Proof of Bunyakovsky Conjecture

Jihyeon Yoon*
(Doctor of Korean Medicine, Shinhwa Geriatric Hospital)

July 2, 2024

Abstract

Bunyakovsky conjecture is proved to be true.

1 Introduction

Bunyakovsky conjecture is as follows[Bouniakowsky(1854)]:

1. The leading coefficient is positive,
2. The polynomial is irreducible over the rationals (and integers), and
3. There is no common factor for all the infinitely many values $f(1), f(2), f(3), \cdots$. (In particular, the coefficients of $f(x)$ should be relatively prime. It is not necessary for the values f(n) to be pairwise relatively prime.)

If $f(x)$ satisfies (1)–(3), then $f(n)$ is prime for infinitely many positive integers n.

2 Proof

As discussed in generalized prine number sequence[Yoon(2023)]:

$$p = (x^3 + 3x^2 + 2x)p' + p''$$

If Bunyakovsky conjecture is true, any polynomial that fits the statement in introduction can be reduced into the form of generalized prime number sequence with given x varies separately.

$$f(x) = ax^n + bx^{n-1} + \cdots z = (x^3 + 3x^2 + 2x)p' + p'' + a'x^n + b'x^{n-1} + \cdots + y'x + z'$$

With expanding generalized prime number sequence:

$$\begin{aligned}
&p = (x^3 + 3x^2 + 2x)p' + p'' \\
&\implies p = k + \alpha_1 M_1 + \alpha_2 M_2 + \alpha_3 M_3 \cdots \\
&({}^{\forall}\alpha \in \mathbb{P} \cup \{0, 1\}, k \in \{0, 1, 3, 5\}) \text{ s.t. } M_x = x(x+1)(x+2)
\end{aligned}$$

If then, with target polynomial function being described in addition or subtraction or multiplication of $x(x+1)(x+2)$ with other $x(x+1)(x+2)$ by arbitrary $x \in \mathbb{Z}$, then Bunyakovsky conjecture gets true. And it is true for the equations in polynomial function that fit the condition, and takes part other than that is presented invariably primes. (e.g. for upper occasion, $a'x^n + b'x^{n-1} + \cdots + y'x + z' = 0$ then leaves only prime form. Note that $a', b', \cdots, y', z'$ can vary with given formulation originated from $x(x+1)(x+2)$.)

*Jihyeon Yoon is a korean medicine doctor. And he is a freelancer programmer also.
E-mail: flyingtext@nate.com, somehowme@gmail.com
Yeongdeungpo-gu, Seoul, 07429,
Seoul, South Korea
ORCiD: https://orcid.org/0000-0001-9610-0994

e.g. $x(x+1)(x+2) = x^3 + 3x^2 + 2x, (x-2)(x-1)x = x^3 - 3x^2 + 2x$
$\implies x(x+1)(x+2) - (x-2)(x-1)x = 6x^2$

For intuitive explanation, $a'x^n + b'x^{n-1} + \cdots + y'x + z'$ inside $f(x)$ works as tank for constituting prime numbers with algebraic form of $x(x+1)(x+2)$. And Bunyakovsky conjecture is proved to be true.

References

[Bouniakowsky(1854)] Viktor Bouniakowsky. *Sur les diviseurs numériques invariables des fonctions rationnelles entieres*. De l'Imprimerie de l'Académie impériale des sciences, 1854.

[Yoon(2023)] Jihyeon Yoon. Generalized prime number sequence with proof. 2023. doi: 10.5281/ZENODO.8222937. URL https://zenodo.org/doi/10.5281/zenodo.8222937.