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A New Maneuvering Target Tracking Algorithm with Input Estimation

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Abstract

In this paper, a new tracking algorithm is proposed. It treats the target acceleration as a nonrandom term, and consists of a constant velocity filter, an input estimator and a maneuver detector implemented in parallel. The new method has the same advantages as the two-stage Kalman estimator, which requires lesser amount of computation and provides even a better performance when compared with an augmented state Kalman filter. At the same time, the new method uses a better tuning parameter and removes a difficulty in implementation of the two-stage Kalman estimator. It is shown that the new filter is a better alternative to the two-stage Kalman estimator on tracking maneuvering targets.

Keywords: *Maneuvering target tracking, input estimation, two-stage Kalman estimator, augmented Kalman filter.*

1 Introduction

Consider the problem of estimating the state of a linear system in the presence of a dynamical bias. Common approaches to this problem treat the bias as a part of the system state. This leads to an augmented state filter whose implementation can be computationally intensive. To maintain the computational cost at a lower level, Freidland [1] proposed a two-stage filter scheme that decouples the augmented filter into two parallel reduced-order filters. The first filter, the "bias-free" filter, is formed by ignoring the bias term. The second filter, the "bias" filter, provides an estimate of the bias term. The output of the "bias-free" filter is compensated by the output of the "bias" filter to reconstruct the augmented state filter. After [1], many researchers have contributed to this problem. Considering the case that a bias is driven by a white noise which is uncorrelated with the system noise, Ignagni [2] proposed a suboptimal two-stage filter. Alouani et al. [3] extended the results of [1] and [2] to include a white Gaussian bias that is correlated with the system noises. It was proven that under an algebraic constraint on the correlation between the system noise and the bias noise, their two-stage Kalman estimator is equivalent to the augmented state Kalman filter. However, since this constraint is not satisfied by almost all real systems, the proposed two-stage Kalman estimator is only suboptimal. Recently, Hsieh [7] proposed an optimal two-stage Kalman estimator, which removes the algebraic constraint but lost the parallel filter structure. The two-stage Kalman estimator has been applied to maneuvering target tracking problems ([4], [5]) by treating the target acceleration as a bias term. It consists of two parallel filters. A constant velocity filter represents the "bias-free" filter and a acceleration filter represents the "bias" filter. However, this approach has disadvantages. First, a restriction on the tuning parameter must be satisfied to gurantee the filter stability. This restriction limits the applicability of this method. Second, changes of the tuning parameter affect both the constant velocity filter and the acceleration filter, which makes it difficult to optimize.

This paper proposes a maneuvering target tracking algorithm with input estimation, and compares its performance with the two-stage Kalman estimator. The new filtering method has a similar structure to the two-stage tracker. It implements a constant velocity filter and an input estimator in parallel. The input estimator is designed to minimize the estimation error covariance for giving input estimator convergence speed. The proposed method uses the input estimator convergence speed as a tuning parameter. This parameter is independent of the system noise, and its physical meaning is more clear than that in the two-stage tracker. This makes the proposed method more suitable for implementation in complex applications. Forthemore, changes of the tuning parameter in the proposed method only affect the input estimator and not the constant velocity filter. This makes the proposed method easier to tune.

Remainder of this paper is organized as follows. Section 2 states the problem. A summary of two-stage tracker is presented in Section 3. The proposed algorithm is derived in Section 4, and the evaluation of its performance is presented in Section 5. A maneuvering target tracking example is shown in Section 6. Section 7 provides conclusions.

2 Statement of the Problem

The problem is to estimate the state of a discrete time system subjected to unknown inputs. The system is described by

$$x_{k+1} = A_k x_k + B_k \gamma_k + w_k \quad (1)$$
$$y_k = C_k x_k + v_k \quad (2)$$

where $x_k \in R^n$ is the system state, $\gamma_k \in R^m$ is the unknown input, $m < n$, $y_k \in R^n$ is the measurement. A_k, B_k, and C_k are time-varying coefficient matrices, and the quantities w_k and v_k are zero-mean uncorrelated random sequences with

$$E\left[w_j {w_k}^T\right] = Q_k \delta_{jk} \quad (3)$$
$$E\left[v_j {v_k}^T\right] = R_k \delta_{jk} \quad (4)$$

This system may represent the dynamics of a maneuvering target, where the system state represents the target position and velocity, and the unknown input represents the target acceleration. Appoaches to this problem fall into two broad categories: modeling the unknown inputs as random processes or nonrandom terms. The two-stage Kalman estimator uses the first approach, and the proposed algorithm uses the second.

3 Two-Stage Kalman Estimator

In this approach, the unknown input is modeled by

$$\gamma_{k+1} = \gamma_k + w_k^\gamma \quad (5)$$

where w_k^γ is a zero-mean random sequence uncorrelated with v_k, and

$$E\left[w_j^\gamma w_k^{\gamma T}\right] = Q_k^\gamma \delta_{jk} \quad (6)$$
$$E\left[w_j w_k^{\gamma T}\right] = Q_k^{x\gamma} \delta_{jk} \quad (7)$$

Based on the models given by (1), (2) and (5), an augmented state Kalman filter may be used to produce the optimal state estimates. However, in order to response quicker to a maneuver, Q_k^γ must maintain at a higher level, and the augmented filter will provide poor noise reduction when the target is not maneuvering. Alouani et al [4] proposed the two-stage Kalman estimator to overcome this problem. The idea of two-stage Kalman estimator is to decouple the augmented Kalman filter into two parallel filters. The first filter uses a constant velocity target model, the second filter produces an estimate of the target acceleration. The output of the acceleration filter is used to correct the output of the constant velocity filter as shown in Fig. 1.

The constant velocity filter is given by

$$\bar{r}_k = y_k - C_k \hat{\bar{z}}_{k|k-1} \quad (8)$$
$$\hat{\bar{z}}_{k|k} = \hat{\bar{z}}_{k|k-1} + \bar{G}_k \bar{r}_k \quad (9)$$
$$\hat{\bar{z}}_{k+1|k} = A_k \hat{\bar{z}}_{k|k} \quad (10)$$

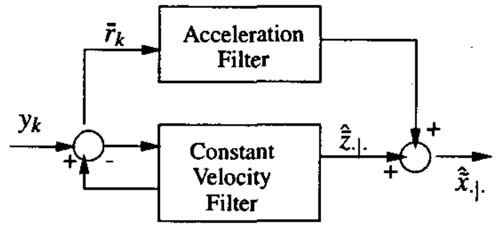

Figure 1: Two-stage Kalman Estimator

$$\bar{G}_k = \bar{P}_{k|k-1} C_k^T \left[C_k \bar{P}_{k|k-1} C_k^T + R_k\right]^{-1} \quad (11)$$
$$\bar{P}_{k|k} = \left(I - \bar{G}_k C_k\right) \bar{P}_{k|k-1} \quad (12)$$
$$\bar{P}_{k+1|k} = A_k \bar{P}_{k|k} A_k^T + \bar{Q}_k \quad (13)$$

where $\hat{\bar{z}}_{\cdot|\cdot}$ represents the estimate of the state process when the unknown input is ignored, $\bar{P}_{\cdot|\cdot}$ is the error covariance of $\hat{\bar{z}}_{\cdot|\cdot}$, and $\bar{Q}_k$ is yet to be determined. The acceleration filter uses the residual sequence of the constant velocity filter to produce an input estimate as follows

$$\hat{\gamma}_{k|k} = \hat{\gamma}_{k|k-1} + \bar{G}_k^\gamma \left(\bar{r}_k - \bar{S}_k \hat{\gamma}_{k|k-1}\right) \quad (14)$$
$$\hat{\gamma}_{k+1|k} = \hat{\gamma}_{k|k} \quad (15)$$
$$\bar{G}_k^\gamma = \bar{P}_{k|k-1}^\gamma \bar{S}_k^T \cdot \left[\bar{S}_k \bar{P}_{k|k-1}^\gamma \bar{S}_k^T + C_k \bar{P}_{k|k-1} C_k^T + R_k\right]^{-1} \quad (16)$$
$$\bar{P}_{k|k}^\gamma = \left(I - \bar{G}_k^\gamma \bar{S}_k\right) \bar{P}_{k|k-1}^\gamma \quad (17)$$
$$\bar{P}_{k+1|k}^\gamma = \bar{P}_{k|k}^\gamma + Q_k^\gamma \quad (18)$$

where

$$\bar{V}_k = \left(I - \bar{G}_k C_k\right) \bar{U}_k \quad (19)$$
$$\bar{S}_k = C_k \bar{U}_k \quad (20)$$
$$\bar{U}_{k+1} = A_k \bar{V}_k + B_k \quad (21)$$

The algorithm for compensating the output of the constant velocity filter with the output of the acceleration filter is described by

$$\hat{\bar{x}}_{k|k} = \hat{\bar{z}}_{k|k} + \bar{V}_k \hat{\gamma}_{k|k} \quad (22)$$
$$\hat{\bar{x}}_{k+1|k} = \hat{\bar{z}}_{k+1|k} + \bar{U}_{k+1} \hat{\gamma}_{k+1|k} \quad (23)$$

The estimation error covariance is given by

$$\bar{P}_{k|k}^{xx} = E\left[\left(x_k - \hat{\bar{x}}_{k|k}\right)\left(x_k - \hat{\bar{x}}_{k|k}\right)^T\right] = \bar{P}_{k|k} + \bar{V}_k \bar{P}_{k|k}^\gamma \bar{V}_k^T \quad (24)$$
$$\bar{P}_{k+1|k}^{xx} = E\left[\left(x_{k+1} - \hat{\bar{x}}_{k+1|k}\right)\left(x_{k+1} - \hat{\bar{x}}_{k+1|k}\right)^T\right] = \bar{P}_{k+1|k} + \bar{U}_{k+1} \bar{P}_{k+1|k}^\gamma \bar{U}_{k+1}^T \quad (25)$$
$$\bar{P}_{k|k}^{x\gamma} = E\left[\left(x_k - \hat{\bar{x}}_{k|k}\right)\left(\gamma_k - \hat{\gamma}_{k|k}\right)^T\right] = \bar{V}_k \bar{P}_{k|k}^\gamma \quad (26)$$

$$\bar{P}^{x\gamma}_{k+1|k} = E\left[\left(x_{k+1}-\hat{\bar{x}}_{k+1|k}\right)\left(\gamma_{k+1}-\hat{\bar{\gamma}}_{k+1|k}\right)^T\right] = \bar{U}_{k+1}\bar{P}^{\gamma}_{k+1|k} \quad (27)$$

$$\bar{P}^{\gamma\gamma}_{k|k} = E\left[\left(\gamma_k-\hat{\bar{\gamma}}_{k|k}\right)\left(\gamma_k-\hat{\bar{\gamma}}_{k|k}\right)^T\right] = \bar{P}^{\gamma}_{k|k} \quad (28)$$

$$\bar{P}^{\gamma\gamma}_{k+1|k} = E\left[\left(\gamma_{k+1}-\hat{\bar{\gamma}}_{k+1|k}\right)\left(\gamma_{k+1}-\hat{\bar{\gamma}}_{k+1|k}\right)^T\right] = \bar{P}^{\gamma}_{k+1|k} \quad (29)$$

It was shown in [6] and [7] that, if

$$Q^{x\gamma}_k = \bar{U}_{k+1}Q^{\gamma}_k \quad (30)$$

$$\bar{Q}_k = Q_k - \bar{U}_{k+1}Q^{\gamma}_k\bar{U}^T_{k+1} \geq 0 \quad (31)$$

the two-stage Kalman estimator is equivalent to the augmented Kalman filter.

For given Q_k and R_k, Q^{γ}_k is the tuning parameter in the two-stage Kalman estimator. As can be seen from (13), (31) and (18), changes of Q^{γ}_k affect both the constant velocity filter and the acceleration filter. Since the algebraic constraint given by (30) is not satisfied by almost all real systems, this two-stage Kalman estimator is only suboptimal. Furthemore, the restriction of (31) introduces an upper bound on the choice of Q^{γ}_k to garantee the filter stability, this yields a difficulty in implementation of the two-stage Kalman estimator.

4 Proposed Tracking Algorithm with Input Estimation

In this approach, the unknown input γ_k is assumed nonrandom and is estimated in real time. If the input γ_k is known, a standard Kalman filter may be used to provide the optimal state estimate as follows

$$\hat{x}^*_{k|k} = \hat{x}^*_{k|k-1} + G_k\left(y_k - C_k\hat{x}^*_{k|k-1}\right) \quad (32)$$

$$\hat{x}^*_{k+1|k} = A_k\hat{x}^*_{k|k} + B_k\gamma_k \quad (33)$$

$$G_k = P_{k|k-1}C^T_k\left[C_kP_{k|k-1}C^T_k + R_k\right]^{-1} \quad (34)$$

$$P_{k|k} = (I - G_kC_k)P_{k|k-1} \quad (35)$$

$$P_{k+1|k} = A_kP_{k|k}A^T_k + Q_k \quad (36)$$

where $\hat{x}^*_{\cdot|\cdot}$ is the optimal state estimate and $P_{\cdot|\cdot}$ is the error covariance of $\hat{x}^*_{\cdot|\cdot}$.

4.1 Derivation of proposed algorithm

Define

$$\hat{x}^*_{k|k} = \hat{z}_{k|k} + V_k\gamma_k \quad (37)$$

$$\hat{x}^*_{k+1|k} = \hat{z}_{k+1|k} + U_{k+1}\gamma_k \quad (38)$$

$$r_k = y_k - C_k\hat{z}_{k|k-1} \quad (39)$$

with

$$V_k = (I - G_kC_k)U_k \quad (40)$$

$$S_k = C_kU_k \quad (41)$$

$$U_{k+1} = A_kV_k + B_k \quad (42)$$

Since $r_k = y_k - C_k\hat{x}^*_{k|k-1} + S_k\gamma_k$, i.e., $E[r_k] = S_k\gamma_k$, the estimate of γ_k can be obtained by

$$\hat{\gamma}_k = \beta_k\hat{E}[r_k] \quad (43)$$

where $\beta_k \in R^{m\times n}$, $m < n$, satisfing

$$\beta_kS_k = I_m \quad (44)$$

is yet to be determined. If there are multiple sensors avaliable to measure the target, it is possible to get $\hat{E}[r_k]$ based on them. However, in most cases there are not enough sensors avaliable to get an reasonable estimate. In the proposed method the estimate of unknown input is given by

$$\hat{\gamma}_k = (I - \Lambda)\hat{\gamma}_{k-1} + \Lambda\beta_kr_k \quad (45)$$

where $\Lambda = \mathrm{diag}(\alpha_1, \cdots, \alpha_m)$, and $0 < \alpha_i < 1$ is the tuning parameter, which actually determines the input estimator convergence speed. Substituting (37)-(39) and (45) into (32) and (33), the proposed algorithm is obtained as

$$\hat{x}_{k|k} = \hat{z}_{k|k} + V_k\hat{\gamma}_{k|k} \quad (46)$$

$$\hat{x}_{k+1|k} = \hat{z}_{k+1|k} + U_{k+1}\hat{\gamma}_{k+1|k} \quad (47)$$

$$\hat{z}_{k|k} = \hat{z}_{k|k-1} + G_kr_k \quad (48)$$

$$\hat{z}_{k+1|k} = A_k\hat{z}_{k|k} \quad (49)$$

$$\hat{\gamma}_{k|k} = \hat{\gamma}_{k|k-1} + \Lambda\beta_k\left(r_k - S_k\hat{\gamma}_{k|k-1}\right) \quad (50)$$

$$\hat{\gamma}_{k+1|k} = \hat{\gamma}_{k|k} \quad (51)$$

Noticed that the filter given by (48)-(49) is actually a constant velocity filter. The input estimator given by (50)-(51) uses the residual sequence of the constant velocity filter to provide an input estimate. The actual state estimate is obtained by compensating the output of the constant velocity filter with the output of the input estimator as shown in (46)-(47), with

$$P^{xx}_{k|k} = E\left[\left(x_k - \hat{x}_{k|k}\right)\left(x_k - \hat{x}_{k|k}\right)^T\right] = P_{k|k} + V_kP^{\gamma\gamma}_{k|k}V^T_k \quad (52)$$

$$P^{xx}_{k+1|k} = E\left[\left(x_{k+1} - \hat{x}_{k+1|k}\right)\left(x_{k+1} - \hat{x}_{k+1|k}\right)^T\right] = P_{k+1|k} + U_{k+1}P^{\gamma\gamma}_{k+1|k}U^T_{k+1} \quad (53)$$

$$P^{x\gamma}_{k|k} = E\left[\left(x_k - \hat{x}_{k|k}\right)\left(\gamma_k - \hat{\gamma}_{k|k}\right)^T\right] = V_kP^{\gamma\gamma}_{k|k} \quad (54)$$

$$P^{x\gamma}_{k+1|k} = E\left[\left(x_{k+1} - \hat{x}_{k+1|k}\right)\left(\gamma_{k+1} - \hat{\gamma}_{k+1|k}\right)^T\right] = U_{k+1}P^{\gamma\gamma}_{k+1|k} \quad (55)$$

$$P^{\gamma\gamma}_{k|k} = E\left[\left(\gamma_k - \hat{\gamma}_{k|k}\right)\left(\gamma_k - \hat{\gamma}_{k|k}\right)^T\right] = (I - \Lambda)P_{k|k-1}(I - \Lambda)^T + \Lambda\beta_k\left(C_kP_{k|k-1}C^T_k + R_k\right)\beta^T_k\Lambda^T \quad (56)$$

$$P^{\gamma\gamma}_{k+1|k} = E\left[\left(\gamma_{k+1} - \hat{\gamma}_{k+1|k}\right)\left(\gamma_{k+1} - \hat{\gamma}_{k+1|k}\right)^T\right] = P^{\gamma\gamma}_{k|k} \quad (57)$$

4.2 Optimal choice of β_k

The optimal β_k is chosen in such a way that it minimizes the error covariance given by (52)-(57). As mentioned previously, the only constraint on β_k is given by (44), this minimizing can be achieved by solving

$$\min_{\substack{\beta_k S_k = I \\ \text{fix}\Lambda}} P^{\gamma\gamma}_{k|k} \tag{58}$$

which yields

$$\beta_k = \left[S_k^T \left(C_k P_{k|k-1} C_k^T + R_k\right)^{-1} S_k\right]^{-1} \cdot S_k^T \left(C_k P_{k|k-1} C_k^T + R_k\right)^{-1} \tag{59}$$

Substituting (59) into (56) yields

$$P^{\gamma\gamma}_{k|k} = (I-\Lambda) P_{k|k-1} (I-\Lambda)^T + \Lambda \left[S_k^T \left(C_k P_{k|k-1} C_k^T + R_k\right)^{-1} S_k\right]^{-1} \Lambda^T \tag{60}$$

4.3 Maneuver detector

Since the constant velocity filter and the input estimator are connected in parallel, the input estimator can be turned on and off as needed. When the target moves with a constant velocity, the input estimator is turned off. Whenever a maneuver is detected, the input estimator is turned on and its ouput is used to correct the estimate of the constant velocity filter until the maneuver ends. Therefore, the proposed method consists of three paralle blocks: a constant velocity filter, an input estimator, and a maneuver detector, as shown in Fig. 2.

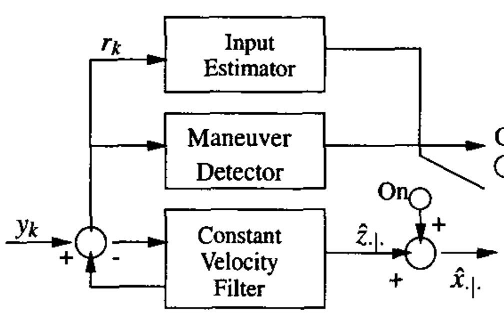

Figure 2: Proposed Tracking Algorithm

Since a maneuver manifests itself as a "large" innovation, it can be detected by watching the normalized innovations squared

$$\varepsilon_k = r_k^T \left(C_k P_{k|k-1} C_k^T + R_k\right)^{-1} r_k \tag{61}$$

A maneuver is declared if $\|\varepsilon_k\|$ exceeds a given threshold, h; and an end of a maneuver is declared if $\|\varepsilon_k\|$ falls below h. The threshold h can be pre-selected in such a way that for non-maneuvering situation

$$P(\|\varepsilon_k\| \le h) = 0.95 \tag{62}$$

4.4 Comments on the proposed algorithm

The proposed algorithm has similar structure to the two-stage Kalman estimator. Since Λ determines the convergence speed of input estimator, from the implementation point of view, the proposed method has advantages over the two-stage estimator in the following senses. First, the same input estimator can be used for a wide range of noise conditions, while the two-stage estimator has to change Q_k^{γ} to match up changes of noise conditions. Second, the tuning of the new method only affects the input estimator, but the tuning of the two-stage estimator affects both the constant velocity filter and the acceleration filter. Third, the restriction given by (31) yields a difficulty in implementation of the two-stage Kalman estimator, and the proposed method is free from such restrictions.

5 Evaluation of Proposed Tracking Method

In this section, the tracking performance of the proposed method is compared with the two-stage tracker. The comparison criterion is, for the same noise condition, i.e., same Q_k and R_k, and the same input estimator convergence speed, the method with smaller estimation error covariance is considered better. Forthemore, since larger convergence speed implies larger estimation error covarince, this also means the better method converges faster to achieve the same level of estimation error covariance. Although the results presented in this section should be true in general, to easily present the results, it is assumed that the system (1) and (2) are pice-wise time-invariant, the noises are pice-wise stationary, and the noises in different channels are uncorrelated. Therefore, we only need to compare both methods in one single channel, i.e.,

$$A = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} \frac{\Delta t^2}{2} \\ \Delta t \end{bmatrix}, \text{rank}(C) = 2 \tag{63}$$

with $\alpha = \Lambda$ and $q^{\gamma} = Q_k^{\gamma}$ be scales, and Δt be the sampling time. Define

$$M^{(\cdot)} = P^{(\cdot)}_{k+1|k}, \quad P^{(\cdot)} = P^{(\cdot)}_{k|k} \tag{64}$$

$$\bar{M}^{(\cdot)} = \bar{P}^{(\cdot)}_{k+1|k}, \quad \bar{P}^{(\cdot)} = \bar{P}^{(\cdot)}_{k|k} \tag{65}$$

$$L = S^T \left(CMC^T + R\right)^{-1} S \tag{66}$$

$$\bar{L} = \bar{S}^T \left(C\bar{M}C^T + R\right)^{-1} \bar{S} \tag{67}$$

The input estimator convergence speed of the proposed method is given by α, the estimation error covariance is given by (52)-(57). From (60)

$$P^{\gamma\gamma} = M^{\gamma\gamma} = \frac{\alpha}{2-\alpha} L^{-1} \tag{68}$$

The input estimator (acceleration filter) convergence speed of two-stage Kalman estimator is given by $\bar{\alpha} = \bar{G}^{\gamma}\bar{S}$. From (16)-(18), the relation between $\bar{\alpha}$ and q^{γ} is obtained as

$$q^{\gamma} = \frac{\bar{\alpha}^2}{1-\bar{\alpha}} \bar{L}^{-1} \tag{69}$$

The corresponding estimation error covariance is given by (24)-(29) with

$$\bar{P}^{\gamma\gamma} = \frac{1-\bar{\alpha}}{\bar{\alpha}} q^{\gamma} \quad (70)$$

$$\bar{M}^{\gamma\gamma} = \frac{1}{\bar{\alpha}} q^{\gamma} \quad (71)$$

Noticed that $\bar{P} = P$ and $\bar{M} = M$ for $q^{\gamma} = 0$, and $q^{\gamma} \ll \|M\|$, $q^{\gamma} \ll \|P\|$ should be true in applications, otherwise the noise in input would overcome any useful signals and the filter can not work properly. Based on above assumptions, $\bar{M}$ and $\bar{P}$ can be expressed as follows:

$$\bar{M} = M + q^{\gamma}\frac{d\bar{M}(0)}{dq^{\gamma}} + \text{h.o.t.} \quad (72)$$

$$\bar{P} = P + q^{\gamma}\frac{d\bar{P}(0)}{dq^{\gamma}} + \text{h.o.t.} \quad (73)$$

Now, we are in the position to state the following

Theorem *Let* $\lambda_1 = |\min(\text{eig}(GC))|$, *if* $q^{\gamma} \ll \|M\|$, $q^{\gamma} \ll \|P\|$, *and* $\alpha < \lambda_1$, *then*

$$M^{xx} < \bar{M}^{xx} \quad (74)$$

Forthemore, if $\alpha < \lambda_2 = \frac{5\lambda+2-\sqrt{\lambda^2+8\lambda+4}}{2(2\lambda+1)}$, *where* $\lambda = \lambda_1(2-\lambda_1)$, *then*

$$P^{xx} < \bar{P}^{xx} \quad (75)$$

Proof: see Appendix.

Since $\lambda_2 < \lambda_1$, we have the following

Corallary *If* $\alpha < \lambda_2$, *with the same input estimator convergence speed, the proposed method has smaller estimation error covariance.*

6 Simulation Results

A maneuvering target tracking example is presented in this section. The tracking performance of the proposed method and the two-stage tracker are compared. Consider a target moving at a constant velocity from $t = 0$ to $t = 5$s, a constant acceleration of 0.5g is applied at $t = 5$s. It completes the acceleration at $t = 10$s, and moves at constant velocity untill $t = 15$s. The initial target position and velocity are $x(0) = 20$m and $\dot{x}(0) = 3$m/s. The sampling time is $\Delta t = 0.1$s, and the observations consist of target position and velocity. The noise covariance matrices are given by

$$Q = \begin{bmatrix} 0.64 & 0 \\ 0 & 0.04 \end{bmatrix} \qquad R = \begin{bmatrix} 1 & 0 \\ 0 & 0.09 \end{bmatrix} \quad (76)$$

This yields $\lambda_1 = 0.48$ and $\lambda_2 = 0.325$. The same Q is used all the time in the proposed method, and in the two-stage Kalman estimator, Q is replaced by $\bar{Q}$ whenever a maneuver is being declared detected. The threshold for maneuver detecting is chosen as $h = 0.5$.

First, the convergence speed of the input estimator is chosen as $\alpha = \bar{\alpha} = 0.3$. The corresponding q^{γ} obtained from (69) is $q^{\gamma} = 0.28$. A Monte-Carlo simulation of 300 runs was done for the two trackers. Fig. 3 shows the average RMS errors in the target position and velocity. As can be seen that the tracking performance of these two trackers are very similar, but the proposed method has slight improvement on the input estimate.

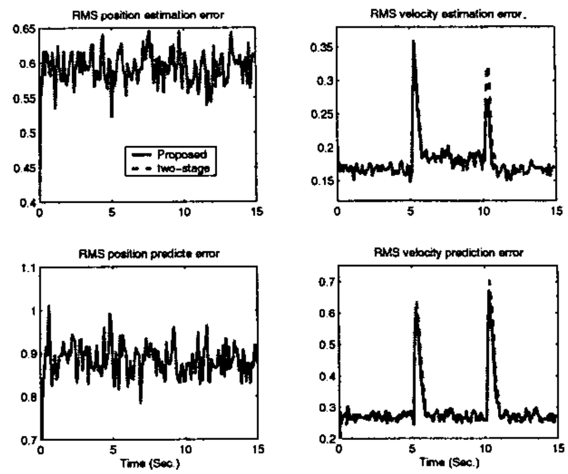

Figure 3: Average RMS Position Error and Velocity Error with $\alpha = 0.3$

Second, the input estimator convergence speed is set as $\alpha = 0.33$, which corresponds to $q^{\gamma} = 0.3$. The simulation result is shown in Fig. 4. The two-stage tracker is unstable because the constraint (31) is not satisfied. The proposed method works well even $\alpha > \lambda_2$ in this case. This implies that using the tuning parameter α has advantage over using q^{γ}.

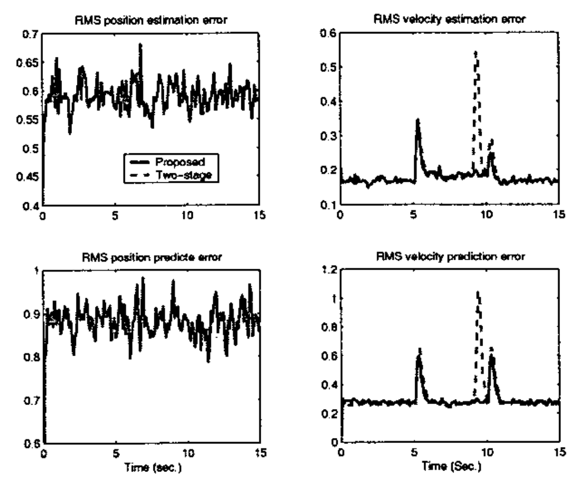

Figure 4: Average RMS Position Error and Velocity Error with $\alpha = 0.33$

7 Conclusions

A maneuvering target tracking algorithm with input estimation has been proposed in this paper. It has been shown that this new method is a better altenative to the two-stage Kalman estimator. The tuning of the new filter is independent of processing and measurement noises, and does not affect the constant velocity filter. This makes the proposed method easy to implement in practical applications. As mentioned in the previous section, the difficulty associated with the two-stage Kalman estimator is the condition given by (31) must be satisfied to guarantee the stability. The new method is free from such conditions.

APPENDIX: Proof of Theorem

From (11)-(13)

$$\begin{aligned}
\frac{d\bar{M}(0)}{dq^{\gamma}} &= A(I-GC)\frac{d\bar{M}(0)}{dq^{\gamma}}(I-GC)^T A^T - UU^T \\
&\geq (1-\lambda_1)^2\frac{d\bar{M}(0)}{dq^{\gamma}} - UU^T \\
&\geq -\frac{UU^T}{\lambda_1(2-\lambda_1)} \qquad (77)
\end{aligned}$$

$$\begin{aligned}
\frac{d\bar{P}(0)}{dq^{\gamma}} &= (I-GC)\frac{d\bar{M}(0)}{dq^{\gamma}}(I-GC)^T \\
&\geq -\frac{VV^T}{\lambda_1(2-\lambda_1)} \qquad (78)
\end{aligned}$$

Substituting (77), (78) into (72), (73), and using (24), (25), (52), (53), (66)-(71), and $\bar{\alpha} = \alpha$, we obtain

$$\begin{aligned}
\tilde{M}^{xx} &= \bar{M} + \bar{U}\bar{M}^{\gamma\gamma}\bar{U}^T \\
&> M + \frac{\alpha}{1-\alpha}\frac{UU^T}{L} + \frac{\alpha^2}{1-\alpha}\frac{1}{L}\frac{d\bar{M}(0)}{dq^{\gamma}} + \text{h.o.t.} \\
&\geq M^{xx} + \frac{\alpha^2 UU^T}{(1-\alpha)L}\left[\frac{1}{\alpha(2-\alpha)} - \frac{1}{\lambda_1(2-\lambda_1)}\right] \\
&\quad + \text{h.o.t.} \\
&> M^{xx} \quad \text{if} \quad \alpha < \lambda_1 \qquad (79)
\end{aligned}$$

$$\begin{aligned}
\tilde{P}^{xx} &= \bar{P} + \bar{V}\bar{P}^{\gamma\gamma}\bar{V}^T \\
&\geq P + q^{\gamma}\frac{d\bar{P}(0)}{dq^{\gamma}} + \alpha VV^T\left(\frac{1}{L} + \frac{1}{\bar{L}}\right) + \text{h.o.t.} \\
&\geq P^{xx} + \frac{VV^T}{\bar{L}}\left[\frac{\alpha(3-2\alpha)}{2-\alpha} - \frac{\alpha^2}{1-\alpha}\frac{1}{\lambda_1(2-\lambda_1)}\right] \\
&\quad + \text{h.o.t.} \\
&> P^{xx} \quad \text{if} \quad \alpha < \lambda_2 \qquad (80)
\end{aligned}$$

End of proof.

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