

# The Quantum SI: A Possible New International System of Units

Peter J. Mohr\*

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## Abstract

The International System of Units is a widely used system of measurement standards that provide the basis for expressing physical quantities, such as the kilogram for mass. This paper will describe a proposed modernization of some of the unit definitions that would provide a system that is more stable over time and more suitable for expressing the values of many fundamental constants.

## 1. INTRODUCTION

It is generally recognized that it is in the interests of science, technology, and commerce to have a universally agreed to set of measurement standards. In fact, efforts in unification and standardization of measurements date back thousands of years.

\* National Institute of Standards and Technology, Gaithersburg, MD 20899-8420, USA

Currently, the International System of Units (SI) provides standards that are officially used by many countries.

As science and technology have advanced, standards have also advanced. A recent change in the SI was made in 1983, when the definition of the meter was revised to be the distance that light travels in a specified time interval. Also, for some time it has been recognized that, after more than 100 years of use, the definition of the kilogram to be the mass of the prototype located near Paris at the International Bureau of Weights and Measures (BIPM), is no longer satisfactory. Moreover, the definition of the ampere has not been used as a practical current standard since 1990. To deal with these shortcomings, redefinitions of the kilogram and ampere are being considered by the metrology community; redefinitions of the kelvin for temperature and the mole for amount of substance are also being discussed [1].

In this paper, the relevant issues and the proposed redefinitions are reviewed. One of the consequences of these possible redefinitions is that values of many of the fundamental constants, when expressed in the new units, would be exact and many others would have reduced uncertainties. Another possibly surprising result is that the new definitions could be made in such a way that the distinction between base units and derived units, presently specified in the SI, would become unnecessary and could be eliminated. This might be called unit democracy, in which all SI units have equal status. In principle, this scheme could be extended to include an analogous redefinition of the second, but this would have to await improved accuracy in the relevant experiments and theory. Constraints on timing of the proposed redefinitions, which could take effect as early as 2011, will be described.

## 2. INTERNATIONAL SYSTEM OF UNITS

The International System of Units is presently defined in terms of seven base units, various coherent derived units, and units that are defined to be in other categories.

The seven base units and symbols are:

- **meter** m (length),
- **kilogram** kg (mass),
- **second** s (time),
- **ampere** A (electric current),
- **kelvin** K (thermodynamic temperature),
- **mole** mol (amount of substance),
- **candela** cd (luminous intensity).

A few of the many SI derived units and symbols are:

- **hertz** Hz (frequency),
- **newton** N (force),
- **joule** J (energy),
- **coulomb** C (electric charge),
- **volt** V (electric potential difference).

The derived units are coherent with the base units and each other, which means that relations between equivalent ways of expressing units have unit coefficients. As an example  $1 \text{ J} = 1 \text{ kg m}^2 \text{ s}^{-2}$ .

Non-SI units and symbols are also recognized in the SI. Two examples are:

- **electron volt** eV (energy),
- **unified atomic mass unit** u (mass).

In general, the non-SI units are not coherent with the SI units; for example,  $1 \text{ eV} \approx 1.6 \times 10^{-19} \text{ J}$ , where the coefficient is not unity, and in this case depends on measured quantities.

The SI system of units provides the international basis for expressing physical quantities that is officially recognized by the 51 states that are members of the Convention of the Meter.

### 3. THE KILOGRAM AND THE AMPERE

In the SI system, the unit of mass is the kilogram, which is defined to be the mass of the prototype kilogram kept at the International Bureau of Weights and Measures (BIPM) near Paris. This definition was adopted by the 3rd General Conference on Weights and Measures (CGPM) in 1901, with some minor subsequent modifications. [Figure 3.1](#) shows a replica of the international prototype of the kilogram. Although it is kept under glass covers, it is not in vacuum. This way of defining the kilogram has a number of limitations:

- The prototype definition is not linked to an unchanging property of nature.
- The mass of the international prototype appears to be changing relative to the mass of its copies.
- The drift of the kilogram prototype together with its copies (relative to an unchanging standard) could be as large as  $20 \times 10^{-9} \text{ kg per year}$  [2,3].
- The prototype and its copies appear to gain mass over time and lose mass when washed for use in comparisons.
- The kilogram mass definition cannot be realized independently of the international prototype.

It is also of interest to consider the definition of the ampere adopted by the 9th CGPM in 1948: *The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed one meter apart in vacuum, would produce between these conductors a force equal to  $2 \times 10^{-7}$  newton per meter of length.* For the precision that is possible with present day technology, this definition is not useful, because it does not provide a measurement standard with comparable precision. In general, it has not been used since 1990, when a conventional electrical system of units was adopted. This modified system is based on the fact that modern voltage measurements are based on the Josephson effect and modern resistance measurements are based on the quantum Hall effect. In this case, voltage and resistance measurements are made in terms of the Josephson constant  $K_J$  and von Klitzing constant  $R_K$ , respectively. The theoretical



**FIGURE 3.1** A replica of the international prototype of the kilogram.

relations,

$$K_J = \frac{2e}{h}, \quad (1)$$

$$R_K = \frac{h}{e^2}, \quad (2)$$

which are often assumed to be exact, relate these constants to the elementary charge  $e$  and the Planck constant  $h$ . However, the values of  $e$  and  $h$  in SI units are of a limited accuracy that is well below the measurement precision that can be achieved. Therefore, in the conventional electrical system, the SI values are replaced by the exact values

$$K_{J-90} = 483\,597.9 \text{ GHz/V}, \quad (3)$$

$$R_{K-90} = 25\,812.807 \, \Omega, \quad (4)$$

which provide an arbitrary, but precise definition of voltage and resistance.

#### 4. POSSIBLE REDEFINITIONS OF THE KILOGRAM

The limitations on the stability of the kilogram defined in terms of the international prototype could be eliminated if the kilogram were defined in terms of a

fundamental constant in analogy with the definition of the meter. This definition of the meter, adopted by the 17th CGPM in 1983, is: *The meter is the length of the path traveled by light in vacuum during a time interval of  $1/299\,792\,458$  of a second.* As a consequence, the velocity of light  $c$  is given by:

$$c = \frac{1 \text{ m}}{1/299\,792\,458 \text{ s}} = 299\,792\,458 \text{ m/s.} \quad (5)$$

An alternative statement of the definition of the meter could be: *The meter is the unit of length scaled such that the velocity of light is  $299\,792\,458 \text{ m/s}$ .*

An analogous definition of the kilogram, based on a certain value of a fundamental constant, can be made by specifying the value of the Planck constant. Such a definition could be used to determine the mass of an object by means of the watt-balance experiment [3–5] which is sketched as follows.

In a recent version of the experiment [5], a horizontal circular coil is suspended in a radial magnetic field produced by superconducting magnets. In the first phase of the experiment, the current in the coil is adjusted so that the net magnetic forces on the coil equal the force of gravity on a 1 kilogram mass. In the second phase of the experiment, the coil is slowly moved through the magnetic field and the induced voltage on the coil and the velocity of the coil are measured to calibrate the magnetic field and geometry factors. The electrical measurements are done in terms of the Josephson and von Klitzing constants, which results in a determination of the combination

$$K_J^2 R_K = \frac{4}{h}, \quad (6)$$

assuming the relations in Eqs. (1) and (2) are exact. The present-day interpretation of this experiment is that the precise kilogram mass together with the watt-balance experiment determine the value of the Planck constant  $h$ .

On the other hand, if the value of the Planck constant were specified in advance, then that value, together with the watt-balance experiment, would determine the mass used in the first phase of the experiment in kilograms. This means that instead of using the example of an object that has the mass of one kilogram in order to define the unit, one could specify a value of the Planck constant and use the watt-balance experiment to determine the mass of a given object in the resulting kilogram units. Of course, the value for the Planck constant would be selected so that the new kilogram unit would be as close as possible to the current artifact based kilogram. In this way, a possible new definition of the kilogram might be: *The kilogram is the unit of mass scaled such that the Planck constant is exactly  $6.626\,069\,3 \times 10^{-34} \text{ J s}$ .*

An alternative definition of the kilogram could be made in terms of an assigned value of the Avogadro constant. In this case the relevant experiment could be the Avogadro Project [6], which is an international effort to measure the Avogadro constant by determining the number of atoms in a kilogram of silicon, along the lines of Ref. [7].

In this experiment, the volume and lattice spacing of a crystalline silicon sphere with a precisely known mass is measured to determine the number of atoms it

contains. This result, together with the relative atomic mass of silicon isotopes and the isotopic composition of the sphere, determines a value of the Avogadro constant.

On the other hand, if a value of the Avogadro constant were specified in advance, then that value, together with the Avogadro Project experiment, would determine the mass of the sphere in kilograms. The corresponding definition of the kilogram would be: *The kilogram is the unit of mass scaled such that the Avogadro constant is exactly  $6.022\,141\,5 \times 10^{23} \text{ mol}^{-1}$ .*

## 5. RELATION BETWEEN THE AVOGADRO CONSTANT AND THE PLANCK CONSTANT

Although the experiments to measure the Planck constant and the Avogadro constant are completely independent, the values of the two constants are closely related theoretically. An expression for the mass of the electron in kilograms can be obtained from the definition of the Rydberg constant  $R_\infty$ :

$$R_\infty = \frac{\alpha^2 m_e c}{2h} \quad \Rightarrow \quad \frac{1}{m_e} = \frac{\alpha^2 c}{2h R_\infty}, \quad (7)$$

where  $\alpha$  is the fine-structure constant and  $m_e$  is the mass of the electron, which taken together with the expression for the electron mass in terms of the unified atomic mass unit  $u$ :

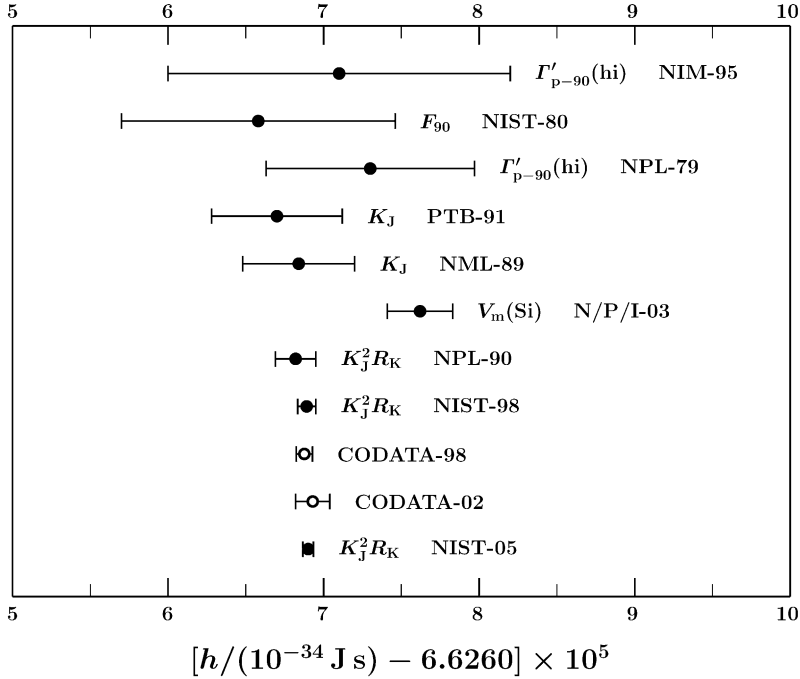
$$m_e = A_r(e)u \quad \Rightarrow \quad \frac{1}{u} = \frac{A_r(e)}{m_e}, \quad (8)$$

yields a relation between the Avogadro constant and the Planck constant given by

$$N_A = \frac{10^{-3} \text{ kg/mol}}{1 u} = A_r(e) \left( \frac{\alpha^2 c}{2h R_\infty} \right) \times 10^{-3} \text{ kg/mol}. \quad (9)$$

This relation contains other constants, but the uncertainties of those constants are much smaller than the uncertainty of either  $N_A$  or  $h$  [8]. The result of this relationship is that in principle, either of the constants could be specified to define the kilogram, and either experiment could be used to realize the definition.

At present, the relation can be used to calculate a value of the Planck constant determined by the Avogadro Project experiment to be compared to the value obtained from watt-balance experiments. Figure 3.2 shows values of the Planck constant determined by various experiments, including watt-balance experiments, labeled  $K_J^2 R_K$ , and the Avogadro Project experiment, labeled  $V_m(\text{Si})$  (see Ref. [8] for an explanation of the other entries). As is evident from the figure, there is disagreement between the two methods of determining the Planck constant. Presumably, this disagreement will be resolved by further work on the experiments.



**FIGURE 3.2** Values of the Planck constant determined by various experiments.

## 6. OTHER REDEFINITIONS

Redefinitions in terms of fundamental constants are also being considered for the ampere, kelvin, and mole [1].

Such a definition of the ampere could be: *The ampere, unit of electric current, is such that the elementary charge is  $1.602\,176\,53 \times 10^{-19}$  coulomb.* This definition determines the ampere, because in principle at least, one could count the number of electrons passing through a surface, and since the amount of charge carried by each electron would be known, the current would also be known. One of the consequences of this definition is that the electric constant  $\epsilon_0$  and the magnetic constant  $\mu_0$  would no longer be exact quantities, but would be defined by experiment through the expressions

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \quad (10)$$

and  $\epsilon_0\mu_0 = c^{-2}$ , for example. Thus, the value of  $\epsilon_0$  would follow directly from the value of  $\alpha$ , since  $e$ ,  $\hbar$ , and  $c$  would have exact values. Another consequence would be that the Josephson constant and the von Klitzing constant, as given in Eqs. (1) and (2) would be exact quantities. This would be valuable for electrical metrology, because voltage and resistance are measured in terms of these constants, and they would have exact values in the new SI. As a result, precise electrical measure-

ments could be made in terms of SI units rather than the arbitrary units defined by Eqs. (3) and (4), as they presently are.

The kelvin temperature unit could be defined in terms of an exact value of the Boltzmann constant  $k$  by the statement: *The kelvin, the unit of thermodynamic temperature, is scaled such that the Boltzmann constant is exactly  $1.380\,650\,5 \times 10^{-23}$  joule per kelvin.*

Similarly, the mole could be defined in terms of an exact value of the Avogadro constant  $N_A$  through the statement: *The mole, the unit of amount of substance, is scaled such that the Avogadro constant is exactly  $6.022\,141\,5 \times 10^{23}$  per mole.*

## 7. CONSEQUENCES FOR OTHER FUNDAMENTAL CONSTANTS

If the SI units were redefined as described above, other fundamental constants, which are functions of the constants used in the new definitions, would also be exact. A partial list is:

- Faraday constant  $\mathcal{F}$
- molar gas constant  $R$
- Stefan–Boltzmann constant  $\sigma$
- eV–joule conversion factor
- Hz–joule conversion factor
- kelvin–joule conversion factor

The effect of these new SI definitions would be to decrease the uncertainties of many other fundamental constants not used in the new definitions. For example, the relative uncertainty of the mass of the electron in kilograms  $m_e$  would change from  $2 \times 10^{-7}$  to  $2 \times 10^{-9}$ . Of course, the mass of the current kilogram artifact mass would no longer be exactly one kilogram, but rather would be determined by experiment, with its uncertainty being of the order of a few parts in  $10^8$ .

## 8. THE QUANTUM SI

The SI system defined in terms of fundamental constants as discussed above is more closely related to atomic scale phenomena than to macroscopic scale standards, as is presently the case. Hence, the name Quantum SI might be appropriate for the new system. It is quantum in the sense that it uses the Planck constant, the quantum of action and angular momentum; the elementary charge, which is quantized; the Boltzmann constant, which appears in the Planck radiation formula; and the mole directly defined as a number of entities, rather than in terms of mass, which emphasizes the role of atoms and molecules. The present day definitions of the kilogram, ampere, and kelvin, and mole are independent of quantum phenomena, since they are based on concepts that predate such knowledge.

In the Quantum SI, the fundamental constants,  $c$ ,  $h$ ,  $e$ ,  $k$ , and  $N_A$ , and at least for the moment, the hyperfine frequency of cesium  $\nu_{\text{Cs}}$ , would have exact values. The SI redefinitions have been described in terms of the base units, such as



the kilogram and ampere to make contact with the present-day system. However, a moment's reflection reveals that associating particular constants with particular units is unnecessary. For example, specifying that the hyperfine frequency of cesium has a particular value and that the speed of light (in meters/second) has a particular value assures that the meter will be a particular length without stating that specification of the speed of light defines the meter. Similarly, the specification of the elementary charge determines the ampere without specifically stating that fact. It is sufficient to define the Coulomb by relating it to the elementary charge, and from that current, which is charge divided by time, follows from the fact that the SI units are coherent. In fact, charge is a derived unit that is defined by specifying the value of the electron charge in coulombs. An explicit association with current is not necessary.

The conclusion that can be drawn from these considerations is that the separation of units into base units and derived units is not necessary in the Quantum SI. This would eliminate the arbitrary selection of base units.

## 9. THE SECOND

Evidently, the present definition of the second is out of character with the rest of the definitions based on constants which could be called universal (with the possible exception of the Avogadro constant). In contrast, the hyperfine frequency of cesium is a property of a specific atom.

A definition of the second that is consistent with those made in terms of universal constants could, in principle at least, be based on specifying a value for the Rydberg constant, which is also a universal constant. If the value of the Rydberg constant in inverse meters were specified, then since the speed of light  $c$  is exact, the corresponding frequency would be fixed. From this, the observed frequencies of particular atomic transitions could be known and used as time standards.

The drawback, at the moment, is that the theory needed to relate the Rydberg constant to observable frequencies is not sufficiently accurate to make this modification without introducing an excessively large uncertainty. This could change in the future, but for now, such a redefinition would not be practical.

## 10. TIME SCALE FOR REDEFINITIONS

At its meeting in October 2004, the International Committee on Weights and Measures (CIPM) asked the Consultative Committee on Units (CCU) to study the possibility of a fundamental constant-based definition of the kilogram. In June, 2005, the CCU requested that the CIPM approve preparation of possible new definitions of the kilogram, ampere, and kelvin in terms of fundamental constants, and also consider redefining the mole at the same time. At its meeting in October, 2005, the CIPM approved, in principle, preparation of the new definitions, as requested by the CCU, for possible adoption by the General Conference on Weights and Measures (CGPM) in 2011, provided the results of experiments over the next few years make this acceptable.

## 11. CONCLUSION

The SI can be improved by modernizing the way units are defined. In particular, the definitions of the kilogram, ampere, kelvin and mole are based on 19th century science and technology and can be replaced by ones that take into account subsequent progress in physics.

If an update of the SI were done by specifying values of fundamental constants, the concepts of base units and derived units would not be necessary.

If there are no persistent problems with experiments, the changes could be made in 2011.

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