General Forms and Properties of Zero Cross-Correlation Radar Waveforms

KARL GERLACH, Member, IEEE
FRANK F. KRETSCHMER, JR., Fellow, IEEE
Naval Research Laboratory

General forms for both the complementary and noncomplementary zero cross-correlation waveform sets are developed. Various properties of these codes and their relationship to zero sidelobe periodic codes are stated and proved. Also, some radar applications and practical considerations of using these codes are briefly discussed.

Manuscript received January 11, 1990; revised March 6, 1991.

IEEE Log No. 9103621.

Authors' address: Naval Research Laboratory, Radar Division, Code 5341, Washington, DC 20375.

0018-9251/92/$3.00 © 1992 IEEE

I. INTRODUCTION

Modern radars generally incorporate pulse compression waveforms to obtain the desired range resolution while avoiding pulses having large peak powers. Pulse compression waveforms are exemplified by the Barker, pseudorandom shift register, chirp, and the polyphase codes [1–3]. New waveforms are described here which have been recently investigated for use in radar systems. Of particular interest are multiple dissimilar waveforms having very low sidelobes after processing. Low sidelobes are desired to prevent the masking of weak targets in the sidelobes of strong targets or clutter returns. The multiple waveforms (whose number we set equal to M) are processed by individually matched filtering, time aligning, and summing the results.

The multiple waveforms considered here are derived from either complementary or noncomplementary waveforms. Complementary waveforms [4–9] are coded sequences (complex numbers in general) having autocorrelation functions (ACFs) (or equivalently the outputs of pulse compressors consisting of filters matched to the coded sequences) which when time aligned and added together, sum to zero everywhere except at the match point. This is illustrated in Fig. 1 for $M = 2$.

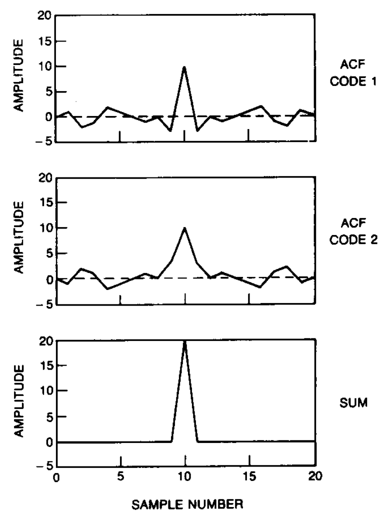


Fig. 1. Complementary code example.

In [2 and 3], new multiple waveforms were discussed which when filtered by filters matched to a different waveform of the set, have zero cross-correlation response after combination of the individual responses. These waveforms have potential applications in cancelling stationary clutter from ambiguous ranges in a medium or high pulse-repetition filter (PRF) radar, and/or in reducing mutual interference between radars in proximity to each other that are operating in the same frequency band.

This paper is an extension of that work presented in [2, 3]. Here we give general forms for both the complementary and noncomplementary zero cross-correlation waveform sets. In addition, various properties of these codes and their relationship to zero sidelobe periodic codes are stated and proved. Also, a radar application of using these codes is presented.

II. DEFINITIONS

In this section we define our nomenclature and review the concept of periodic coded waveforms. A code word $\mathbf{a}$ is defined as a vector of length N and

$$\mathbf{a} = (a_0, a_1, \ldots, a_{N-1}) \tag{1}$$

where a_n, $n = 0,1,\ldots,N-1$ are the elements of the code word. This code word modulates a carrier frequency and is match filtered at baseband upon reception. The aperiodic ACF of $\mathbf{a}$ is given by the expressions

$$\begin{aligned} r_a(k) &= \sum_{i=0}^{N-1-k} a_i^* a_{i+k}, \qquad k = 0,1,\ldots,N-1 \\ r_a(-k) &= \sum_{i=0}^{N-1-k} a_{i+k}^* a_i, \qquad k = 1,2,\ldots,N-1 \end{aligned} \tag{2}$$

where * in the superscript denotes complex conjugation. The $k=0$ value of $r_a(k)$ corresponds to the match point and the $k \neq 0$ values correspond to the right and left sidelobes of the compressed pulse.

A periodic code is one that repeats the code word $\mathbf{a}$ indefinitely. Hence, if $\mathbf{a}_{\text{pc}}$ is the periodic code associated with $\mathbf{a}$ then

$$\mathbf{a}_{\text{pc}} = \mathbf{a} \circ \mathbf{a} \circ \mathbf{a} \ldots \tag{3}$$

where the symbol "$\circ$" denotes concatenation. On reception, a periodic code is match filtered with its code word. The output of the correlation process is also periodic with a period N. Hence, the matched peak response repeats every N unit time delays as does the sidelobe response. We define the N point periodic ACF as

$$r_p(k) = \sum_{i=0}^{N-1} a_i^* a_{(i+k)\bmod N}, \qquad k = 0,1,\ldots,N-1. \tag{4}$$

Note that the $i+k$ subscript is taken modulo N. Thus we are computing the residue of $i+k$ with respect to the number of subpulses contained in the code word. For our development, we always compute the subscript with respect to the code order and drop the mod N notation from the subscript, thus $a_{N+i} = a_i$.

Define the vectors, $\mathbf{h}_k$, $k = 0,\ldots,N-1$ as

$$\begin{aligned} \mathbf{h}_0 &= (a_0, a_1, \ldots, a_{N-1}) \\ \mathbf{h}_1 &= (a_1, a_2, \ldots, a_{N-1}, a_0) \\ \mathbf{h}_2 &= (a_2, a_0, \ldots, a_{N-1}, a_0, a_1) \\ &\vdots \\ \mathbf{h}_{N-1} &= (a_{N-1}, a_0, a_1, \ldots, a_{N-2}) \end{aligned} \tag{5}$$

where these vectors are the circular rotations of $\mathbf{a}$. Equation (4) can be rewritten as

$$r_p(k) = \mathbf{h}_0^* \mathbf{h}_k^{\mathrm{T}}, \qquad k = 0,1,\ldots,N-1 \tag{6}$$

where T denotes the vector transpose operation.

A zero sidelobe periodic code (ZSPC) has the property that

$$r_p(k) = \mathbf{h}_0^* \mathbf{h}_k^{\mathrm{T}} = 0, \qquad \text{for} \quad k \neq 0. \tag{7}$$

If all the code elements of a ZSPC have unit amplitude, then the code is called a perfect periodic code [1, 2].

We now consider multiple waveforms. Define the code matrix C as an $M \times N$ matrix of code words:

$$C = \begin{bmatrix} c_{00} & c_{01} & \cdots & c_{0,N-1} \\ c_{10} & c_{11} & \cdots & c_{1,N-1} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ c_{M-1,0} & c_{M-1,1} & \cdots & c_{M-1,N-1} \end{bmatrix}. \tag{8}$$

Let there be M code words of length N where the mth code word ($m = 0,1,\ldots,M-1$) is defined by the $m+1$th row of C or

$$\mathbf{c}_m = (c_{m0}, c_{m1}, \ldots, c_{m,N-1}). \tag{9}$$

We define the aperiodic cross-correlation vector (CCV) between $\mathbf{c}_m$ and $\mathbf{c}_n$ as

$$\mathbf{c}_m^* \ast \tilde{\mathbf{c}}_n = (r_{-(N-1)}^{(mn)}, r_{-(N-2)}^{(mn)}, \ldots, r_0^{(mn)}, r_1^{(mn)}, \ldots, r_{N-1}^{(mn)}) \tag{10}$$

where the bold asterisk $\ast$ denotes the linear convolution operation, $\sim$ denotes the time reversal of the sequence $\mathbf{c}_n$ and

$$r_k^{(mn)} = \sum_{i=0}^{N-1-k} c_{mi}^* c_{n,i+k}, \qquad k \geq 0 \tag{11}$$

$$r_{-k}^{(mn)} = \sum_{i=0}^{N-1-k} c_{m,i+k}^* c_{n,i}, \qquad k > 0. \tag{12}$$

Note that in general $|r_k^{(mn)}| \neq |r_{-k}^{(mn)}|$ unless $m = n$.

In addition, the summed CCV is defined as

$$\sum_{m=0}^{M-1} \mathbf{c}_m^* \bullet \bar{\mathbf{c}}_{m+l} = (q_{-(N-1)}^{(l)}, q_{-(N-2)}^{(l)}, \ldots, q_0^{(l)}, q_1^{(l)}, \ldots, q_{N-1}^{(l)}). \tag{13}$$

We note that if

$$\sum_{m=0}^{M-1} \mathbf{c}_m^* \bullet \bar{\mathbf{c}}_m = (0,0,\ldots,\underset{\substack{\uparrow\\ N\text{th position}}}{1},0,0,\ldots,0) \tag{14}$$

then the code words of C form a complementary code set. If $|q_{-k}^{(l)}| = |q_k^{(l)}|$, then the summed CCV is called "magnitude symmetric." Furthermore, if

$$\sum_{m=0}^{M-1} \mathbf{c}_m^* \bullet \bar{\mathbf{c}}_{m+l} = \mathbf{0}, \qquad l \neq 0 \tag{15}$$

where $\mathbf{0}$ is a vector of $2N-1$ zeros, then we call the code words of C a zero cross-correlation code (ZCC).

In the following sections we consider codes which are formed by concatenating the M rows of C. Thus a code word $\mathbf{a}$ is formed as

$$\mathbf{a} = \mathbf{c}_0 \circ \mathbf{c}_1 \circ \cdots \circ \mathbf{c}_{M-1}. \tag{16}$$

III. PROPERTIES OF ZCC COMPLEMENTARY WAVEFORMS

There is a relationship between ZCC complementary codes and their associated periodic code which is stated in the following theorem.

THEOREM 1. *If the rows of C form a ZSPC, are a complementary code, and the summed CCV is magnitude symmetric, then the rows of C form a ZCC code.*

PROOF. Let us form the periodic code associated with C

$$\mathbf{h}_0 = (c_{00}, c_{01}, \ldots, c_{0,N-1}, c_{10}, c_{12}, \ldots, c_{1,N-1}, c_{20}, \ldots, c_{M-1,N-1}). \tag{17}$$

The circular rotations of $\mathbf{h}_0$ are defined by (5).

Let $l = l_1 N + l_2$ where $0 \leq l_2 < N$ and set $r_{\pm N}^{(mn)} = 0$ for all m,n. It is straightforward to show that for a ZSPC, $l \neq 0$

$$\mathbf{h}_0^* \mathbf{h}_l^{\mathrm{T}} = \sum_{m=0}^{M-1} r_{l_2}^{(m,m+l_1)} + \sum_{m=0}^{M-1} r_{-(N-l_2)}^{(m,m+l_1+1)} = 0 \tag{18}$$

where $m + l_1$ and $m + l_1 + 1$ are taken modulo M.

Using (13) we know that

$$q_j^{(i)} = \sum_{m=0}^{M-1} r_j^{(m,m+i)}. \tag{19}$$

Note that $q_{\pm N}^{(i)} = 0$ because $r_{\pm N}^{(mn)} = 0$ for all m,n. It is instructive to write (18) out for successive values of l using (19)

$$\begin{aligned}
\mathbf{h}_0^*\mathbf{h}_1^{\mathrm{T}} &= q_1^{(0)} + q_{-(N-1)}^{(1)} = 0\\
\mathbf{h}_0^*\mathbf{h}_2^{\mathrm{T}} &= q_2^{(0)} + q_{-(N-2)}^{(1)} = 0\\
&\vdots\\
\mathbf{h}_0^*\mathbf{h}_{N-1}^{\mathrm{T}} &= q_{N-1}^{(0)} + q_{-1}^{(1)} = 0\\
\mathbf{h}_0^*\mathbf{h}_N^{\mathrm{T}} &= q_N^{(0)} + q_0^{(1)} = 0\\
\mathbf{h}_0^*\mathbf{h}_{N+1}^{\mathrm{T}} &= q_1^{(1)} + q_{-(N-1)}^{(2)} = 0\\
\mathbf{h}_0^*\mathbf{h}_{N+2}^{\mathrm{T}} &= q_2^{(1)} + q_{-(N-2)}^{(2)} = 0\\
&\vdots\\
\mathbf{h}_0^*\mathbf{h}_{2N-1}^{\mathrm{T}} &= q_{N-1}^{(1)} + q_{-1}^{(2)} = 0\\
\mathbf{h}_0^*\mathbf{h}_{2N}^{\mathrm{T}} &= q_N^{(1)} + q_0^{(2)} = 0\\
\mathbf{h}_0^*\mathbf{h}_{2N+1}^{\mathrm{T}} &= q_1^{(2)} + q_{-(N-1)}^{(3)} = 0.\\
&\vdots
\end{aligned} \tag{20}$$

We note that every Nth equation of (20) is of the form, $\mathbf{h}_0^*\mathbf{h}_{nN}^{\mathrm{T}} = q_N^{(n-1)} + q_0^{(n)} = 0$, $n = 1,2,\ldots,N-1$. Since $q_N^{(n-1)} = 0$ it follows that $q_0^{(n)} = 0$ for $n = 1,2,\ldots,N-1$. It is seen that if the code words of C are complementary then $q_j^{(0)} = 0$ for $j \neq 0$. Thus using the first $N-1$ equations of (20) imply that $q_{-j}^{(1)} = 0$ for $j = 1,2,\ldots,N-1$. If the summed CCV is magnitude symmetric then $q_j^{(1)} = 0$ for $j = 1,2,\ldots,N-1$. Hence using the $(N+1)$th through $(2N-1)$th equation of (20), it follows that $q_{-j}^{(2)} = 0$. This argument can be repeated to show that $q_j^{(i)} = 0$ for all i,j except for when $i = j = 0$. Hence the theorem follows.

The following two theorems can be shown using the same arguments:

THEOREM 2. *If C is a ZCC code and complementary then C is also a ZSPC.*

THEOREM 3. *If C is a ZCC code and a ZSPC then C is complementary.*

Next, consider the following matrix

$$C_{\text{aug}} = \begin{bmatrix} c_{00} & c_{01} & c_{02} & \cdots & c_{0,N-1} & 00\ldots0 \\ c_{10} & c_{11} & c_{12} & \cdots & c_{1,N-1} & 00\ldots0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \cdots & \cdot & \\ \cdot & \cdot & \cdot & \cdot & \cdot & \\ c_{M-1,0} & c_{M-1,1} & c_{M-1,2} & \cdots & c_{M-1,N-1} & 00\ldots0 \end{bmatrix} \quad (K \text{ zeros}) \tag{21}$$

where K is an arbitrary positive integer. This C_{aug} matrix is merely the original C matrix augmented with an $M \times K$ block of zeros. We show the following theorem.

THEOREM 4. *If C is a ZCC code and complementary then C_{aug} is a ZSPC.*

PROOF. It is elementary to show that if C is a ZCC code and complementary then C_{aug} is a ZCC code and complementary. Hence using Theorem 2 the theorem follows.

IV. GENERAL FORM OF ZCC COMPLEMENTARY WAVEFORMS

Consider the following $N \times N$ code matrix C where an element of C is defined by

$$c_{mi} = \lambda^m d_{i+1} W_N^{M'mi}, \qquad m,\ i = 0,1,\ldots,N-1 \tag{22}$$

where

$$W_N = e^{j(2\pi/N)}, \qquad \lambda \in \{1, W_N, W_N^2, \ldots, W_N^{N-1}\}, \tag{23}$$

$d_1, d_2, \ldots, d_{N-1}$ are arbitrary complex numbers, and M' is an integer relatively prime to N. We show the following theorem.

THEOREM 5. *The matrix C as defined by (22) is a ZCC complementary code.*

PROOF. Using (11) and (12), we can show that

$$\begin{aligned} r_k^{(mn)} &= \sum_{i=0}^{N-1-k} (\lambda^m d_{i+1} W_N^{M'mi})^* (\lambda^n d_{i+k+1} W_N^{M'n(i+k)}) \\ &= \lambda^{n-m} W_N^{M'nk} \sum_{i=0}^{N-1-k} d_{i+1}^* d_{i+k+1} W_N^{M'(n-m)i} \end{aligned} \tag{24}$$

$$\begin{aligned} r_{-k}^{(mn)} &= \sum_{i=0}^{N-1-k} (\lambda^m d_{i+k+1} W_N^{M'm(i+k)})^* (\lambda^n d_{i+1} W_N^{M'ni}) \\ &= \lambda^{n-m} W_N^{-M'mk} \sum_{i=0}^{N-1-k} d_{i+k+1}^* d_{i+1} W_N^{M'(n-m)i}. \end{aligned} \tag{25}$$

Thus if we set $n = m + l$, then

$$r_k^{(m,m+l)} = \lambda^l W_N^{M'(m+l)k} \sum_{i=0}^{N-1-k} d_{i+1}^* d_{i+k+1} W_N^{M'li} \tag{26}$$

$$r_{-k}^{(m,m+l)} = \lambda^l W_N^{-M'mk} \sum_{i=0}^{N-1-k} d_{i+1} d_{i+k+1}^* W_N^{M'li}. \tag{27}$$

From these equations, it can be shown that

$$q_k^{(l)} = \sum_{m=0}^{N-1} r_k^{(m,m+l)} = \left(\sum_{m=0}^{N-1} W_n^{M'mk}\right) \times \lambda^l W_N^{M'lk} \sum_{i=0}^{N-1-k} d_{i+1}^* d_{i+k+1} W_N^{M'li} \tag{28}$$

$$q_{-k}^{(l)} = \sum_{m=0}^{N-1} r_{-k}^{(m,m+l)} = \left(\sum_{m=0}^{N-1} W_N^{-M'mk}\right) \times \lambda^l \sum_{i=0}^{N-1-k} d_{i+1} d_{i+k+1}^* W_N^{M'li}. \tag{29}$$

Since

$$\sum_{m=0}^{N-1} W_N^{M'mk} = 0 \tag{30}$$

for M' relatively prime to N and $k \neq 0$, it follows that $q_k^{(l)} = q_{-k}^{(l)} = 0$ for $k, l \neq 0$. For $k = 0$ and $l \neq 0$, the second summation in both (28) and (29) is of the same form as (30). Thus $q_k^{(l)} = q_k^{(-l)} = 0$ for $k = 0$ and $l \neq 0$. Hence the theorem is proven.

We note that for $\lambda = d_1 = d_2 \cdots = d_N = M' = 1$ that the general form reduces to the Frank matrix which was shown in [2] to be a ZCC complementary waveform. In addition, if the Lewis-Kretschmer P4 code [1] has a length that is a square integer N^2, and the elements of this code are put into square matrix form where the concatenation of the rows generate the P4 code, then it is straightforward to show that this code also fits the general form given by (22) and hence is a ZCC complementary code.

V. ZCC NONCOMPLEMENTARY WAVEFORMS

In this section the following theorem is proved.

THEOREM 6. *If C has the form*

$$C = \begin{bmatrix} a_0 b_0 & a_0 b_1 & \cdots & a_0 b_{N-1} \\ a_1 b_0 & a_1 b_1 & \cdots & a_1 b_{N-1} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ a_{M-1} b_0 & a_{M-1} b_1 & \cdots & a_{M-1} b_{N-1} \end{bmatrix} \tag{31}$$

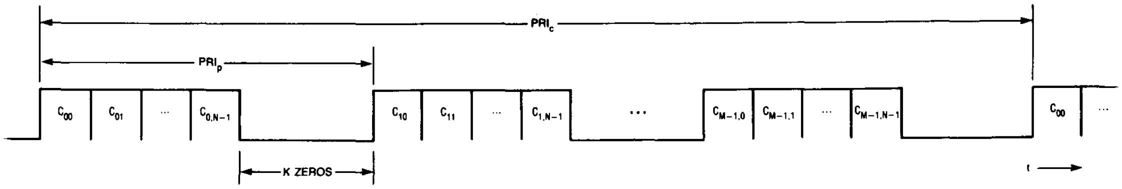


Fig. 2. Transmitted multiple codel.

and $\mathbf{a} = (a_0, a_1, \ldots, a_{M-1})$ *is a ZSPC, then the rows of* C *form a ZCC code.*

We call the code given by (31) an inner–outer code, because a given inner code of subpulses represented by $b_0, b_1, \ldots, b_{N-1}$ is modulated on a pulse-to-pulse basis by an outer code given by $a_0, a_1, \ldots, a_{M-1}$.

PROOF. The individual code elements are given by

$$c_{mi} = a_m b_i, \qquad m,\ i = 0, 1, \ldots, N-1. \tag{32}$$

Using (11) and (12), it follows that

$$r_k^{(mn)} = \sum_{i=0}^{N-1-k} a_m^* b_i^* a_n b_{i+k}, \qquad k \geq 0, \tag{33}$$

$$r_{-k}^{(mn)} = \sum_{i=0}^{N-1-k} a_m^* b_{i+k}^* a_n b_i, \qquad k > 0. \tag{34}$$

Thus setting $n = m + l$

$$q_k^{(l)} = \sum_{m=0}^{M-1} r_k^{(m,m+l)} = \left(\sum_{m=0}^{M-1} a_m^* a_{m+l}\right)\left(\sum_{i=0}^{N-1-k} b_i^* b_{i+k}\right), \qquad k \geq 0, \tag{35}$$

$$q_{-k}^{(l)} = \sum_{m=0}^{M-1} r_{-k}^{(m,m+l)} = \left(\sum_{m=0}^{M-1} a_m^* a_{m+l}\right)\left(\sum_{i=0}^{N-1-k} b_{i+k}^* b_i\right), \qquad k > 0, \tag{36}$$

Since $\mathbf{a}$ is a ZSPC,

$$\sum_{m=0}^{M-1} a_m^* a_{m+l} = 0 \qquad \text{for} \quad l \neq 0.$$

Hence $q_k^{(l)}$ and $q_{-k}^{(l)}$ are equal to zero and the theorem follows.

We note that the advantage that the noncomplementary ZCC waveforms have over the complementary ZCC waveforms is that the code matrix does not necessarily have to be square, i.e. $M \neq N$. Hence, there is inherently more flexibility in transmitting and receiving these waveforms.

VI. RADAR APPLICATION EXAMPLE

In this section a radar application using the complementary or inner–outer waveforms described with elements in the previous sections is briefly discussed. Only codes that are unit amplitude or zero (if the code element is turned off) are considered. These codes have the practical advantage that they are energy efficient on transmit. Thus for the general form of the ZCC complementary code given by (22), we stipulate that $d_1, d_2, \ldots, d_{N-1}$ must be on the unit circle.

Most radar waveforms do not have 100 percent duty cycles but have off-times which are used to listen for or receive the waveform. Hence the actual pulse train associated with the matrix C may look as shown in Fig. 2. Here each row of C forms a pulse (or group of subpulses). We define the code of the mth subpulse associated with the $m+1$ row or pulse as

$$\mathbf{c}_m = (c_{m0}, c_{m1}, \ldots, c_{m,N-1}). \tag{37}$$

Each pulse is separated by a given pulse-repetition interval (PRI$_p$) where there are 0s transmitted between the end of one pulse and the next. Normally this "off" time is greater than the pulse "on" time. All of the code words are transmitted in PRI$_c$ seconds. Thereafter, they may be repeated with a period PRI$_c$ for multiple burst processing.

One application of the ZCC complementary codes, which was first presented in [2 and 3] and is also applicable to ZCC inner–outer codes, is in removing ambiguous range radar returns for medium or high PRF radars. An example of this for a single burst is shown in Fig. 3 for $N = 4$. The waveforms are transmitted as shown in Fig. 3 according to the rows in C, but the return signals are processed only during the indicated processing interval in multiple channels having filters matched to the indicated codes in each PRI. That is, after transmitting $\mathbf{c}_0$ in the processing interval, all received signals are processed by filters matched to $\mathbf{c}_0$, $\mathbf{c}_3$, $\mathbf{c}_2$, and $\mathbf{c}_1$ in channels 0 to 3 respectively, and so on. The result is that channel 0 is matched to the first unambiguous range interval and rejects stationary returns (those that have almost zero Doppler shift) from the 2nd, 3rd, and 4th time around range intervals. Likewise, channels 1, 2, and 3 are matched to the 2nd, 3rd, and 4th time around returns and reject stationary clutter from the other range intervals. If the waveforms are complementary, stationary targets in the matched intervals have no sidelobes. Note that the fill pulses $\mathbf{c}_1$, $\mathbf{c}_2$, and $\mathbf{c}_3$ are necessary for this processing scheme (as they would be for any ambiguous range radar). However, if multiple bursts were used in a particular look direction

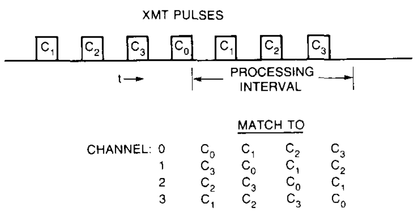


Fig. 3. Example of orthogonal waveform processing for $N = 4$.

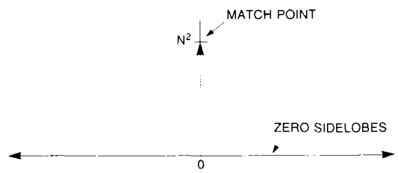


Fig. 4. ACF for ZCC complementary waveforms.

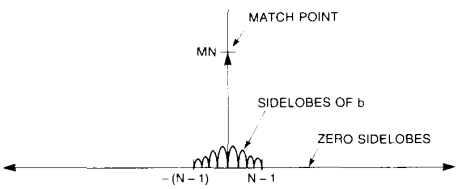


Fig. 5. ACF for noncomplementary ZCC waveforms.

then these fill pulses would be unnecessary for the succeeding bursts, because the preceding single burst would provide the fill pulses for the current burst.

For example, the matched filter response for a single burst of ZCC complementary waveforms is shown in Fig. 4 and for noncomplementary ZCC waveforms, in Fig. 5. From Fig. 4, we see that there are no sidelobes for the ZCC complementary waveforms. From Fig. 5, we observe that the sidelobes are non-zero only in the first $N - 1$ near-in right and left sidelobes about the match point for the noncomplementary ZCC waveforms. In fact, these sidelobes correspond to the sidelobes of the ACF of the codeword **b** times M where the sidelobes level is measured relative to the match point gain MN. Finally, we note that for clutter having a small spectral spread about zero Doppler, the nonambiguous range clutter can be reduced using multiple target indicator (MTI) processing. The PRI of the MTI canceler would equal PRI_c.

VII. SUMMARY

In this paper we have described the properties of zero cross-correlation waveform codes, i.e., the cross-correlation responses sum to zero everywhere. These codes, in turn, are related to periodic codes having zero sidelobe ACFs. These ideal periodic codes are important in themselves because the underlying aperiodic codes usually have useful attributes such as low sidelobes and/or good Doppler tolerance. This is exemplified by the Frank, P4, and shift register codes.

Two general forms of the ZCCs were described. The first form consists of a sequence of dissimilar waveforms that have the additional property of being complementary. The second form consists of a sequence of identically coded waveforms except for an outer code that results in a different phase being associated with each repetitive waveform.

A processing scheme using multiple waveforms was described that utilizes the ZCCs to eliminate zero Doppler ambiguous range clutter that might occur in a medium or high PRF radar. For clutter having a small spectral spread about zero Doppler, the nonambiguous range clutter is reduced in a manner similar to MTI processing. A detailed assessment of the tradeoffs, and the ability to resolve the true range of a target is the subject of future work.

REFERENCES

[1] Lewis, B. L., Kretschmer, F. F., Jr., and Shelton, W. W. (1986)
Aspects of Radar Signal Processing.
Norwood, MA: Artech House, 1986.

[2] Kretschmer, F. F., Jr., and Gerlach, K. (1991)
Low sidelobe radar waveforms derived from orthogonal matrices.
IEEE Transactions on Aerospace and Electronic Systems, **AES-27**, 1 (Jan. 1991).

[3] Kretschmer, F. F., Jr., and Gerlach, K. (1988)
New radar pulse compression waveforms.
Presented at the National Radar Conference, 1988.

[4] Golay, M. J. E. (1961)
Complementary series.
IRE Transactions on Information Theory, **IT-17** (Apr. 1961), 82–87.

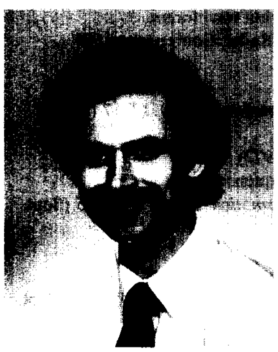
[5] Tseng, C. C., and Liu, C. L. (1972)
Complementary sets of sequences.
IEEE Transactions on Information Theory, **II-18**, 5 (Sept. 1972), 644–651.

[6] Sivaswami, R. (1978)
Multiphase complementary codes.
IEEE Transactions on Information Theory, **IT-24**, 5 (Sept. 1978), 546–552.

[7] Weathers, G., and Holiday, E. M. (1983)
Group-complementary array coding for radar clutter rejection.
IEEE Transactions on Aerospace and Electronic Systems, **AES-19**, 3 (May 1983), 369–379.

[8] Frank, R. L. (1980)
Polyphase complementary codes.
IEEE Transactions on Information Theory, **IT-26** (1980), 641–647.

[9] Sivaswami, R. (1978)
Digital and analog subcomplementary sequences for pulse compression.
IEEE Transactions on Aerospace and Electronic Systems, **AES-14**, 2 (Mar. 1978), 343–350.



Karl Gerlach (M'81) was born in Chicago, IL. He received his B.S. in 1972 from the University of Illinois, Urbana, and his M.S. and D.Sc. from George Washington University, Washington, DC, in 1975 and 1981, respectively. All degrees are in electrical engineering.

Since 1972, he has been employed by the Naval Research Laboratory in Washington, DC. From 1972 to 1976, he worked on experimental submarine communications systems and from 1976 to the present he has been with the Radar Division where his research interests include adaptive signal processing and space-based radar.

Dr. Gerlach was the 1986 recipient of the Radar Systems Panel award.

Frank F. Kretschmer, Jr. (M'64—SM'77—F'88) was born in Philadelphia, PA. He received the B.S. degree in 1957 from the Pennsylvania State University, University Park, the M.S. degree in 1961 from Drexel Institute of Technology, Philadelphia, PA, and the Ph.D. degree in 1970 from The Johns Hopkins University, Baltimore, MD, all in electrical engineering.

He was employed by the Burroughs Corporation, Paoli, PA, from 1957 to 1958. From 1958 to 1964 he was employed by Bendix Radio in Towson, MD, where he was engaged in radar systems work. From 1964 to 1970 he was a Research Associate at The Johns Hopkins University. Since 1970 he has been with the Naval Research Laboratory, Washington, DC, where his research interests include adaptive signal processing, clutter processing, and waveform coding.

Dr. Kretschmer is a member of Sigma Xi.