

Generalized Minimum Variance Control for Time Varying Systems without Diophantine equation

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Abstract

This paper presents a strategy of Generalized Minimum Variance Control (GMVC) for time varying systems (TVS). In general, GMVC has developed in time invariant Systems (TIS). GMVC for TVS is designed by referring the time varying polynomials of the plant at various control periods. However, the time function of the polynomials deviates from the true references due to the Diophantine equation. This paper proposes a GMVC strategy that predicts the future output by without applying the Diophantine equation. The predictive method is on the basis of O. P. Palsson proposed prediction. The future output is derived from recursive referring with only the time varying polynomials of the plant. Furthermore, this paper develops the servomechanism against the reference signal changing and the load-disturbance. The simulation shows the effective results of the proposed GMVC in TVS.

Keywords: Diophantine equation, discrete time systems, disturbance rejection, generalized minimum variance control, predictive control, servo systems, time delay, time varying systems

1 Introduction

GMVC [1, 2, 3] is an effective control strategy in plants including a time delay. It predicts the future output by referring to the polynomials of the plant model. The conventional GMVC has developed in time invariant systems (TIS). Therefore, in time varying systems (TVS), GMVC must refer to the time varying polynomials at various control periods. However, GMVC still can not provide stable control in TVS, even if referring to it, because the time function of the polynomials deviates from the true references by directly predicting the future output with the Diophantine equation. In other words, the unique solutions of the Diophantine equation can not be derived accurately [4, 5]. For TVS, this paper proposes a GMVC strategy, based on O. P. Palsson proposed prediction [6, 7, 8]. The method does not apply the Diophantine equation. The prediction

is calculated by recursive referring to the time varying polynomials of the plant at various control periods.

On the other hand, the offsets remain due to the reference signal changing and the load-disturbance in GMVC when the control weight of the cost function is set. K. Takahashi proposed the servo method [9, 10], based on the internal model principle, to eliminate these offsets in TIS. The servo method means that the offset due to the reference signal changing is removed by setting the difference filter $\Delta = 1 - q^{-1}$ into the cost function, while setting it into the Diophantine equation against the load-disturbance. However, the GMVC for TVS can not apply the K. Takahashi proposed servo method because it does not have the Diophantine equation. Therefore, the filter $\Delta = 1 - q^{-1}$ is directly substituted for the CARMA model.

Furthermore, in this paper, the time delay and the degree of the polynomials are described as generalized form, while the conventional predictive method [6, 7] has developed on the TVS that specifies the time delay and the degree of polynomials.

2 GMVC with Diophantine equation in TVS

This chapter introduces the GMVC strategy, which arranges the conventional GMVC for TIS with the Diophantine equation for TVS. The result is verified from the simulation in TVS.

2.1 Control law

Consider Single-Input Single-Output (SISO) systems, described with the Controlled Auto-Regressive and Moving Average (CARMA) model

$$A_k(q^{-1})y(k) = q^{-j}B_k(q^{-1})u(k) + C_k(q^{-1})\xi(k), \quad (1)$$

which has time-varying coefficients with

$$\begin{aligned} A_k(q^{-1}) &= 1 + a_1(k)q^{-1} + \cdots + a_n(k)q^{-n}, \\ B_k(q^{-1}) &= b_0(k) + b_1(k)q^{-1} + \cdots + b_m(k)q^{-m}, \end{aligned} \quad (2)$$

(3)

$$C_k(q^{-1}) = 1 + c_1(k)q^{-1} + \dots + c_l(k)q^{-l}, \quad (4)$$

where $u(k)$ and $y(k)$ are the control signal and the output signal, q^{-j} represents a time delay of the plant and $\xi(k)$ is white noise with zero mean and variance σ^2 . In TVS, the coefficients of polynomials are expressed as a function of time k , such as equations (2), (3) and (4).

The generalized output in the cost function $J = E\{h(k+j)^2\}$ is expressed as follows:

$$\begin{aligned} h(k+j) &= P(q^{-1})y(k+j) - R(q^{-1})w(k+j) + S(q^{-1})u(k), \end{aligned} \quad (5)$$

$$P(q^{-1}) = 1 + p_1q^{-1} + \dots + p_{n_p}q^{-n_p}, \quad (6)$$

$$R(q^{-1}) = r_0 + r_1q^{-1} + \dots + r_{n_r}q^{-n_r}, \quad (7)$$

$$S(q^{-1}) = s_0 + s_1q^{-1} + \dots + s_{n_s}q^{-n_s}, \quad (8)$$

where $w(k+j)$ is the reference signal. The polynomials $P(q^{-1})$, $R(q^{-1})$ and $S(q^{-1})$ are the weights of the cost function. In particular, GMVC sets the control weight $S(q^{-1})$. However, $y(k+j)$ in the generalized output (5) must be predicted, because it can not be observed at the present time k . The following Diophantine equation is introduced

$$\begin{aligned} P(q^{-1})C(q^{-1}) &= E_k(q^{-1})A_k(q^{-1}) + q^{-j}F_k(q^{-1}), \end{aligned} \quad (9)$$

$$\begin{aligned} E_k(q^{-1}) &= 1 + e_1(k)q^{-1} + \dots + e_{j-1}(k)q^{-(j-1)}, \end{aligned} \quad (10)$$

$$\begin{aligned} F_k(q^{-1}) &= f_0(k) + f_1(k)q^{-1} + \dots + f_h(k)q^{-h}, \end{aligned} \quad (11)$$

$$h_1 = \max\{n, n_p + l - j\}. \quad (12)$$

When both sides of the CARMA model (1) are multiplied by $E_k(q^{-1})$ and the Diophantine equation (9) is substituted for $E_k(q^{-1})A_k(q^{-1})$, the j -steps-ahead optimal prediction is derived from

$$\begin{aligned} \hat{y}(k+j|k) &= \frac{E_k(q^{-1})B_k(q^{-1})u(k) + F_k(q^{-1})y(k)}{P(q^{-1})C_k(q^{-1})}. \end{aligned} \quad (13)$$

Substituting equation (13) for equation (5) and minimizing the cost function provide

$$u(k) = \frac{C_k(q^{-1})R(q^{-1})w(k+j) - F_k(q^{-1})y(k)}{E_k(q^{-1})B_k(q^{-1}) + C_k(q^{-1})S(q^{-1})}. \quad (14)$$

However, in the GMVC (14), the offsets remain due to the reference signal changing and the load-disturbance.

Then, a pre-compensator $1/\Delta$ ($\Delta = 1 - q^{-1}$) [9, 10], based on the internal model principle, is set in the

closed-loop systems. First, a difference filter Δ is set into the generalized output (5) [2]

$$\begin{aligned} h(k+j) &= P(q^{-1})y(k+j) - R(q^{-1})w(k+j) \\ &\quad + S(q^{-1})\Delta u(k), \end{aligned} \quad (15)$$

which removes the offset due to the reference signal changing. Second, Δ is set into the Diophantine equation

$$P(q^{-1})C_k(q^{-1}) = \Delta E_k(q^{-1})A_k(q^{-1}) + q^{-j}F_k(q^{-1}), \quad (16)$$

which removes the offset due to the load-disturbance. Equation (15) and equation (16) provide the control law

$$u(k) = \frac{1}{\Delta} \frac{C_k(q^{-1})R(q^{-1})w(k+j) - F_k(q^{-1})y(k)}{E_k(q^{-1})B_k(q^{-1}) + C_k(q^{-1})S(q^{-1})}, \quad (17)$$

that have servo characteristics against the reference signal changing and the load-disturbance.

2.2 Simulation 1

This section discusses the simulation of the following model, used in ref.[6, 7]

$$A_k(q^{-1}) = 1 - 0.56q^{-1}, \quad (18)$$

$$B_k(q^{-1}) = b_0(k) + b_1(k)q^{-1} + b_2(k)q^{-2}, \quad (19)$$

$$C_k(q^{-1}) = 1. \quad (20)$$

The dynamics of this plant, such as the energy systems [11] of area heating, periodically varies only according to polynomial $B_k(q^{-1})$ as follows:

$$\begin{aligned} b_0(k) &= 0.35 + 0.32 \sin \frac{\pi}{12}(k-2) - 0.24 \cos \frac{\pi}{12}(k-2), \end{aligned} \quad (21)$$

$$\begin{aligned} b_1(k) &= 0.18 + 0.31 \sin \frac{\pi}{12}(k-3) - 0.25 \cos \frac{\pi}{12}(k-3), \end{aligned} \quad (22)$$

$$\begin{aligned} b_2(k) &= 0.18 + 0.09 \sin \frac{\pi}{12}(k-4) - 0.09 \cos \frac{\pi}{12}(k-4). \end{aligned} \quad (23)$$

Suppose that the future polynomials are known on the basis of equations (21), (22) and (23). Simulation 1 is the time response of the closed-loop systems. In simulation 1, the reference signal is changing at $k = 50$ from 0 to 1 and a load-disturbance of magnitude 0.2 occurs at $k = 150$. The variance of noise $\xi(k)$ is 0.01. The weights of cost function are $P(q^{-1}) = R(q^{-1}) = 1$ [12, 13] and $S(q^{-1}) = 0.7$. The time delay is extended to 5 steps, although the Palsen proposed plant sets 2 steps, because this paper deals with the generalized

formed plant regarding the time delay and the degree of polynomials.

Figure 1 shows a time response of GMVC (17) in TVS. The dotted line is the control signal $u(k)$ and the solid line is the output signal $y(k)$. The output and control signals of GMVC with the Diophantine equation oscillate, even if the varying polynomials are referred at various control periods. The reason is caused of no deriving the accurate unique solutions of the Diophantine equation in TVS.

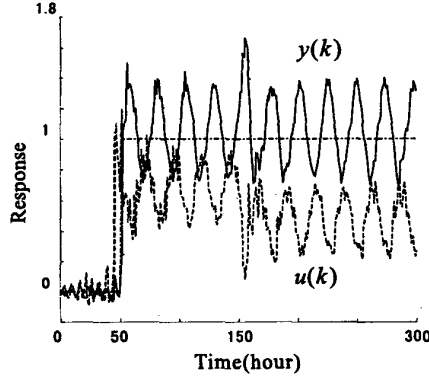


Figure 1: Time response of GMVC with a pre-compensator in TVS

3 GMVC without the Diophantine equation for TVS

This chapter introduces the strategy of GMVC without the Diophantine equation for TVS. O. P. Palsson proposed the method [6, 7] to predict the future output without the Diophantine equation in TVS. However, this O. P. Palsson proposed method specifies the time delay and the degree of polynomials. Further, the offset remains due to the load-disturbance. This chapter describes the generalized formed GMVC strategy and reject the load-disturbance in TVS.

Figure 2 verifies the difference of both predictive methods. Figure 2(a) shows the conventional prediction with the Diophantine equation and Figure 2(b) shows the proposed prediction without the Diophantine equation. In Figure 2(a), the conventional method can not predict the future output $y(k+5)$ because the solution of the Diophantine equation, which is to directly predict the output, can not be solved in TVS. On the other hand, in Figure 2(b), the proposed method can predict the output because the Diophantine equation is not used, though the predictive algorithm is more complicated. Here, this chapter describes the process

to derive the proposed GMVC strategy without the Diophantine equation.

First, the future output

$$\begin{aligned} y(k+j) &= -\{a_1(k+j)q^{-1} + a_2(k+j)q^{-2} \\ &\quad + \dots + a_n(k+j)q^{-n}\}y(k+j) \\ &\quad + \{b_0(k+j) + b_1(k+j)q^{-1} \\ &\quad + \dots + b_m(k+j)q^{-m}\}u(k) \\ &\quad + C_{k+j}(q^{-1})\xi(k+j), \end{aligned} \quad (24)$$

is derived without the Diophantine equation by referring to the CARMA model (1) directly. Then, the prediction $\hat{y}(k+j)$ is expressed as

$$\begin{aligned} \hat{y}(k+j) &= -\sum_{i=1}^{j-1} \{a_i(k+j)y(k+j-i)\} \\ &\quad + b_0(k+j)u(k) + h_p(k+j|k), \end{aligned} \quad (25)$$

$$\begin{aligned} h_p(k+j|k) &= -\sum_{p=j}^n \{a_p(k+j)y(k+j)\} \\ &\quad + \sum_{p=1}^m \{b_p(k+j)u(k-p)\}, \end{aligned} \quad (26)$$

which eliminates the future noise term $C_{k+j}(q^{-1})\xi(k+j)$ of equation (24). $h_p(k+j|k)$ of equation (25) comprises only the past and present data of the output signal and the control signal. Since $\sum_{i=1}^{j-1} \{a_i(k+j)y(k+j-i)\}$ of equation (25) is the future term that is not observed, the prediction

$$\begin{aligned} \hat{y}(k+j-i|k) &= -\sum_{i'=1}^{j-i-1} \{a_{i'}(k+j-i)y(k+j-i-i')\} \\ &\quad - \sum_{p=j-i}^n \{a_p(k+j-i)y(k+j-i-p)\} \\ &\quad + \sum_{p=0}^m \{b_p(k+j-i)u(k-i-p)\}, \\ &\quad i = 1, \dots, j-1 \end{aligned} \quad (27)$$

is calculated by recursive referring to the CARMA model (1). Then equation (25) is arranged as

$$\begin{aligned} \hat{y}(k+j|k) &= b_0(k+j)u(k) + h'_p(k+j|k), \end{aligned} \quad (28)$$

which comprises only the past and present data. $h'_p(k+j|k)$ of equation (28) is expressed as

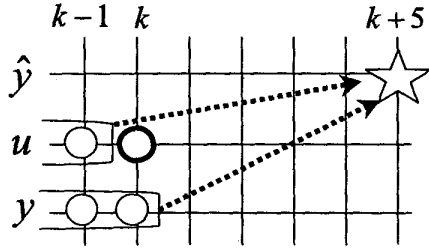
$$h'_p(k+j|k)$$

$$= - \sum_{i=1}^{j-1} \{a_i(k+j)\hat{y}(k+j-i|k)\} + h_p(k+j|k). \quad (29)$$

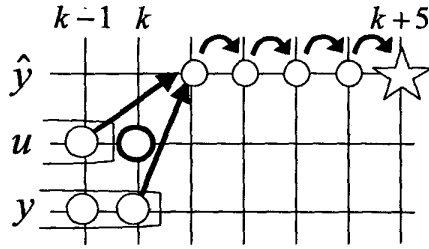
If the equation (28) is substituted for the generalized output (5), GMVC without the Diophantine equation is derived from

$$\{S(q^{-1}) + P(q^{-1})b_0(k+j)\}u(k) = R(q^{-1})w(k+j) - P(q^{-1})h(k+j). \quad (30)$$

Figure 3 shows the block diagram of the closed-loop systems by GMVC (30) without the Diophantine equation.



(a) Prediction with Diophantine equation



(b) Prediction without Diophantine equation

Figure 2: The structures to calculate the output predic-

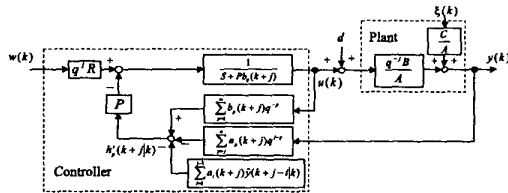


Figure 3: Block diagram of GMVC without Diophantine equation

4 Design for the servo systems

In the conventional GMVC, the offsets due to the reference changing and the load-disturbance remain when the control weight is set in the cost function. The proposed GMVC without the Diophantine equation is also similar. This chapter presents the servo systems of the GMVC without the Diophantine equation.

4.1 The cost function with a difference filter

GMVC without the Diophantine equation also uses the generalized output (15) with a difference filter $1 - q^{-1}$ in the control weight of the cost function to remove the offset due to the reference signal changing. Then the control law is

$$\{\Delta S(q^{-1}) + P(q^{-1})b_0(k+j)\}u(k) = R(q^{-1})w(k+j) - P(q^{-1})h(k+j), \quad (31)$$

which is derived by substituting the prediction (28) for the generalized output (15).

4.2 Simulation 2

Figure 4 shows the response of the closed-loop systems of GMVC whose cost function includes the difference filter $1 - q^{-1}$. The condition of the simulation is the same as Section 2.2. The result removes the offset due to the reference signal changing, although the offset due to the load disturbance remains.

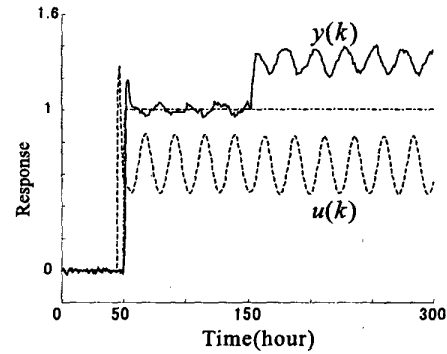


Figure 4: Time responses of Palsson type GMVC with a filter

4.3 A pre-compensator based on the internal model principle

For the conventional GMVC, the offset due to the load-disturbance is removed by setting the filter $1 - q^{-1}$ into the Diophantine equation. However, the proposed GMVC can not set the filter because it does not have the Diophantine equation. This section presents the servo systems against both offsets due to the reference signal changing and the load disturbance. First, the generalized output (15) is also used as in Section 2.1.

4.5 Rejection of the load-disturbance

If the systems including the load-disturbance are considered, then the future output is expressed by the CARMA model (1), such as

$$y(k+j) = \frac{B_{k+j}(q^{-1})}{A(q^{-1})}\{u(k)+d\} + \frac{C(q^{-1})}{A(q^{-1})}\xi(k+j), \quad (40)$$

which model varies only according to the polynomial B_{k+j} , based on the O. P. Palsson proposed simulation. On the other hand, the prediction $\hat{y}(k+j|k)$ is derived from model (32) as

$$\begin{aligned} \hat{y}(k+j|k) &= \frac{B_{k+j}(q^{-1})}{A(q^{-1})}u(k) + \hat{y}(k+j-1|k) \\ &\quad - \frac{B_{k+j-1}(q^{-1})}{A(q^{-1})}u(k-1). \end{aligned} \quad (41)$$

Consequently, equation (41) is arranged to

$$\hat{y}(k+j|k) = \frac{B_{k+j}(q^{-1})}{A(q^{-1})}u(k) + \frac{B_k(q^{-1})}{A(q^{-1})}d, \quad (42)$$

where the prediction (42) has the load-disturbance term because the observed data $y(k)$ includes the effect of the previous load-disturbance. Subtracting equation (42) from equation (40) provides the predictive error

$$\begin{aligned} y(k+j) - \hat{y}(k+j|k) &= \frac{B_{k+j}(q^{-1}) - B_k(q^{-1})}{A(q^{-1})}d + \frac{C(q^{-1})}{A(q^{-1})}\xi(k+j). \end{aligned} \quad (43)$$

The predictive error (43) confirms that only the noise term remains in TIS because the second term of equation (43) is the same time polynomials such as $B(q^{-1}) - B(q^{-1}) = 0$; while, in TVS, the variance due to the different time polynomials $B_{k+j}(q^{-1})$, $B_k(q^{-1})$ further remains. For the simulation of Section 4.4, the variance of the output after $k = 150$ with the load-disturbance occurs because of equation (43).

5 Conclusion

This paper presented the GMVC without the Diophantine equation for TVS. Previously, the conventional GMVC with the Diophantine equation reformed for the expression of TVS. However, the conventional GMVC with the Diophantine equation can not track the reference signal even if it refers to the time varying polynomials at various time. Because the time function of the polynomials deviates from the true reference due to the Diophantine equation. This paper introduced the prediction, based on the O. P. Palsson proposed predictive method, and the servo mechanism of GMVC developed for TVS. Especially, the offset due to the

load-disturbance was removed by setting the difference filter $1 - q^{-1}$ into the CARMA model. Furthermore, the delay time of plants and the degree of polynomials are described as generalized form. The simulation results confirmed that the proposed GMVC realizes the stable servo systems for TVS.

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