Payne-Scott, "Solar radition at radio frequencies and its relation to sun spots," *Proceedings of the Royal Society*, **A190**, 357-375, 1947.

[13] P. H. van Cittert, "Die Wahrscheinliche Schwingungsverteilung in einer von einer Lightquelle Direkt oder mittels einer Linse Beleuchteten Ebene," *Physica*, **1**, 201-210, 1934.

[14] F. Zernike, "The concept of Degree of Coherence and its Application to Optical Problems," *Physica*, **5**, 785-795, 1938.

[15] M. Ryle and A. Hewish, "The Synthesis of Large Radio Telescopes," *Monthly Notices of the Royal Astronomical Society*, **120**, 220-230, 1960.

[16] M. Ryle, "The New Cambridge Radio Telescope," *Nature*, **194**, 517-518, 1962.

[17] R. Hanbury Brown and R. Q. Twiss, "A Test of a New Type of Stellar Interferometer on Sirius," *Nature*, **178**, 1046-1048, 1956.

[18] R. Hanbury Brown, J. Davis and L. R. Allen, "The Angular Diameters of 32 Stars," *Monthly Notices of the Royal Astronomical* Society, **167**, 121-136, 1974.

[19] A. Labeyrie, "Interference Fringes obtained on Vega with two Optical Telescopes," *Astrophysical Journal*, **196**, L71-L75, 1975.

[20] G. P. Di Benedetto and G. Conti, "Stellar Diameter Measurements by Two-Aperture Interferomerty in the Infrared," *Astrophysical Journal*, **268**, 309-318, 1983.

[21] R. C. Dennison, "A phase sensitive interferometer technique for the measurement of the Fourier transforms of spatial brightness distributions of small angular extent," *Monthly Notices of the Royal Astronomical* Society, **118**, 276-284, 1958.

[22] A. C. S. Readhead,, T. S. Nakajima, T. J. Pearson, G. Neugebauer, J. B. Oke, and W. W, Sargent, "Diffraction-limited imaging with ground-based optical telescopes," *Astronomical Journal*, **95**, 1278-1296, 1988.

[23] J. E. Baldwin, C. A. Haniff, C. D. MacKay and P. J. Warner, "Closure phase in high-resolution optical imaging," *Nature*, **320**, 595-597, 1986.

[24] C. A. Haniff, C. D. MacKay, D. J. Titterington, D. Sivia, J. E. Baldwin and P. J. Werner, "The first images from optical aperture synthesis," *Nature*, **328**, 694-696, 1987.

[25] M. Shao and D. H. Staelin, "First fringe measurements with a phase-tracking stellar interferometer," *Applied Optics*, **19**, 1519-1522, 1980.

[26] M. Shao, M. M. Colavita, B. E. Hines, D. H. Staelin, D. J. Hutter, K. J. Johnston, D. Mozurkewich, R. S. Simon, J. L. Hershey, J. A. Hughes, G. H. Kaplan, "The Mark III stellar interferometer," *Astronomy and Astrophysics*, **193**, 357-371, 1988.

[27] J. E. Baldwin, R. C. Boysen, G. C. Cox, C. A. Haniff, J. Rogers, P. J. Warner, D. M. A. Wilson and C. D. MacKay, "Design and performance of COAST," *Proceedings of SPIE*, **2200**, 118-128, 1994.

[28] J. T. Armstrong, D. Mozurkewich, L. J Rickard, D. J. Hutter, J. A. Benson, P. F. Bowers, N. M. Ellias II, C. A. Hummel, K. J. Johnston, D. F. Buscher, J. H. Clark III, L. Ha, L.-C. Ling, N. M. White and R. S. Simon, "The Navy Prototype Optical Interferometer," *Astrophysical Journal*, **496**, 550-571, 1998.

[29] J. E. Baldwin, M. G. Beckett, R. C. Boysen, D. Burns, D. F. Buscher, G. C. Cox, C. A. Haniff, C. D. MacKay, N. S. Nightingale, P. A. G. Scheuer, T. R. Scott, P. G. Tuthill, P. J. Warner, D. M. A. Wilson and R. W. Wilson, "The first images from an optical aperture synthesis array: mapping of Capella with COAST at two epochs," *Astronomy and Astrophysics*, **306**, L13-L16, 1996.

[30] J. A. Benson, D. J. Hutter, N. M. Elias II, P. F. Bowers, K. J. Johnston, A. R. Hajian, J. T. Armstrong, D. Mozurkewich, T. A. Pauls, L. J Rickard, C. A. Hummel, N. M. White, D. Black and C. S. Denison, "Multichannel optical aperture synthesis imagining of ζ^1 Ursae Majoris with the Navy Prototype Optical Interferometer," *Astronomical Journal*, **114**, 1221-1226, 1997.

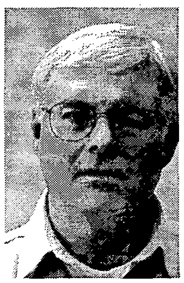
[31] P. R. Lawson ed., "Selected Papers on Long Baseline Stellar Interferometry," *SPIE Milestone Series*, **MS 139**, Bellingham: SPIE Press, 1997.

[32] R. D. Reasenberg ed., "Astronomical Interferometry," *Proceedings of SPIE*, **3350**, Bellingham: SPIE Press, 1998.

[33] P. J. Léna and A. Quirrenbach eds., "Interferometry in Optical Astronomy," *Proceedings of SPIE*, **4006**, Bellingham: SPIE Press, 2000.

[34] M. Shao and M. M. Colovita, "Long-baseline Optical and Infrared Stellar Interferometry," *Annual Reviews of Astronomy and Astrophysics*, **30**, 457-498, 1992.

[35] P. R. Lawson, *Principles of Long-Baseline Stellar Interferometry*, Pasadena: JPL Publication 00-009 07/00, 2000.



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