Distributed Fault Diagnosis for Input-Output Continuous-Time Nonlinear Systems

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Abstract: In this paper, new results on distributed fault diagnosis of continuous–time nonlinear systems with partial state measurements are proposed. Following an overlapping decomposition framework, the dynamics of a nonlinear uncertain large-scale dynamical systems is described as the interconnection of several subsystems. Each subsystem is monitored by its own Local Fault Diagnoser, based on a set of local estimators. A consensus-based protocol is used to improve the detectability and the isolability of faults affecting variables shared among different subsystems because of the overlapping decomposition. A sufficient condition assuring the convergence of the estimation errors is derived. Time-varying threshold functions guaranteeing no false-positive alarms and theoretical results containing detectability and isolability conditions are presented.

Keywords: Distributed Fault Detection and Isolation, Non-linear systems, Large-scale systems.

1. INTRODUCTION

The growing scientific interest for distributed systems and networks is testified by the wealthy amount of research, cited by surveys such as Baillieul and Antsaklis (2007) and Abdallah and Tanner (2007). The greater interest of the control community is focused on the design of distributed systems: the goal is to find distributed solutions to problems that are difficult or impossible to solve in a centralized framework, because of structural, efficiency, computational, and robustness issues. Practical engineering examples of large–scale and/or distributed systems are abundant; e.g., large–scale communication networks, water distribution systems or traffic networks, energy, pulp-and-paper, or steel-making plants, multi–vehicle formations, and so on. Decentralized control methods suited to these systems were proposed since at least the 1970s. Although many enhancements have been achieved in the design and analysis of decentralized and, later, distributed control and estimation schemes, the design of fault diagnosis schemes specifically for distributed and large–scale systems is still a challenging field. Research activity in this field led to effective distributed algorithms suited to discrete event systems (see, among many others, Baroni et al. (1999) and Preparata et al. (1967)). A notable contribution in the field of decentralized hybrid systems fault diagnosis is the work of Fagiolini et al. (2007). Furthermore, analysis of fault scenarios and effects in distributed systems were addressed in Teixeira et al. (2010) and in Jafari et al. (2010). However, as far as distributed discrete–time or continuous–time systems are concerned, qualitative fault diagnosis schemes were attempted only recently (see as example Lechevin and Rabbath (2009)), or quantitative methods that were formulated for linear systems only (like Stankovič et al. (2010)), with Zhang (2010) being one of the few contributions on decentralized fault detection for large–scale nonlinear systems. In this paper, we propose a fault detection and isolation scheme for a class of nonlinear uncertain continuous-time systems, where, furthermore, the system states are only partially measurable (in the recent paper Boem et al. (2011a), the full-state case has been dealt with). Though several papers dealing with centralized fault diagnosis schemes for input-output systems have appeared in the literature (Zhang et al. (1998, 2001); Subbarao and Vemuri (2007); Zhang and Jaimoukha (2009)), to the best of the authors knowledge this is the first contribution addressing distributed schemes for input-output continuous–time nonlinear systems. More specifically, the main contributions of this paper are the formulation of a distributed architecture specifically for input–output continuous-time systems, with a sufficient condition guaranteeing the convergence of the estimators, and the derivation of rigorous analytical conditions for detectability and isolability.

2. PROBLEM FORMULATION

Similar to Boem et al. (2011b), we consider a multi-input multi-output uncertain nonlinear system:

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$$\mathscr{S}:\begin{cases}\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{f}(\boldsymbol{x},\boldsymbol{u}) + \boldsymbol{\eta}_x(\boldsymbol{x},\boldsymbol{u},t) + \beta(t-T_0)\boldsymbol{\phi}(\boldsymbol{x},\boldsymbol{u})\\ \boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} + \boldsymbol{\eta}_y(\boldsymbol{x},\boldsymbol{u},t),\end{cases} \tag{1}$$

where $\boldsymbol{x} \in \mathbb{R}^n$, $\boldsymbol{u} \in \mathbb{R}^m$ and $\boldsymbol{y} \in \mathbb{R}^p$ denote the state, the control input and the measured output vectors respectively [2], the matrix $\boldsymbol{A} \in \mathbb{R}^{n\times n}$ and the vector field $\boldsymbol{f} : \mathbb{R}^n \times \mathbb{R}^m \mapsto \mathbb{R}^n$ describe the nominal healthy dynamics, $\boldsymbol{C} \in \mathbb{R}^{p\times n}$ is the nominal output matrix, $\boldsymbol{\eta}_x$ and $\boldsymbol{\eta}_y$ are the uncertainties in the state and in the output equations. The term $\beta(t-T_0)\boldsymbol{\phi}(\boldsymbol{x},\boldsymbol{u})$ represent the fault function dynamics: $\boldsymbol{\phi}(\boldsymbol{x},\boldsymbol{u})$ denotes the functional structure and $\beta(t-T_0)$ characterizes the time profile of the fault:

$$\beta(t-T_0) \triangleq \begin{cases}0 & \text{if} \quad t < T_0\\ 1-e^{-\alpha(t-T_0)} & \text{if} \quad t \geq T_0\end{cases}. \tag{2}$$

where T_0 is the unknown fault occurrence time and $\alpha > 0$ represents the unknown fault-evolution rate, modeling either *incipient* faults or *abrupt* faults, as $\alpha \to \infty$. The following assumptions are needed.

Assumption 1. The state variables $\boldsymbol{x}$ and control variables $\boldsymbol{u}$ are bounded before and after the occurrence of a fault: $\exists \mathcal{R}$, compact region of $\mathbb{R}^n \times \mathbb{R}^m : (\boldsymbol{x}(t),\boldsymbol{u}(t)) \in \mathcal{R}$, $\forall t \geq 0$.

Assumption 2. The fault–evolution rate parameter α is unknown, but lower bounded by a known constant $\bar{\alpha}$.

Assumption 3. The measuring uncertainty term $\boldsymbol{\eta}_y$ is an unstructured and unknown nonlinear function, bounded by a known function: $\left|\boldsymbol{\eta}_y^{(k)}(\boldsymbol{x},\boldsymbol{u},t)\right| \leq \bar{\boldsymbol{\eta}}_y^{(k)}(\boldsymbol{x},\boldsymbol{u},t)$, $\forall k = 1,\ldots,p$, $(\boldsymbol{x},\boldsymbol{u}) \in \mathcal{R}$ and $\forall t \geq 0$.

In this paper, the methodology presented in Boem et al. (2011b) for discrete–time systems is adapted to the continuous–time context, based on the decomposition of the monolithic system $\mathscr{S}$ into N subsystems $\mathscr{S}_I, I = 1,\ldots,N$, allowing the overlapping of certain states. After the decomposition, the I-th subsystem $\mathscr{S}_I$ dynamics can be described by:

$$\begin{cases}\dot{x}_I = A_I x_I + f_I(x_I,u_I) + g_I(C_I x_I, u_I, z_I)\\ \qquad + \beta(t-T_0)\phi_I(C_I x_I, z_I, u_I)\\ y_I = C_I x_I + \eta_{y,I}(x_I,u_I,t),\end{cases} \tag{3}$$

where $x_I \in \mathbb{R}^{n_I}$, $u_I \in \mathbb{R}^{m_I}$ and $y_I \in \mathbb{R}^{p_I}$ are the local state, the local control input, and the local measured output vectors respectively, and $z_I \in \mathbb{R}^{q_I}$ is the vector of the interconnection variables [3]. The matrix $A_I \in \mathbb{R}^{n_I \times n_I}$ and the vector field $f_I : \mathbb{R}^{n_I} \times \mathbb{R}^{m_I} \mapsto \mathbb{R}^{n_I}$ represent the local nominal healthy dynamics, $C_I \in \mathbb{R}^{p_I \times n_I}$ is the nominal local output matrix, $g_I : \mathbb{R}^{p_I} \times \mathbb{R}^{m_I} \times \mathbb{R}^{q_I} \mapsto \mathbb{R}^{n_I}$ is the interconnection function that incorporates the effects of the local state modeling uncertainty term $\eta_{x,I}$. The term $\eta_{y,I}$ is the local output uncertainty function that takes into account the measurement error, while $\phi_I : \mathbb{R}^{p_I} \times \mathbb{R}^{m_I} \times \mathbb{R}^{q_I} \mapsto \mathbb{R}^{n_I}$ is the local fault function.

Assumption 4. The decomposition of the monolithic system (1) is such that z_I is made of measurable variables.

This assumption is needed in order to allow the learning of the interconnection function and of the fault function.

[2] The use of boldface letters indicates that a given variable is related to the system (1).

[3] The interconnection variables z_I are the states of the neighboring subsystems nodes in the structure graph having a connection with the subsystem I

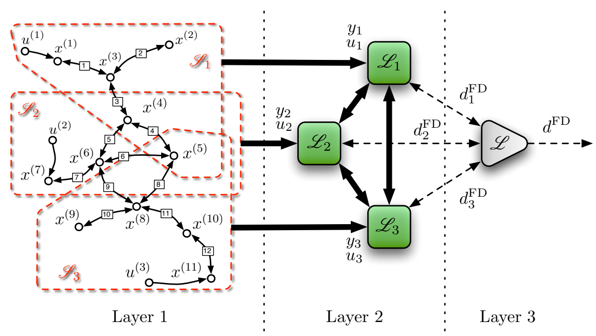

Fig. 1. A scheme of the proposed DFDI architecture.

Assumption 5. The structural graph and the decomposition are the same before and after the fault event.

Assumption 6. (A_I, C_I) is an observable pair, $\forall I$.

Assumption 7. The interconnection function g_I is unstructured, uncertain and nonlinear, but bounded by a known function, i.e., $\left|g_I^{(k)}(C_I x_I, z_I, u_I)\right| \leq \bar{g}_I^{(k)}(C_I x_I, z_I, u_I)$, for all $I = 1,\ldots,N$, $k = 1,...,n_I$ and for all $(\boldsymbol{x},\boldsymbol{u}) \in \mathcal{R}$.

3. DISTRIBUTED DETECTION ARCHITECTURE

In Ferrari et al. (2012), a *Distributed Fault Detection and Isolation* (DFDI) methodology is proposed, consisting of N agents called *Local Fault Diagnosers* (LFDs) $\mathscr{L}_I$, $I \in \{1\ldots N\}$. For detection purposes, each LFD is equipped with a non-linear adaptive estimator, called *Fault Detection Approximation Estimator* (FDAE), based on the nominal model, that computes the estimate of the local state x_I and of the local output y_I. The difference between the estimated output $\hat{y}_I$ and the measurements y_I is the output estimation error $\epsilon_{y,I} \triangleq y_I - \hat{y}_I$, which plays the role of a residual and will be compared, component by component, to a suitable detection threshold $\bar{\epsilon}_{y,I} \in \mathbb{R}^{p_I}$.

$$\left|\epsilon_{y,I}^{(k)}(t)\right| \leq \bar{\epsilon}_{y,I}^{(k)}(t),\ \forall k = 1,...,p_I \tag{4}$$

is a necessary (but generally not sufficient) condition for the fault-free hypothesis $\mathcal{H}_I$: "The system $\mathscr{S}_I$ is healthy". If the condition is violated at some time instant t, then the hypothesis $\mathcal{H}_I$ is falsified and we can say that a fault has occurred.

The local FDAE estimation, in the case of non-shared state variables, can be computed as:

$$\dot{\hat{x}}_I = A_I \hat{x}_I + f_I(\hat{x}_I,u_I) + \hat{g}_I(y_I,u_I,v_I,\hat{\vartheta}_I) + L_I(y_I - \hat{y}_I)$$
$$\hat{y}_I = C_I \hat{x}_I,$$

where $\hat{g}_I$ is the output of an adaptive approximator designed to learn the unknown interconnection function g_I and $\hat{\vartheta}_I \in \hat{\Theta}_I$ denotes its adjustable parameters vector. Due to the uncertain output measurements, each LFD receives from its neighbors the vector $v_I = z_I + \varsigma_I$, where ς_I is made with the components of $\eta_{y,J}$ that affect the relevant components of the neighboring subsystems measurements y_J. In the case of variables $x^{(s)}$ shared among more than one LFD, we take advantage of the redundancy obtained by means of the overlap. We propose a deterministic consensus protocol defined on a generic communication graph $\mathcal{G}_s \triangleq (\mathcal{O}_s, \mathcal{E}_s)$, whose nodes are the LFDs in the overlap set $\mathcal{O}_s$ of $x^{(s)}$ (see Ferrari et al. (2012)):

$$\dot{\hat{x}}_I^{(s_I)} = \sum_{J\in\mathcal{O}_s} W_s^{(I,J)} \Big[A_J^{(s_J)}\hat{x}_J + f_J^{(s_J)}(\hat{x}_J,u_J) + \hat{g}_J^{(s_J)}(y_J,u_J,v_J,\hat{\vartheta}_J) + L_J^{(s_J)}(y_J-\hat{y}_J)\Big] \quad (5)$$

where the terms $W_s^{(I,J)}$ are the components of a doubly stochastic weighted adjacency matrix, as for instance the Metropolis matrix (Xiao et al. (2007)). Other choices are possible, but in all cases the weights $W_s^{(I,J)}$ can be viewed as a measure of how much the I-th subsystem is confident about the information received from subsystem J. It is important to note that, in order to implement (5), the I-th LFD does not need the information about the expressions of $A_J^{(s_J)}$, $f_J^{(s_J)}$, $\hat{g}_J^{(s_J)}$ and of $L_J^{(s_J)}$: it is sufficient that each LFD computes locally the term $A_J^{(s_J)}\hat{x}_J + f_J^{(s_J)} + \hat{g}_J^{(s_J)} + L_J^{(s_J)}(y_J-\hat{y}_J)$ and communicates it to other LFDs according to the communication graph $\mathcal{G}_s$.

We now analyze the dynamics of the FDAE estimation error before the occurrence of a fault. In the non-shared case, the i-th state estimation error component is:

$$\dot{\epsilon}_{x,I}^{(i)} = A_{0,I}^{(i)}\epsilon_{x,I} + \Delta f_I^{(i)} + \Delta g_I^{(i)} - L_I^{(i)}\eta_{y,I},$$

where $A_{0,I} \triangleq A_I - L_I C_I$ is a stable matrix (thanks to Assumption 6), $\Delta f_I^{(i)} \triangleq f_I^{(i)}(x_I,u_I) - f_I^{(i)}(\hat{x}_I,u_I)$ and $\Delta g_I^{(i)} \triangleq g_I^{(i)}(C_I x_I,u_I,z_I) - \hat{g}_I^{(i)}(y_I,u_I,v_I,\hat{\vartheta}_I)$. We denote with $A^{(i)}$ the i-th row of the matrix A.

In the case of shared variables, the dynamics of the LFD state estimation error component can be written as:

$$\dot{\epsilon}_{x,I}^{(s_I)} = \dot{x}_I^{(s_I)} - \dot{\hat{x}}_I^{(s_I)} = A_I^{(s_I)}x_I + f_I^{(s_I)}(x_I,u_I) + g_I^{(s_I)}(C_I x_I,u_I,z_I) - \sum_{J\in\mathcal{O}_s} W_s^{(I,J)}\Big[A_J^{(s_J)}\hat{x}_J + f_J^{(s_J)}(\hat{x}_J,u_J) + \hat{g}_J^{(s_J)}(y_J,u_J,v_J,\hat{\vartheta}_J) + L_J^{(s_J)}(y_J-\hat{y}_J)\Big].$$

By assumption it holds $\sum_{J\in\mathcal{O}_s} W_s^{(I,J)} = 1$ and, thanks to the way the model decomposition was obtained, the state estimation error component can be rewritten as:

$$\dot{\epsilon}_{x,I}^{(s_I)} = \sum_{J\in\mathcal{O}_s} W_s^{(I,J)}\Big[A_{0,J}^{(s_J)}\epsilon_{x,J} + \Delta f_J^{(s_J)} + \Delta g_J^{(s_J)} - L_J^{(s_J)}\eta_{y,J}\Big].$$

We now introduce a general formulation of the state error equation for analysis purpose. To this end we define the extended state estimation error vector $\epsilon_{x,E} \in \mathbb{R}^{n_E\times 1}$, with $n_E = \sum_{J=1}^N n_J$, that is a column vector collecting the state estimation error vectors of the N sub-systems: $\epsilon_{x,E} \triangleq \mathrm{col}\,(\epsilon_{x,J} : J = 1,...,N)$. The dynamics of $\epsilon_{x,E}$ are:

$$\dot{\epsilon}_{x,E} = W\left[A_{0,E}\epsilon_{x,E} + \Delta f_E + \Delta g_E - L_E\eta_{yE}\right] \quad (6)$$

where W is a $N\times N$ block matrix

$$W \triangleq \begin{bmatrix} W_{1,1} & \dots & W_{1,N} \\ \dots & \dots & \dots \\ W_{N,1} & \dots & W_{N,N} \end{bmatrix},$$

such that each block $W_{I,J}$, with $J = 1,...,N$ and $I = 1,...,N$ collects the consensus weights of the subsystem I with regard to the subsystem J. The diagonal blocks $W_{I,I}$ are square diagonal matrices in $\mathbb{R}^{n_I\times n_I}$, whose s_I–th diagonal element, with $s_I = 1,...,n_I$, is equal to the weight $W_s^{(I,I)}$if $x_I^{(s_I)}$ is a shared variable, and is equal to 1 otherwise. The matrices $W_{I,J} \in \mathbb{R}^{n_I\times n_J}$, with $J \neq I$, have non-null elements only in positions (s_I, s_J) corresponding to shared variables x_s, and here they take the value of the consensus weight $W_s^{(I,J)}$. This results in W being a symmetrical, sparse and doubly–stochastic $n_E \times n_E$ matrix. $A_{0,E}$ is a $N\times N$ diagonal block matrix:

$$A_{0,E} \triangleq \begin{bmatrix} A_{0,1} & 0 & 0 & 0 \\ 0 & A_{0,2} & 0 & 0 \\ \ddots & \ddots & \ddots & \ddots \\ 0 & 0 & 0 & A_{0,N} \end{bmatrix},$$

where the generic block is $A_{0,J} = A_J - L_J C_J \in \mathbb{R}^{n_J\times n_J}$, for $J = 1,..,N$, resulting in $A_{0,E}$ being a sparse $n_E \times n_E$ matrix. $\Delta f_E(t)$ is a $n_E \times 1$ matrix, collecting the values $\Delta f_J^{(s_J)}(t)$, for each $s_J = 1,...,n_J$ and for every $J = 1,...,N$. $\Delta g_E(t)$ is defined in an analogous way as $\Delta f_E(t)$. Furthermore, $L_E \triangleq \mathrm{blkdiag}(L_J : J = 1,..,N)$ is a $N\times N$ diagonal block matrix with dimension $n_E \times p_E$, where $p_E \triangleq \sum_{J=1}^N p_J$, while $\eta_{y,E}(t)$ is a $p_E \times 1$ column vector collecting the uncertainty terms of the N subsystems: $\eta_{y,E} \triangleq \mathrm{col}\,(\eta_{y,I} : J = 1,...,N)$.

In order to guarantee the convergence of the state estimation error, the matrix $WA_{0,E}$ has to be a stable matrix. We derived a sufficient condition assuring that all the eigenvalues of the matrix are in the negative semi-plane.

Proposition 3.1. If $A_{0,E}$ is a diagonalizable matrix (which is not a restrictive assumption since we can choose L_I, $\forall I$ so that this assumption is guaranteed) and if W is made so that the elements on the diagonal $W_{i,i} > 0.5$, then $WA_{0,E}$ is a stable matrix.

Proof 3.1. Since $A_{0,E}$ is a diagonal block matrix where the single blocks $A_{0,I}$, $I = 1,\dots N$ are stable matrices, it is a stable matrix in turn. If it is a diagonal or a diagonalized matrix, the elements on the diagonal are the negative eigenvalues $-\lambda_i$. Using Gerschgorin circles on the columns, it is possible to prove that all the eigenvalues of $WA_{0,E}$ are trapped in the collection of circles centered at $-W_{i,i}\lambda_i$, with radii $\sum_{k\neq i} W_{k,i}\lambda_i = (1 - W_{i,i})\lambda_i$, where $W_{k,i}$ is the element of W corresponding to the k-th row and i-th column, with $k, i = 1,\dots,n_E$. The condition $-W_{i,i}\lambda_i+(1-W_{i,i})\lambda_i < 0$ assures that all the eigenvalues of $WA_{0,E}$ have negative real part. This condition is satisfied if $W_{i,i} > 0.5$ for all $i = 1,\dots,n_E$.

After the convergence of the state estimation error has been proved, the solution of the differential equation (6) can be written as:

$$\epsilon_{x,E}(t) = \int_0^t e^{WA_{0,E}(t-\tau)}\left[W\Delta f_E(\tau) + W\Delta g_E(\tau) - WL_E\eta_{y,E}(\tau)d\tau\right] + e^{WA_{0,E}(t)}\epsilon_{x,E}(0) \quad (7)$$

The extended output estimation error is then defined as:

$$\epsilon_{y,E} \triangleq C_E\epsilon_{x,E} + \eta_{y,E} \quad (8)$$

where $C_E \triangleq \mathrm{blkdiag}(C_J : J = 1,..,N)$ is a $N\times N$ diagonal block matrix, with dimension $p_E \times n_E$. From (6), (8) and the definition of C_E, the following learning law for the adjustable parameter vector $\hat{\vartheta}_I$ of the adaptive approximator $\hat{g}_I$, $I \in 1,\dots,N$ can be derived:

$$\dot{\hat{\vartheta}}_I = P_{\hat{\Theta}_I}\left[\Gamma_I H_I^\top W_{I,I}^\top C_I^\top \epsilon_{y,I}\right] \quad (9)$$

where $H_I^\top = \partial\hat{g}_I/\partial\hat{\vartheta}_I$ and $P_{\hat{\Theta}_I}$ is a projection operator restricting $\hat{\vartheta}_I$ within $\hat{\Theta}_I$ (Polycarpou (1998)), Γ_I is a symmetric and positive definite learning rate matrix (see for details Boem et al. (2011a)). In the general form, the component-wise output estimation error can be bounded by the following threshold, that can be computed in a distributed way:

$$\left|\epsilon_{y,E}^{(k)}(t)\right| \le \left|C_E^{(k)}\epsilon_{x,E}(t)\right| + \left|\eta_{y,E}^{(k)}(t)\right|$$
$$\le \left|C_E^{(k)}\right| \Big\{ \int_0^t \left\|e^{WA_{0,E}(t-\tau)}\right\| W\left[\bar{\Delta}f_E(\tau) + \bar{\Delta}g_E(\tau)\right.$$
$$\left. + |L_E|\,\bar{\eta}_{y,E}(\tau)d\tau\right] + \left\|e^{WA_{0,E}t}\right\|\bar{\epsilon}_{x,E}(0)\Big\} + \bar{\eta}_{y,E}^{(k)}(t) \quad (10)$$

where we denote with $|A|$ the element by element absolute value of the matrix A. Furthermore,

$$\bar{\Delta}f_E^{(s)}(t) = \max_{x^{(s)}\in R^{x^{(s)}}} \left\{\left|\Delta f_E^{(s)}(t)\right|\right\},$$
$$\bar{\epsilon}_{x,E}^{(s)}(0) = \max_{x^{(s)}\in R^{x^{(s)}}} \left\{\left|x^{(s)} - \hat{x}^{(s)}(0)\right|\right\},$$

for every $s = 1,...,n_E$. With regards to $\bar{\Delta}g_E$, some considerations are expressed in Boem et al. (2011b) and Ferrari et al. (2012): Δg_I can be upper bounded by $\bar{\Delta}g_I \triangleq \|H_I\|\,\kappa_I(\hat{\vartheta}_I) + \bar{\nu}_I + \max_{\eta_{yI}}\max_{\varsigma_I}|\Delta\hat{g}_I|$, where κ_I is such that $\kappa_I(\hat{\vartheta}_I) \ge \left\|\tilde{\vartheta}_I\right\|$. In fact, by defining the parameter estimation error $\tilde{\vartheta}_I \triangleq \hat{\vartheta}_I^* - \hat{\vartheta}_I$ and the function $\Delta\hat{g}_I \triangleq \hat{g}_I(C_I x_I, z_I, u_I, \hat{\vartheta}_I) - \hat{g}_I(y_I, v_I, u_I, \hat{\vartheta}_I)$, we can write $\Delta g_I = H_I\tilde{\vartheta}_I + \nu_I + \Delta\hat{g}_I$, where $\nu_I \triangleq g_I(C_I x_I, z_I, u_I) - \hat{g}_I(C_I x_I, z_I, u_I, \hat{\vartheta}_I^*)$ is the *Minimum Functional Approximation Error*, with

$$\hat{\vartheta}_I^* \triangleq \arg\min_{\hat{\vartheta}_I}\sup_{x_I,z_I,u_I}\left\|g_I(C_I x_I, z_I, u_I) - \hat{g}_I(C_I x_I, z_I, u_I, \hat{\vartheta}_I\right\|$$

is the optimal weight vector. The extended upper bound $\bar{\Delta}g_E$ simply collects the upper bounds of the N subsystems. The threshold in Eq. (10) guarantees that no false-positive alarms will be issued until T_0 because of the uncertainties. This, of course and in rough terms, comes at the cost of the impossibility of detecting faults "hidden by the uncertainties in the system dynamics".

3.1 Fault Detectability Analysis

Let us assume that at time $t = T_0$ a fault $\boldsymbol{\phi}$ occurs in the monolithic system. ϕ_E denotes the extended fault function vector collecting the N subsystems fault functions. After the occurrence of the fault, for $t > T_0$, the state estimation error dynamics becomes

$$\dot{\epsilon}_{x,E} = W\left[A_{0,E}\epsilon_{x,E} + \Delta f_E + \Delta g_E - L_E\eta_{y,E}(t)\right] + (1 - e^{-\alpha(t-T_0)})\phi_E$$

and the output estimation error equation for the k-th component is: $\epsilon_{y,E}^{(k)}(t) =$

$$C_E^{(k)}\Big\{\int_0^t e^{WA_{0,E}(t-\tau)}\Big[W(\Delta f_E(\tau) + \Delta g_E(\tau) - L_E\eta_{y,E}(\tau))$$
$$+ (1 - e^{-\alpha(\tau-T_0)})\phi_E(\tau)d\tau\Big] + e^{WA_{0,E}t}\epsilon_{x,E}(0)\Big\} + \eta_{y,E}^{(k)}(t)$$

Now, we are able to state and prove a sufficient condition for the off-line characterization, in a non-closed form, of a class of faults that can be detected by the proposed FD methodology. Due to space constraints the proof of the theorem is omitted, but the steps are similar to those presented in the fault detectability theorem in Boem et al. (2011a).

Theorem 1. (Fault Detectability). If there exists a time instant $t_1 > T_0$ such that the fault ϕ_E satisfies the inequality

$$\left|\int_{T_0}^{t_1} C_E^{(k)} e^{WA_{0,E}(t_1-\tau)}(1 - e^{-\alpha(\tau-T_0)})\phi_E(\tau)d\tau\right| > 2\bar{\epsilon}_{y,E}^{(k)}(t_1)$$

for at least one component $k \in \{1,...,p_E\}$, then the fault will be detected at time t_1, that is $\left|\epsilon_{y,E}^{(k)}(t_1)\right| > \bar{\epsilon}_{y,E}^{(k)}(t_1)$.

4. DISTRIBUTED ISOLATION ARCHITECTURE

After a fault is detected by any of the N LFDs, the *Global Fault Diagnoser* (GFD) receives the corresponding local fault decision and switches each LFD from fault detection to fault isolation operating mode, stopping the learning of the parameter $\hat{\vartheta}_I$. Consistently with Ferrari et al. (2012), for isolation purposes we assume that the fault function $\boldsymbol{\phi}$ may belong to a known global fault set $\mathcal{F}$ or be unknown:

$$\mathcal{F} \triangleq \{\boldsymbol{\phi_1}(\boldsymbol{Cx},\boldsymbol{u}),\ldots,\boldsymbol{\phi_{N_\mathcal{F}}}(\boldsymbol{Cx},\boldsymbol{u})\},$$

It is possible that not all the subsystems are affected by a given fault function $\boldsymbol{\phi_l}$, but only those contained in the corresponding *fault influence set* $\mathcal{U}_{\boldsymbol{l}} \triangleq \{I : \exists t, \exists\boldsymbol{s}, \boldsymbol{s} \in \mathcal{I}_I, \boldsymbol{\phi_l^{(s)}}(\boldsymbol{Cx},\boldsymbol{u}) \neq 0\}$, for the l-th fault function $\boldsymbol{\phi}_l$, with $\boldsymbol{l} = 1,...,\boldsymbol{N_\mathcal{F}}$. A *local fault set* $\mathcal{F}_I$ can be defined for each subsystem $\mathscr{S}_I$, collecting the local fault functions $\phi_{I,l}$ such that $I \in \mathcal{U}_{\boldsymbol{l}}$: $\mathcal{F}_I \triangleq \{\phi_{I,1}(C_I x_I, z_I, u_I),...,\phi_{I,N_{\mathcal{F}_I}}(C_I x_I, z_I, u_I)\}$. A more detailed description can be found in Ferrari et al. (2012), where a scheme of the isolation algorithm is presented. Besides the FDAE, in the isolation mode each LFD uses other $N_{\mathcal{F}_I}$ estimators called *Fault Isolation Estimators* (FIE), one for each fault in the local fault set $\mathcal{F}_I$, in order to locally isolate the fault that is acting on the subsystem I. In this way, it is not necessary that the I–th LFD knows the global fault influence set: it is only able to isolate the *local* part of a fault that influences the subsystem $\mathscr{S}_I$. For each LFD $\mathscr{L}_I$, with $I = 1,...,N$, the generic l–th FIE, with $l \in \{1,\ldots,N_{\mathcal{F}_I}\}$, monitors the corresponding fault function $\phi_{I,l}$, belonging to the local fault set $\mathcal{F}_I$. We assume that each fault function in $\mathcal{F}_I$ can be expressed as:

$$\phi_{I,l} = \text{col}(\vartheta_{I,l,k}^\top H_{I,l,k}(C_I x_I, z_I, u_I), k = 1,...,n_I),$$

where $H_{I,l,k} : \mathbb{R}^{p_I} \times \mathbb{R}^{q_I} \times \mathbb{R}^{m_I} \mapsto \mathbb{R}^{q_{I,l,k}}$, with $k \in \{1,\ldots,n_I\}$, $l \in \{1,\ldots,N_{\mathcal{F}_I}\}$, are the known functions describing the functional structure of the fault and $\vartheta_{I,l,k} \in \Theta_{I,l,k} \subset \mathbb{R}^{q_{I,l,k}}$ are the unknown parameter vectors providing its "magnitude", where the parameter domains $\Theta_{I,l,k}$ are assumed to be origin–centered hyper–spheres with radius $M_{\Theta_{I,l,k}}$, without much loss of generality. After $t = T_d$, the generic l–th FIE estimator is activated and monitors its subsystem, computing a local state estimate $\hat{x}_{I,l}$ and a local output estimate $\hat{y}_{I,l}$. The difference between the estimate $\hat{y}_{I,l}$ and the measurements y_I is the estimation error $\epsilon_{y,I,l} \triangleq y_I - \hat{y}_{I,l}$, used as a residual and compared, component by component, to an appropriate isolation threshold $\bar{\epsilon}_{y,I,l} \in \mathbb{R}_+^{p_I}$. The condition $|\epsilon_{y,I,l}^{(k)}(t)| \le$

$\bar{\epsilon}^{(k)}_{y,I,l}(t)\ \forall\, k = 1, \ldots, p_I$ is associated to the l–th fault hypothesis $\mathscr{H}_{I,l}$: "The subsystem $\mathscr{S}_I$ is affected by the l-th fault". Should the condition be violated at some time instant t, then the hypothesis $\mathscr{H}_{I,l}$ will be falsified excluding the fault $\phi_{I,l}$ and the *local fault isolation signature* $\mathcal{S}_{I,l}$ will become non–empty:

$$\mathcal{S}_{I,l}(t) \triangleq \{k \in \{1, \ldots, p_I\} : \exists\, t_1,\, t \geq t_1 > 0 \quad \text{such that } |\epsilon^{(k)}_{y,I,l}(t_1)| > \bar{\epsilon}^{(k)}_{y,I,l}(t_1)\}.$$

The aim of the isolation task is to exclude every but one fault: a fault $\phi_{I,\rho} \in \mathcal{F}_I$ is *locally isolated* at time t iff $\forall l,\ l \in \{1, \ldots, N_{\mathcal{F}_I}\} \setminus \rho\,, \mathcal{S}_{I,l}(t) \neq \emptyset$ and $\mathcal{S}_{I,\rho}(t) = \emptyset$. We can say that it actually occurred if we assume that only faults belonging to the set $\mathcal{F}_I$ may occur. After the fault $\phi(t)$ has occurred, the s_I–th component of the I–th local state equation becomes

$$\dot{x}^{(s_I)}_I = A^{(s_I)}_I x_I + f^{(s_I)}_I(x_I, u_I) + g^{(s_I)}_I(C_I x_I, z_I, u_I) + \beta(t - T_0)\phi^{(s)}(Cx, u)\,.$$

The l-th FIE computes a local estimate, that, in the case of non-shared state variables, can be defined as:

$$\begin{cases} \dot{\hat{x}}_{I,l} = A_I \hat{x}_{I,l} + f_I(\hat{x}_{I,l}, u_I) + \hat{g}_I(y_I, u_I, v_I, \hat{\vartheta}_{I,0}) \\ \qquad + L_I(y_I - \hat{y}_{I,l}) + \hat{\phi}_{I,l}(y_I, v_I, u_I, \hat{\vartheta}_{I,l}) \\ \hat{y}_{I,l} = C_I \hat{x}_{I,l}, \end{cases}$$

where $L_I \in \mathcal{R}^{n_i \times p_I}$ is the local output error gain, $\hat{\phi}^{(s_I)}_{I,l}(y_I, v_I, u_I, \hat{\vartheta}_{I,l}) \triangleq (\hat{\vartheta}_{I,l,s_I})^\top H_{I,l,s_I}(y_I, v_I, u_I)$ is the s_I–th component of a linearly-parameterized function that learns the structure of the l–th fault function $\phi_{I,l}$, where the vector $\hat{\vartheta}_{I,l} \triangleq \mathrm{col}(\hat{\vartheta}_{I,l,k}, k \in \{1, \ldots, n_I\})$ contains its adjustable parameters: $\dot{\hat{\vartheta}}_{I,l} = P_{\hat{\Theta}_{I,l}}\left[\Gamma_{I,l} H^\top_{I,l} W^\top_{I,I} C^\top_I \epsilon_{y,I,l}\right]$, where $H^\top_{I,l} = \partial\hat{\phi}_{I,l}/\partial\hat{\vartheta}_{I,l}$. The dynamics of the l–th FIE estimator for the most general case of distributed fault are

$$\dot{\hat{x}}^{(s_I)}_{I,l} = \sum_{J \in \mathcal{O}_s} W^{(I,J)}_s \Big[A^{(s_J)}_J \hat{x}_{J,l} + f^{(s_J)}_J(\hat{x}_{J,l}, u_J) + \hat{g}^{(s_J)}_J(y_J, u_J, v_J, \hat{\vartheta}_{J,0}) + L^{(s_J)}_J(y_{J,l} - \hat{y}_J) + \hat{\phi}^{(s_J)}_{J,l}\Big]$$

The i-th estimation error component is

$$\dot{\epsilon}^{(i)}_{x,I,l} = A^{(i)}_{0,I}\epsilon_{x,I,l} + \Delta f^{(i)}_I + \Delta g^{(i)}_I - L^{(i)}_I \eta_{y,I} + (1 - e^{-\alpha(t-T_0)})\phi^{(i)} - \hat{\phi}^{(i)}_{J,l},$$

On the other hand, the dynamics of the state estimation error component for shared variables can be described as:

$$\dot{\epsilon}^{(s_I)}_{x,I,l} = A^{(s_I)}_I x_I + f^{(s_I)}_I(x_I, u_I) + g^{(s_I)}_I(C_I x_I, u_I, z_I) + (1 - e^{-\alpha(t-T_0)})\phi^{(s)} - \sum_{J \in \mathcal{O}_s} W^{(I,J)}_s \Big[A^{(s_J)}_J \hat{x}_{J,l} + f^{(s_J)}_J(\hat{x}_{J,l}, u_J) + \hat{g}^{(s_J)}_J(y_J, u_J, v_J, \hat{\vartheta}_{J,0}) + L^{(s_J)}_J(y_J - \hat{y}_{J,l}) + \hat{\phi}^{(s_J)}_{J,l}\Big]\,.$$

When we consider a matched fault, that is $\phi^{(s)} = \phi^{(s_J)}_{J,l}(x_J, z_J, u_J, \vartheta_{J,l}), \forall\, J \in \mathcal{O}_{\boldsymbol{s}}$, the error dynamics are:

$$\dot{\epsilon}^{(s_I)}_{x,I,l} = \sum_{J \in \mathcal{O}_s} W^{(I,J)}_s \Big[A^{(s_J)}_{0,J}\epsilon_{x,J,l} + \Delta f^{(s_J)}_J + \Delta g^{(s_J)}_J - L^{(s_J)}_J \eta_{y,J} + \Delta\phi^{(s_J)}_{J,l}\Big]$$

where

$$\Delta\phi^{(s_J)}_{J,l} \triangleq (1 - e^{-\alpha(t-T_0)})\phi^{(s)} - \hat{\phi}^{(s_J)}_{J,l} = (1 - e^{-\alpha(t-T_0)}) (H_{J,l,s_J}(t)^\top \vartheta_{J,l,s_J} + \Delta H^\top_{J,l,s_J}\vartheta_{J,l,s_J}) - H_{J,l,s_J}(t)^\top \hat{\vartheta}_{J,l,s_J}$$

with $\Delta H^\top_{J,l,s_J} \triangleq H_{J,l,s_J}(x_J, z_J, u_J) - H_{J,l,s_J}(y_J, v_J, u_J)\,.$ It can be rewritten as

$$\Delta\phi^{(s_J)}_{J,l} = -e^{-\alpha(t-T_0)} H_{J,l,s_J}(t)^\top \hat{\vartheta}_{J,l,s_J} + (1 - e^{-\alpha(t-T_0)})(H_{J,l,s_J}(t)^\top \tilde{\vartheta}_{J,l,s_J} + \Delta H_{J,l,s_J}(t)^\top \vartheta_{J,l,s_J})$$

if we introduce the parameter estimation errors $\tilde{\vartheta}_{J,l,s_J} \triangleq \vartheta_{J,l,s_J} - \hat{\vartheta}_{J,l,s_J}$. Using the general formulation, we can express the dynamics of the estimation error in the case of a matched fault, as:

$$\dot{\epsilon}_{x,E,l} = W\left[A_{0,E}\,\epsilon_{x,E,l} + \Delta f_E + \Delta g_E - L_E\eta_{yE} + \Delta\phi_{E,l}\right].$$

We can now compute the state estimation error solution:

$$\epsilon_{x,E,l}(t) = \int_{T_d}^{t} e^{WA_{0,E}(t-\tau)}\left[W\Delta f_E(\tau) + W\Delta g_E(\tau) - WL_E\eta_{y,E}(\tau) + W\Delta\phi_{E,l}(\tau)d\tau\right] + e^{WA_{0,E}(t-T_d)}\epsilon_{x,E,l}(T_d)$$

The output estimation error in the case of a matched fault can be written componentwise as: $\epsilon^{(k)}_{y,E,l} \triangleq C^{(k)}_E \epsilon_{x,E,l} + \eta^{(k)}_{y,E}$, for all $k = 1, ..., p_E$. It can be bounded by:

$$\left|\epsilon^{(k)}_{y,E,l}(t)\right| \leq \left|C^{(k)}_E\right| \Big\{\int_{T_d}^{t} \left\|e^{WA_{0,E}(t-\tau)}\right\| W\left[\bar{\Delta}f_E(\tau) + \bar{\Delta}g_E(\tau) + |L_E|\,\bar{\eta}_{y,E}(\tau) + \bar{\Delta}\phi_{E,l}(\tau)d\tau\right] + \left\|e^{WA_{0,E}(t-T_d)}\right\| \bar{\epsilon}_{x,E,l}(T_d)\Big\} + \bar{\eta}^{(k)}_{y,E}(t) \quad (11)$$

where

$$\bar{\Delta}\phi_{E,l} = \mathrm{col}(\|H_{I,l,s_I}(t)\|\kappa_{I,l,s_I}(\hat{\vartheta}_{I,l,s_I}) + \bar{\Delta}H_{I,l,s_I}(t)\bar{\vartheta}_{I,l,s_I} - e^{-\bar{\alpha}(t-T_d)}\|H_{I,l,s_I}(t)\|\|\hat{\vartheta}_{I,l,s_I}\|, s_I = 1, ..., n_I, I = 1, ..., N)$$

where $\kappa_{I,l}(\hat{\vartheta}_{I,l}) \geq \left\|\tilde{\vartheta}_{I,l}\right\|$. The threshold (11) can be computed in a distributed way and guarantees that no matched fault will be excluded due to the presence of uncertainties or to the effect of the parameter estimation error $\tilde{\vartheta}_{I,l}$.

4.1 Fault isolability analysis

We now consider the case of a non–matched fault $\phi^{(s_I)}_I(x_I, z_I, u_I) = \phi^{(s_I)}_{I,\rho}(x_I, z_I, u_I, \vartheta_{I,\rho})$, with $\rho \neq l$. In this case, the dynamics of the shared s_I–component of the estimation error for the l–th FIE of the I–th LFD can be written as

$$\dot{\epsilon}^{(s_I)}_{x,I,l} = \sum_{J \in \mathcal{O}_s} W^{(I,J)}_s [A^{(s_J)}_{0,J}\epsilon_{x,J,l} + \Delta f^{(s_J)}_J + \Delta g^{(s_J)}_J - L^{(s_J)}_J \eta_{y,J} + (1 - e^{-\alpha(t-T_0)})\phi^{(s_I)}_{I,\rho}(x_I, z_I, u_I, \vartheta_{I,\rho}) - \hat{\phi}^{(s_J)}_{J,l}(y_J, v_J, u_J, \hat{\vartheta}_{J,l})].$$

and considering the vector $\epsilon_{x,E,l}$:

$$\dot{\epsilon}_{x,E,l} = W\left[A_{0,E}\,\epsilon_{x,E,l} + \Delta f_E + \Delta g_E - L_E\eta_{yE} + \Delta_l\phi_{E,\rho}\right],$$

where the *mismatch vector* is introduced

$$\Delta_l\phi_{E,\rho} \triangleq \mathrm{col}((1 - e^{-\alpha(t-T_0)})\phi^{(s_I)}_{I,\rho} - \hat{\phi}^{(s_I)}_{I,l}, \quad s_I = 1, ..., n_I, I = 1, ..., N).$$

The solution can then be written as

$$\epsilon_{x,E,l}(t) = \int_{T_d}^{t} e^{WA_{0,E}(t-\tau)} [W\Delta f_E(\tau) + W\Delta g_E(\tau)$$
$$-WL_E\eta_{y,E}(\tau) + W\Delta_l\phi_{E,\rho}(\tau)d\tau] + e^{WA_{0,E}(t-T_d)}\epsilon_{x,E,l}(T_d)$$

and the output residual can be computed componentwise:

$$\epsilon^{(k)}_{y,E,l}(t) = \eta^{(k)}_{y,E}(t) + C^{(k)}_E \epsilon_{x,E,l}(t)$$

At this point, a sufficient condition for fault isolability can be proved. Due to page limitations, the proof is omitted.

Theorem 2. (Fault Isolability). Given a fault $\phi_{I,\rho} \in \mathcal{F}_I$, if for each $l \in \{1, \ldots, N_{\mathcal{F}_I}\}\backslash\rho$, the following inequality holds

$$\left| \int_{T_d}^{T_l} C^{(k)}_E e^{WA_{0,E}(T_l-\tau)} \Delta_l\phi_{E,\rho}(\tau)d\tau \right| > \bar{\epsilon}^{(k)}_{y,E,l}(T_l)$$
$$+ \left|C^{(k)}_E\right| \left\{ \int_{T_d}^{T_l} \left\|e^{WA_{0,E}(T_l-\tau)}\right\| W \left[\bar{\Delta} f_E(\tau) + \bar{\Delta} g_E(\tau)\right.\right.$$
$$\left.\left. + \left|L_E\right| \bar{\eta}_{y,E}(\tau)d\tau\right] + \left\|e^{WA_{0,E}(T_l-T_d)}\right\| \bar{\epsilon}_{x,E,l}(T_d) \right\} + \bar{\eta}^{(k)}_{y,E}(T_l)$$

at some time instant $T_l > T_d$, for some $k \in \{1, \ldots, p_I\}$, then the ρ–th fault will be isolated.

4.2 Global fault isolation logic

In this subsection, we analyze the global fault isolation logic. The GFD, which is assumed to know both the global fault set $\mathcal{F}$ and the fault influence sets of all the global fault functions, receives the local fault decisions d^{FD}_I from each LFD and determines which one of the faults, if any, in the global set $\mathcal{F}$ affects the system $\mathscr{S}$. As proposed in Ferrari et al. (2012), it is important to make a distinction between local and distributed faults. For a local fault, it is sufficient that the corresponding LFD excludes every but that fault for concluding that it is isolated. Instead, in the case of distributed faults, the isolation requires that all the LFDs in the influence set of that fault, exclude all other faults.

5. CONCLUDING REMARKS

In this paper, the problem of distributed fault detection and isolation with partial state measurement was addressed in the case of continuous–time systems, by extending a diagnosis architecture developed by the same authors in Boem et al. (2011b) for discrete-time systems. Future research efforts will be devoted to relax some of the detectability and isolability conditions and to illustrate the effectiveness of the proposed technique by validation on practically-relevant distributed use-cases, both in simulation and in actual experiments.

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