Against tachyonic neutrinos

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We argue that there is strong circumstantial evidence against the proposal that at least one neutrino is a tachyon.

In 1985 Chodos, Hauser and Kostelecky observed [1] that experimental results on the muon neutrino mass appeared to prefer negative values of the mass squared. Taking such results at face value, they raised the question of whether the muon neutrino might indeed be a tachyon (faster-than-light particle). While this suggestion is counter-intuitive to most physicists, none the less, in this paper we pursue the kinematical and dynamical implications of this idea a little further and find that it is fraught with unphysical consequences. The possible existence of faster-than-light particles was first raised in pre-relativity times by Sommerfeld [2], and the kinematical aspects were subsequently elaborated upon in a relativistic context by Bilaniuk, Deshpande, and Sudarshan [3]. The properties of tachyonic representations of the Poincaré group were studied by Wigner [4], and Feinberg [5] investigated the possibility of tachyonic quantum field theory (see ref. [6] for a review).

In special relativistic kinematics the energy, p_0, and three-momentum, $\boldsymbol{p}$, of a particle are given in terms of its speed, v, and rest mass, m, by

$$p_0=\frac{mc^2}{(1-v^2/c^2)^{1/2}}, \tag{1}$$

and

$$|\boldsymbol{p}|=\frac{mv}{(1-v^2/c^2)^{1/2}}. \tag{2}$$

Particle speeds $v>c$ are then only consistent with real values for the four-momentum observables if the rest mass is imaginary,

$$m^2<0. \tag{3}$$

The speed of light is a lower bound for the particle speed, v, and corresponds to infinite four-momentum. Conversely, in the limit $v\to\infty$ we find,

$$p_0\to 0, \tag{4}$$

and

$$|\boldsymbol{p}|\to|m|c, \tag{5}$$

the minimum value of three-momentum.

An important point is that, under the action of a Lorentz transformation which is continuously connected to the identity, a positive energy tachyon may be transformed into a negative energy one, since p^μ is space-like (we put $c=1$, henceforth),

$$p_\mu p^\mu=m^2<0. \tag{6}$$

This is quite different from the $m^2>0$ case for which the sign of the time component for p^μ is a Lorentz invariant. One is of course free to re-interpret a negative energy tachyon as a positive energy anti particle travelling along the reversed world-line [3], but this re-interpretation is frame-dependent, unlike the case of negative energies with $m^2>0$ [7]. Therefore, when considering the kinematics of a decay with a tachyon in the final state, one cannot forbid negative tachyon energies, because to do so would violate Lorentz invariance. The possibility of negative tachyon ener-

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gies has a significant effect on a decay process.

We start with the decay of a particle of rest mass M into two particles of rest masses m_1, m_2. In the rest frame of M the magnitude of the three-momentum, p, of either particle 1 or 2 is

$$p^2=\frac{(M^2-m_1^2-m_2^2)^2-4m_1^2m_2^2}{4M^2}, \tag{7}$$

and the energies of the decay products are

$$E_1=\frac{M^2+m_1^2-m_2^2}{2M}, \tag{8}$$

and

$$E_2=\frac{M^2+m_2^2-m_1^2}{2M}.$$

When 1 and 2 are both normal particles ($m_{1,2}^2>0$) there is a threshold for the decay at

$$M=m_1+m_2. \tag{9}$$

For the case when 1 is a normal particle and 2 is a tachyon ($m_2^2<0$) there is the mass shell requirement

$$p^2\leqslant -m_2^2, \tag{10}$$

which is satisfied for any $M^2>0$. However, for $M^2<m_1^2-m_2^2$ we have $E_2<0$. Therefore, if we accept the interpretation of ref. [1] for the decay [8] $\pi^+\to\mu^+\nu_\mu$ in which $M_{\nu_\mu}^2<0$, we would also have to accept that kinematically the "decay" $\mu^-\to\pi^-\nu_\mu$ is also allowed.

The N-body Lorentz invariant phase space factor

$$R_N(P_i^\mu P_{i\mu})=\prod_{j=1}^{N}\int d^4p_j\,\delta(p_j^2-m_j^2)$$
$$\times\delta^{(4)}\left(\sum_{j=1}^{N}p_j^\mu-P_j^\mu\right), \tag{11}$$

where p_j^μ is the initial state four-momentum, will be different when tachyons are allowed. This is because, for normal particles, the integrals in eq. (11) are restricted by $p_j^0\geqslant 0$, whereas for tachyons the analogous restriction would be $|\boldsymbol{p}_j|^2\geqslant -m_j^2$. For the two-body case, we have

$$R_2(M^2)=\frac{\pi[(M^2-m_1^2-m_2^2)^2-4m_1^2m_2^2]^{1/2}}{2M^2}, \tag{12}$$

for $m_{1,2}^2>0$, with the threshold condition $M\geqslant m_1+m_2$, whereas when we allow $m_2^2<0$, we find instead

$$R_2(M^2)=\frac{\pi[(M^2-m_1^2-m_2^2)^2-4m_1^2m_2^2]^{1/2}}{M^2+m_1^2-m_2^2}, \tag{13}$$

with $0<M^2<\infty$. Eq. (12) and (13) show that the phase space factors with tachyons are *not* obtained by the replacement $m^2\to -m^2$ in the phase space for normal particles.

This result, that there is no threshold for the invariant mass of a tachyon and a normal particle, has interesting consequences for three-body decays too. For instance, consider the decay of a particle of rest mass M into three bodies of rest masses m_1, m_2 and m_3. When all three final state particles are normal, the three-body phase space factor is

$$\frac{dR_3}{dp_3}=\pi^2p_3^2\frac{[(S_2-m_1^2-m_2^2)^2-4m_1^2m_2^2]^{1/2}}{E_3S_2}, \tag{14}$$

where

$$p_3=|\boldsymbol{p}_3|,\quad E_3=\sqrt{p_3^2+m_3^2}, \tag{15}$$

and

$$S_2=M^2+m_3^2-2E_3M=(p_1^\mu+p_2^\mu)^2, \tag{16}$$

in the M rest frame. In the tritium beta-decay experiments [9] for the ν_e mass, we have

$$m_1=M-\Delta M,\quad m_2=m_\nu,\quad m_3=m_e, \tag{17}$$

and we take the limit where the tritium mass goes to infinity, $M\to\infty$, giving

$$\frac{dR_3}{dp_e}\simeq\pi\frac{p_e^2}{E_e}\frac{2}{M}[(\Delta M-E_e)^2-m_\nu^2]^{1/2}, \tag{18}$$

which clearly shows the end point in the electron spectrum at

$$E_e=\Delta M-m_\nu, \tag{19}$$

from which m_ν is determined. [The M^{-1} in eq. (18) is removed by the wavefunction normalization in the matrix element.] However, when $m_2^2<0$ (tachyonic neutrino) the phase-space factor is quite different:

$$\frac{dR_3}{dp_3}=\sqrt{2}\frac{\pi^2p_3^2}{E_3}\left(\frac{(S_2-m_1^2-m_2^2)^2-4m_1^2m_2^2}{(S_2-m_2^2)^2+m_1^4-2m_1^2m_2^2}\right)^{1/2}. \tag{20}$$

There is clearly no end point in energy for particle 3 in eq. (20) because the numerator is positive definite, and there is a large amount of phase space for electrons of arbitrarily large energies. For instance, with the definitions of eq. (17) and the limit $M \to \infty$ for tritium beta decay, we find

$$\lim_{E_e \to \infty} \frac{dR_3}{dp_3} \simeq \sqrt{2}\,\frac{\pi^2 p_e^2}{E_e} . \tag{21}$$

So, kinematical arguments give results for the electron energies in tritium beta decay which are in obvious contradiction with experience.

Yet another kinematical phenomenon in the three-body decay can be seen from eq. (16), where for normal particles we have

$$(p_1^\mu + p_2^\mu)^2 \geqslant (m_1 + m_2)^2 , \tag{22}$$

and, hence,

$$M \geqslant m_1 + m_2 + m_3 . \tag{23}$$

However, if particle 2 is tachyonic, no such restriction can be made. Indeed, $(p_1^\mu + p_2^\mu)$ can even be space-like, so that if the ν_e in the decay

$$\mu^+ \to e^+ \nu_e \bar{\nu}_\mu , \tag{24}$$

is a tachyon, then the "decay"

$$e^- \to \bar{\mu} \bar{\nu}_\mu \nu_e \tag{25}$$

would be kinematically allowed, again in clear contradiction with experience.

So far, we have focused on tachyon kinematics, but the difficulties which we have found could disappear if they were forbidden for some dynamical reason. One should therefore consider the possible properties of the matrix elements involving tachyonic neutrinos. To do so, we would need to study the wave equation appropriate for a free tachyon, and we turn to this next.

The authors of ref. [1] have suggested that

$$(i\gamma_5 \partial\!\!\!/ - m)\psi = 0 \tag{26}$$

is an appropriate wave equation for a tachyonic neutrino (m real). This becomes clear when we introduce the new γ matrices

$$\bar{\gamma}_\mu \equiv i\gamma_5 \gamma_\mu , \tag{27}$$

with

$$\gamma_5 = i\gamma_0 \gamma_1 \gamma_2 \gamma_3 , \tag{28}$$

which satisfy the same Clifford algebra

$$\{\bar{\gamma}_\mu, \bar{\gamma}_\mu\} = 2\eta_{\mu\nu} , \tag{29}$$

have the same hermiticity properties,

$$\bar{\gamma}_\mu^\dagger = \bar{\gamma}_0 \bar{\gamma}_\mu \bar{\gamma}_0 , \tag{30}$$

and γ_5,

$$\bar{\gamma}_5 = \gamma_5, \quad \bar{\gamma}_5^\dagger = \bar{\gamma}_5 , \quad \bar{\gamma}_5^2 = 1 , \tag{31}$$

as the γ_μ. In terms of the new representation the wave equation (26) may be re-written as

$$(i\bar{\partial\!\!\!/} - im)\psi = 0 , \tag{32}$$

where now,

$$\bar{\partial\!\!\!/} \equiv \bar{\gamma}_\mu \partial^\mu , \tag{33}$$

and so for real m, the rest-mass of ψ is pure-imaginary (tachyonic). At first sight there appears to be a hermiticity problem with eq. (32), but this is compensated for formally in the conserved Dirac scalar product

$$\langle \psi_1 | \psi_2 \rangle \equiv \int d^3x\, \psi_1^\dagger \gamma_5 \psi_2 \tag{34}$$

for two solutions ψ_1, ψ_2 of eq. (32).

We turn now to the properties of such solutions under the Poincaré group. The Poincaré group has two Casimir invariants,

$$P^2 = P_\mu P^\mu , \tag{35}$$

and

$$W^2 = W_\mu W^\mu , \tag{36}$$

where the Pauli–Lubanski vector is

$$W_\mu = -\tfrac{1}{2}\epsilon_{\mu\nu\alpha\beta} J^{\nu\alpha} P^\beta . \tag{37}$$

Here, P_μ are the generators of translations,

$$P_\mu \equiv i\partial_\mu , \tag{38}$$

and the $J^{\nu\alpha}$ are the generators of rotations and Lorentz boosts,

$$J_{\mu\nu} = \tfrac{1}{2}\sigma_{\mu\nu} + i(x_\mu \partial_\nu - x_\nu \partial_\mu) , \tag{39}$$

with

$$\sigma_{\mu\nu} = \tfrac{1}{2} i [\gamma_\mu, \gamma_\nu] . \tag{40}$$

From the Dirac equation (20) or (32), it is easy to prove that

$$P^2=-m^2<0\,, \tag{41}$$

so that the representation is indeed tachyonic, and that, formally,

$$W^2=+\tfrac{3}{4}m^2\,, \tag{42}$$

which has the same magnitude, but opposite sign as for conventional Dirac particles. The value of W^2 is related to spin, so it would appear that eq. (26) could describe the quantum mechanics of a tachyonic, spin-$\frac{1}{2}$ particle.

However, we must also look at the properties of the representation of the little group. Consider the set of zero energy characteristics states, ψ_0, for which

$$P_\mu=(0,0,0,m)\,, \tag{43}$$

and

$$\Psi_0=e^{-imz}\chi_0\,. \tag{44}$$

The spinors χ_0 transform amongst themselves under the action of the little group, which has three generators, namely, rotations about the z-axis, generated by

$$L_3=\tfrac{1}{2}\sigma_{12}\,, \tag{45}$$

and boosts along the x-axis,

$$K_1=\tfrac{1}{2}\sigma_{01}\,, \tag{46}$$

and along the y-axis,

$$K_2=\tfrac{1}{2}\sigma_{02}\,. \tag{47}$$

These generators satisfy the non-compact SO(2, 1) algebra,

$$[L_3,K_1]=+iK_2\,,\quad [L_3,K_2]=-iK_1\,,$$
$$[K_1,K_2]=iL_3\,. \tag{48}$$

The problems with this representation become apparent when we consider the hermiticity properties of the generators:

$$L_3^\dagger=L_3\,,\quad K_{1,2}^\dagger=-K_{1,2}\,. \tag{49}$$

The representation is non-unitary which has catastrophic consequences for the probability interpretation of the theory. Indeed, the conserved current is,

$$J^\mu=\bar\psi\gamma^\mu\gamma_5\psi\,, \tag{50}$$

and for momentum eigenspinors

$$\psi_p=\exp(ip^\mu x_\mu)\,\chi_p\,, \tag{51}$$

the current becomes

$$J_p^\mu\equiv\bar\psi_p\gamma^\mu\gamma_5\psi_p=\frac{p^\mu}{m}\bar\psi_p\psi_p\,. \tag{52}$$

The characteristic states (43) and (44) are therefore zero-norm, since

$$J^0=0\,, \tag{53}$$

but even worse for these states is that

$$\bar\psi_p\psi_p\equiv 0\,, \tag{54}$$

which means that all four components of J^μ vanish. However, since any state can be reached by Lorentz transformation from the characteristic state, the result (54) means that all states are zero norm, and so there is no probability interpretation for this theory. This serious problem arises with the non-unitary representation of the little group, so we must study instead the properties of a unitary representation which has the appropriate value of the invariant W^2 to describe a spin-$\frac{1}{2}$ tachyon.

The algebra (48) may be rewritten as

$$[L_3,K_\pm]=\pm K_\pm\,,\quad [K_+,K_-]=2L_3\,, \tag{55}$$

where

$$K_\pm\equiv K_1\pm iK_2\,, \tag{56}$$

and we have

$$W^2=-m^2[\tfrac{1}{2}(K_+K_-+K_-K_+)-L_3^2]\,. \tag{57}$$

If we now introduce a set of normalized helicity eigenstates with

$$L_3|\lambda\rangle=|\lambda\rangle\,,\quad \langle\lambda|\lambda\rangle=1\,, \tag{58}$$

we find from eqs. (54) and (56)

$$|\langle\lambda|K_+|\lambda-1\rangle|=\left(\lambda(\lambda-1)-\frac{W^2}{m^2}\right)^{1/2}\,. \tag{59}$$

The eigenvalues λ must be either integers or half-odd integers, so if the Casimir invariant is chosen to be [4]

$$W^2=m^2S(S-1)\,, \tag{60}$$

with $S>0$ integer or half-odd integer, the values of λ are either $\lambda=s,\ s+1,\ s+2,\ \ldots$, or $\lambda=-s,\ -s-1,$

$-s-2$, ..., and the representations are infinite-dimensional [4,10]. For the tachyonic neutrino, we clearly would need the case $s=\frac{1}{2}$, which raises the question of how the higher helicity states of the neutrino would contribute to weak processes. For instance, in the classic experiment on parity violation [11], the final state electron is left-handed and the anti-neutrino right-handed with helicity $+\frac{1}{2}$, so that the electron asymmetry arises from angular momentum conservation as shown in fig. 1a. However, if the antineutrino were tachyonic, it could be present with a right-handed helicity of $+\frac{3}{2}$, so that the electron could still be left-handed but moving in the opposite direction from the first case (see fig. 1b). This would have spoiled the electron asymmetry, and unless there was some dynamical reason for this final state to be suppressed, we must regard the observed asymmetry as further evidence against the tachyonic neutrino hypothesis.

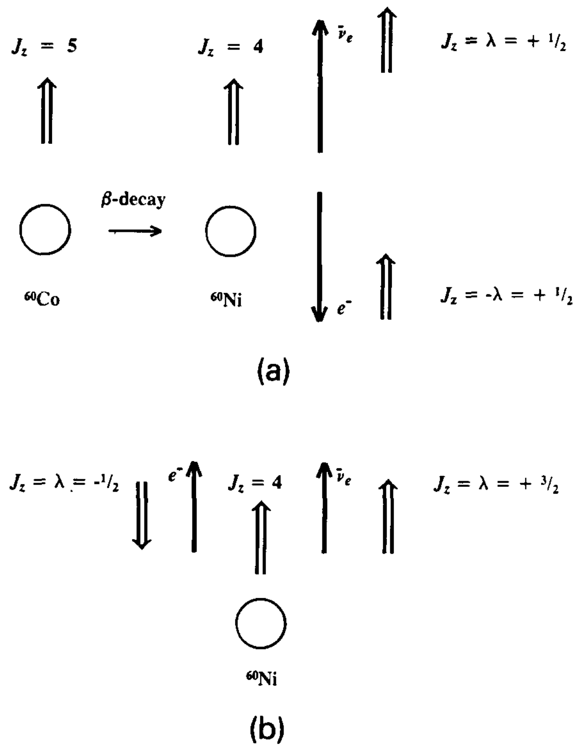

Fig. 1. (a) Conventional arrangement of spins and helicities in the experiment of Wu et al. [11]. (b) A second final state when the antineutrino is tachyonic.

Obviously there is very much more to constructing a tachyonic wave equation for a neutrino then the replacement $m \to im$ in the Dirac equation [10]. However, the requirements of Poincaré symmetry which show that tachyonic neutrinos would occur with all half-odd integer helicities is another prediction which appears to not be borne out in practice.

The problems of constructing weak currents out of an infinite-dimensional tachyonic neutrino wave function and a four-dimensional electron spinor are formidable. Furthermore, the second quantization of such a theory poses remarkable problems because of the absence of momentum components of the tachyon below the modulus of the tachyon mass. This lack of completeness of the tachyon plane waves is a well-known, serious difficulty [5].

In conclusion, we believe that there is strong circumstantial evidence of a kinematical and dynamical nature against the notion that the neutrino is a tachyon (a result which is certainly not surprising). The analyses of experimental results in terms of conventional Lorentz invariant phase space which appear to indicate negative mass squared for the neutrino would have to be re-done to take account of the unusual phase space for tachyons which we have described here. The wave equation (26) proposed in ref. [1] does not describe a tachyonic helicity one-half particle. Finally, we note that the best test of the neutrino speed V_ν comes from SN1987A [12]. The transit times of photons and light from the Large Magellanic Cloud to the earth are $\sim 1.6 \times 10^5$ years, whereas the relative uncertainties is these times are of the order of ± 3 hours [12], leading to the bound

$$\left|\frac{V_\nu - c}{c}\right| \lesssim 2 \times 10^{-9} . \qquad (61)$$

For a 15 MeV $\bar{\nu}_e$, this bound translates into

$$-1\,\mathrm{keV}^2 < m_\nu^2 < +1\,\mathrm{keV}^2 , \qquad (62)$$

so that this direct test does not rule out the tachyonic neutrino.

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