1 November 1999

PHYSICS LETTERS A

ELSEVIER

Physics Letters A 262 (1999) 257–260

www.elsevier.nl/locate/physleta

Comment on ‘‘The speed of gravity’’

Gerald E. Marsh [a,*], Charles Nissim-Sabat [b]

[a] Argonne National Laboratory, USA
[b] Northeastern Illinois University, USA

Received 18 March 1999; accepted 22 September 1999
Communicated by P.R. Holland

Abstract

Comment on a recent article by Van Flandern [Phys. Lett. A 250 (1998) 1]. © 1999 Published by Elsevier Science B.V. All rights reserved.

In a provocative paper, Van Flandern [1] argues that the speed of propagation of the inverse-square gravitational force must be, if not infinite, at least greater than $2 \times 10^{10}\ c$. The core of the argument is the statement that ‘‘the consequences of introducing a delay into gravitational interactions ... is usually disastrous because conservation of angular momentum is destroyed’’. Thus, Van Flandern argues that if the speed of propagation for gravitational fields is limited to the velocity of light, two gravitating masses would experience a torque which accelerates the two masses so as to add angular momentum to the system. Since the effects of such a torque have not been observed, Van Flandern concludes that the speed of gravitational interactions must be far greater than that of light to minimize retardation effects.

Van Flandern quotes Sir Arthur Eddington’s description of the argument and goes on to note that ‘‘the speed of gravity in Newtonian Universal Law is necessarily infinite. But [general relativity] reduces to Newtonian gravity in the low velocity, weak-field limit, which raises the obvious question of how that can be true if the propagation speed in one model is the speed of light, and in the other model it is infinite’’. Eddington’s version of the argument, with reference to Fig. 1, is:

‘‘If the Sun attracts Jupiter towards its present position S, and Jupiter attracts the Sun towards its present position J, the two forces are in the same line and balance. But if the Sun attracts Jupiter towards its previous position S', and Jupiter attracts the Sun towards its previous position J', when the force of attraction started out to cross the gulf, then the two forces give a couple. This couple will tend to increase the angular momentum of the system, and, acting cumulatively, will soon cause an appreciable change of period, disagreeing with observation if the speed is at all comparable with that of light’’.

Note that Eddington chooses a non-rotating frame of reference with origin at the center of mass. If one were to choose a frame of reference centered on one of the masses, say S, the force on J due to S is always directed radially toward S. The force on S due to J is also always radial and directed towards J. But since J is moving in the non-rotating refer-

* Corresponding author. E-mail: gmarsh@anl.gov

PII: S0375-9601(99)00675-1

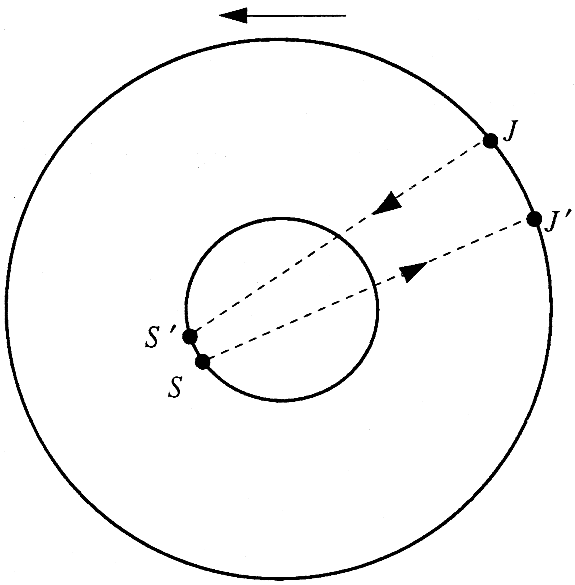

Fig. 1. Orbit stability and finite propagation speeds.

ence frame of S, one must consider retardation effects, and in this case there is no couple but linear momentum is not conserved. This is due to the fact that the force of S on J is not collinear with the force of J on S.

In Eddington's book [2], the very next sentence after Van Flandern's quote states that "The argument is fallacious, because ..." and there follows some discussion and a reference to Note 6 in Eddington's Appendix. While Eddington's comments as to why the argument is fallacious are cursory at best, he does speak of the potential of Liénard and Wiechert. He *implies* that if one first computes the retarded potentials of the two masses, one obtains the gravitational equivalent of the Liénard and Wiechert potentials and the latter are well known to yield, for a charge q with a uniform velocity v, electric and magnetic fields at an observation point O *that point back to the present position, not the retarded position*. As a result, Eddington concludes that there is no torque.

The no-torque result can be obtained from the Lorentz transformations when they are applied directly to the static Newtonian gravitational force. We will do the equivalent computation below in terms of the gravitational version of the Liénard and Wiechert potentials. Consider also the Trouton and Noble [3] experiment where two charges are held at the opposite ends of a horizontal rod. Before special relativity, because of the Earth's rotation and revolution about the sun, it was believed that the axis of the rod *will inevitably be inclined with respect to its direction of motion through the ether*. Thus the two charges would produce currents that repel (attract) each other if the charges are opposite (alike), and consequently there will be a torque on the rod. Special relativity, where there is no ether, predicts no torque and none is observed in the charges' rest frame.

The Van Flandern argument predicts the wrong result when applied to Trouton and Noble type experiments: For the sake of simplicity, consider two opposite charges such that the line connecting them is instantaneously perpendicular to the velocity, v, of an observer moving with respect to the charges. One would predict, using Van Flandern's argument, electric fields pointing back to retarded positions of the charges. If F is the force along the line connecting the charges in their rest frame, there would be a force Fv/c on each charge opposing v. This force cannot be canceled by the v^2/c^2 magnetic force between the charges.

The above considerations yield two conclusions: (1) rather mysteriously, retardation is enough to obtain the right potentials, as Liénard and Wiechert did before special relativity, but retardation (as used by Van Flandern) is not enough to obtain the right fields; and (2), the Lorentz transformations predict that an observer with respect to whom two masses are moving will observe a 'magnetic' velocity dependent gravitational force. We now turn to the gravitational equivalent of the Liénard and Wiechert potentials, and present what we believe Eddington had in mind when he declared that the argument given by Van Flandern was fallacious. The only difference between the potential theory of gravitational and electrostatic fields is that the electrostatic potential may have either sign. This being the case, one may compute the Liénard and Wiechert potentials for a particle of mass m moving with velocity $\boldsymbol{v}$. A particularly simple and clear derivation in the case of electromagnetics is given by Reitz and Milford [4], and a clear discussion and concise exposition by Panofsky and Phillips [5]. The formulas obtained are, of course, identical to those obtained by applying a Lorentz transformation to the fields of a static charge. In the gravitational case, one obtains for the potentials

$$\phi = G\frac{m}{s}, \quad A = \frac{1}{c^2}\phi\boldsymbol{v}, \tag{1}$$

where

$$s = r_0\left[1 - \frac{v^2}{c^2}\sin^2\psi\right]^{1/2}, \tag{2}$$

and the geometry is shown in Fig. 2. From the figure, it is clear that

$$\sin\psi = \frac{\left(y_0^2 + z_0^2\right)^{1/2}}{r_0}, \tag{3}$$

so that s may be written as

$$s = \left[x_0^2 + Y_0^2 - \frac{v^2}{c^2}\left(y_0^2 + z_0^2\right)\right]^{1/2}. \tag{4}$$

We have retained c for the velocity of propagation but additional discussion, given below, is needed to assert this is actually the velocity of light.

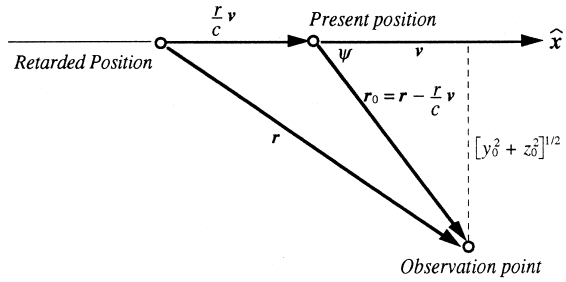

Fig. 2. Field coordinates and geometry for a particle of mass m moving in the x_0-direction with velocity $\boldsymbol{v}$.

The gravitational equivalent of the electric field is then given by

$$\boldsymbol{g} = -\nabla\phi - \frac{\partial \boldsymbol{A}}{\partial t} = -\nabla\phi - \frac{\partial \boldsymbol{A}}{\partial x_0}\boldsymbol{v}. \tag{5}$$

Since the motion is in the x_0-direction, the gradient becomes

$$\frac{\mathrm{d}}{\mathrm{d}x_0} = \frac{\mathrm{d}s}{\mathrm{d}x_0 s}\frac{\mathrm{d}}{\mathrm{d}s} = \frac{x_0}{s}\frac{\mathrm{d}}{\mathrm{d}s}.$$

Straightforward substitution gives

$$\boldsymbol{g} = Gm\frac{\boldsymbol{r}_0}{r_0^3}\frac{1 - \frac{v^2}{c^2}}{\left(1 - \frac{v^2}{c^2}\sin^2\psi\right)^{3/2}}. \tag{6}$$

The key point, particularly with regard to Van Flandern's argument, is that *the the field is directed along $\boldsymbol{r}_0$, the line joining the observation point to the present position, not the retarded position*. We believe this result is what motivated Eddington to claim the argument presented by Van Flandern is fallacious.

There is also a gravitational equivalent of the magnetic field called the 'gravitomagnetic' field in general relativity. And it is the fact that one obtains similar expressions in the weak field 3 + 1 formulation of general relativity [6] that allows one to identify the velocity of propagation, given above as c, with the velocity of light.

References

[1] T. Van Flandern, Phys. Lett. A 250 (1998) 1.

[2] A. Eddington, Space, Time and Gravitation, Cambridge University Press, Cambridge, 1966; Van Flandern's quote is on p. 94, and Note 6 on p. 204.

[3] See S.A. Teukolsky, Am. J. Phys. 64 (1996) 1104, and the references cited therein. The actual experiments used a charged parallel plate capacitor.

[4] J.R. Reitz, F.J. Milford, Foundations of Electromagnetic Theory, Addison-Wesley, Reading, MA, 1960, p. 354.

[5] W.K.H. Panofsky, M. Phillips, Classical Electricity and Magnetism Addison-Wesley, Reading, MA, 1956, p. 286.

[6] V.B. Braginsky, C.M. Caves, K.S. Thorne, Phys. Rev. D 15 (1977) 2047, Section 3B; I. Ciufolini, J.A. Wheeler, Gravitation and Inertia, Princeton University Press, Princeton, NJ, 1995, Ch. 6; K.S. Thorne, R.H. Price, D.A. Macdonald, Black Holes: The Membrane Paradigm, Yale University Press, New Haven 1986, Section 3. The discussion of Salisbury and Menzel [Nature 252 (1974) 664] and the response by Lorrain [Nature 257 (1975) 161] may also be of interest.