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EOQ Model with Ramp type Demand for three parameter Weibully Distributed Deterioration and Time Dependent Holding Cost

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ABSTRACT

In this paper, we have dealt with an inventory model with ramp type demand, starting with shortage and three – parameter Weibull distribution deterioration. Shortages are allowed and are completely backlogged. Our objective is to minimize the average total cost. A brief analysis of the costs involved has been carried out.

1. Introduction

A number of inventory models were developed by researchers assuming the demand of the items to be constant, linearly increasing or decreasing demand or exponentially increasing or decreasing with time. Later it was experienced that above demand patterns do not precisely depict the demand of certain items such as newly launched fashion goods and cosmetics, garments, automobiles etc. for which the demand increases with time as they are launched into the market and after some time it becomes constant. In order to consider the demand of such items, the concept of ramp type demand is introduced. Ramp type demand function depicts a demand, which increases up to a certain time after which it stabilizes and becomes constant.

Deterioration of items in inventory systems has become an interesting feature for its practical importance. Deterioration refers to damage, spoilage, vaporization or obsolescence of the products. There are several types of products that will deteriorate if stored for extended periods of time. Examples of deteriorating items include iron parts which are prone to corrosion and rusting and food items which are subject to spoilage and decay. Electronic components and fashion goods also fall into this category because they can become obsolete over time and their demand will decrease drastically.

Manna and Chiang (2010) developed an EPQ model for deteriorating items with ramp type demand. Teng and Chang (2005) considered the economic production quantity model for deteriorating items with stock level and selling price dependent demand. Jain et al. (2007) developed an economic production quantity model with shortages by incorporating the deterioration effect and stock dependent demand rate. Roy and Chaudhary (2009) developed two production rates inventory model for deteriorating items when the demand rate was assumed to be stock dependent.

In the research of Sana et al. (2004) shortages are allowed to occur at the end of a cycle. With the consideration of time varying demand and constant deteriorating rate, the optimal production inventory policy was studied. Raman Patel (2014) developed a production inventory model for Weibull deteriorating items with price and quantity dependent demand and varying holding cost with shortages. Both Skouri and Papachristos (2003) and Chen et al. (2002) developed a production inventory model in which the shortages are allowed at the beginning of a cycle. In contrast, in Manna and Chaudhari (2006) shortages are allowed but occur at the end of each cycle. Goyal's (2003) production inventory problem of a product with time varying demand, production and deterioration rates in which the shortages occur at the beginning of the cycle.

The work of the researchers, who used Ramp type demand function for the formulation of economic order quantity inventory model, is summarized in the following table:

Reference	Objectives	Constraints	Contributions	Limitations
Mandal & Pal (1998)	Finding EOQ	Ramp type demand, constant rate of deteriorations, shortage not allowed	An approximate Solution for EOQ is obtained	Approximate Solution, Constant rate of deterioration
Kun-Shan & Ouyang (2000)	Finding EOQ	Ramp type demand, constant rate of deteriorations, shortage allowed	An exact solution is obtained	Constant rate of deterioration
Jalan, Giri S. Chaudhari (2001)	Finding EOQ	Ramp type demand, Weibull deteriorations, shortage allowed	EOQ given by Numerical technique	Approximate Solution, Constant rate of

				deterioration
Kun-Shan (2001)	Finding EOQ	Ramp type demand, Weibull deteriorations, shortage allowed, Partial backlogging	EOQ obtained by 3 numerical examples	Approximate Solution, Constant rate of deterioration
Skouri & Konstantaras (2009)	Finding EOQ	Ramp type demand, Weibull deteriorations, with shortages and without shortages	An exact Solution for EOQ is obtained	Holding cost is constant
Skouri Konstantaras, Papachristos & Ganas (2009)	Finding EOQ	Ramp type demand, Weibull deteriorations, shortages are allowed, Partially backlogging.	An exact Solution for EOQ is obtained	Holding cost is constant
Jain & Mukesh (2010)	Finding EOQ	Ramp type demand, Weibull deteriorations, Starting with shortage	An exact Solution for EOQ model	Approximate Solution, Constant rate of deterioration

Recently, Kirtan Parmar and U. B. Gothi (2015) have developed an order level lot size inventory model for deteriorating items under quadratic demand with time dependent inventory holding cost and partial backlogging. Devyani Chatterji and U. B. Gothi (2015) have developed an EOQ model for deteriorating items under two and three parameter Weibull deterioration rate and constant inventory holding cost with partially backlogged shortages. Ankit Bhojak and U. B. Gothi (2015) have developed an EOQ model for deteriorating items under time dependent demand and Weibull distributed deterioration. The Inventory system for deteriorating rate of deteriorating items with power demand pattern and time depending holding cost with two parameter Weibull distribution and Pareto Type-I distribution was recently developed by Pooja D. Khatri and U. B. Gothi (2017).

In this paper, an EOQ model for deteriorating items with Ramp type demand is developed. Weibull distribution with three parameters is regarded for the deterioration rate. We employ Weibull Distribution because it is widely used in reliability and survival analysis. In order to justify this model, shortages are allowed to occur with complete backlogging. An analytical solution of the model is discussed.

2. Notations

The following notations are used to develop the model:

1. T: Length of the replenishment cycle.
2. R(t): Demand rate.
3. Z(t): Deterioration Rate.
4. Q(t): Instantaneous rate of the inventory level at any time t.
5. S: The maximum inventory level for each ordering cycle.
6. A: Ordering cost per order during the cycle period.
7. C_p: The purchase cost per unit.
8. C_s: The shortage cost per unit per unit time.
9. C_d: The cost of deterioration for single unit.
10. TC: The average total cost.

3. Assumptions

The mathematical model developed in this paper is based on the following assumptions:

1. The inventory system is considered over an infinite time horizon.
2. Shortages in inventory are allowed and are completely backlogged.
3. Rate of replenishment is assumed to be infinite.
4. Lead-time is practically assumed to be zero.
5. The deterioration rate function Z(*t*) for two-parameter Weibull distribution is $Z(t) = \alpha\beta(t-\gamma)^{\beta-1}$

   where α (0 < α <<1) is the scale parameter, β (> 0) is the shape parameter, γ ($t \geq \gamma$) is the location parameter and *t* (*t* > 0) is the time of deterioration.
6. The demand function *R* (*t*) is taken to be a ramp type function of time:

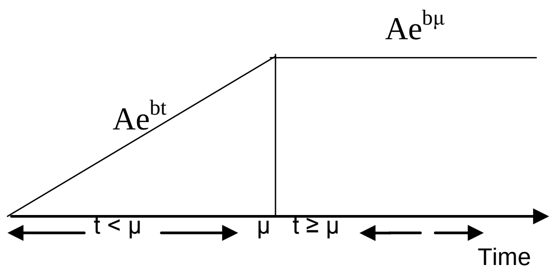
$$R(t) = Ae^{b[t-(t-\mu)H(t-\mu)]}$$

where H(t – μ) is the well-known Heaviside's function as following:

$$H(t-\mu) = \begin{cases} 1, & t \geq \mu \\ 0, & t < \mu \end{cases}$$

A = initial demand rate, b = a constant governing the exponential demand rate.

The pictorial presentation of demand function R(t) is shown in figure



7. Inventory model is developed starting with shortages.

4. Mathematical Model and Analysis

The inventory system starts with zero inventory at t = 0. Shortages are allowed to accumulate up to time t_1. At time t_1 inventory is replenished. The quantity received at t_1 is partly used to meet the shortages which accumulated from time 0 to t_1, leaving a balance of S items at time t_1. As time passes, the inventory level S declines due to demand during the period [t_1, γ], and mainly due to demand and partly due to deterioration of items during the period [γ, T]. At time T the inventory level gradually falls to zero.

The pictorial presentation is shown in the Figure – 1.

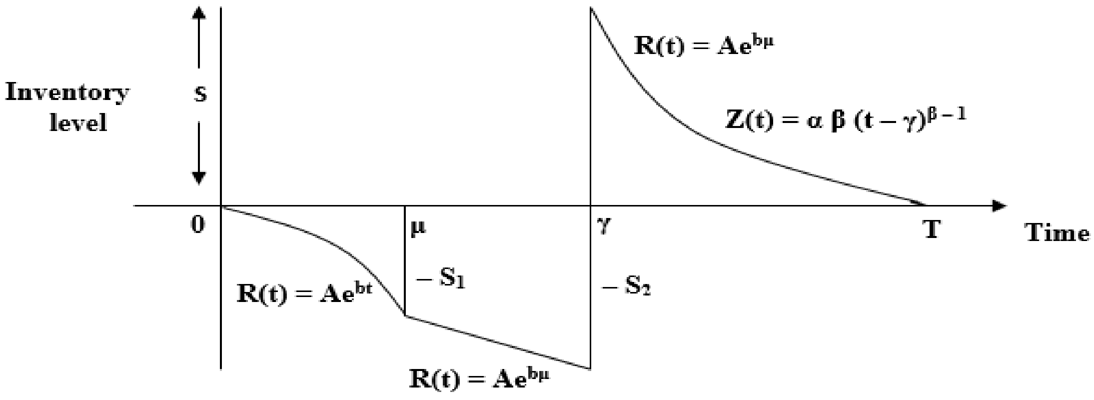


Figure 1: Inventory level-time relationship

The differential equations which describe the instantaneous state of Q(t) over the period (0, T) are given by

$$\frac{dQ(t)}{dt} = -Ae^{bt} \qquad (0 \le t \le \mu) \qquad \text{\_\_\_\_\_\_\_(1)}$$

$$\frac{dQ(t)}{dt} = -Ae^{b\mu} \qquad (\mu \le t \le \gamma) \qquad \text{\_\_\_\_\_\_\_(2)}$$

$$\frac{dQ(t)}{dt} + \alpha\beta(t-\gamma)^{\beta-1}Q(t) = -Ae^{b\mu} \qquad (\gamma \le t \le T) \qquad \text{\_\_\_\_\_\_\_(3)}$$

Under the boundary conditions Q(0) = 0, Q(μ) = – S_1, Q(T) = 0, Q(γ) = – S_2, and Q(γ) = S solutions of equations (1) to (3) are given by

$$Q(t) = \frac{A}{b}\left(1-e^{bt}\right) \qquad (0 \le t \le \mu) \qquad \text{\_\_\_\_\_\_\_(4)}$$

$$Q(t) = Ae^{b\mu}(\mu - t) + \frac{A}{b}\left(1-e^{b\mu}\right) \qquad (\mu \le t \le \gamma) \qquad \text{\_\_\_\_\_\_\_(5)}$$

$$Q(t) = Ae^{b\mu}\left[(T-t) + \frac{\alpha}{\beta+1}\left\{(T-\gamma)^{\beta+1} - (t-\gamma)^{\beta+1}\right\} - \alpha(T-t)(t-\gamma)^{\beta}\right] \qquad (\gamma \le t \le T) \qquad \text{\_\_\_\_\_\_\_(6)}$$

Putting Q(γ) = – S_1 in equation (4), we get

$$S_1 = -\frac{A}{b}\left(1-e^{b\mu}\right) \qquad \text{\_\_\_\_\_\_\_(7)}$$

Putting Q(γ) = – S_2 in equation (5), we get

$$S_2 = -Ae^{b\mu}(\mu-\gamma) - \frac{A}{b}\left(1-e^{b\mu}\right) \qquad \text{\_\_\_\_\_\_\_(8)}$$

Taking Q(γ) = S in equation (6), we get

$$S = Ae^{b\mu}\left[(T-\gamma) + \frac{\alpha}{\beta+1}\left\{(T-\gamma)^{\beta+1}\right\}\right] \qquad \text{\_\_\_\_\_\_\_(9)}$$

The total cost comprises of the following costs:

1) The ordering cost

OC = A _______(10)

2) The deterioration cost during the period [γ, T]

$$DC = c_d \int_{\gamma}^{T} \alpha\beta(t-\gamma)^{\beta-1} Q(t)\,dt$$

$$\Rightarrow DC = C_d\left[S - Ae^{b\mu}(T-\gamma)\right] \qquad \_\_\_\_\_\_\_(11)$$

3) The inventory holding cost during the period [γ, T]

$$IHC = \int_{\gamma}^{T}(h+rt)Q(t)dt$$

$$= \int_{\gamma}^{T} Ae^{b\mu}(\mu-t) + \frac{A}{b}\left(1-e^{b\mu}\right)dt$$

$$\Rightarrow IHC = Ae^{b\mu}\left[(r\gamma+h)\left(\frac{(T-\gamma)^2}{2} + \frac{\alpha\beta(T-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}\right) + r\left(\frac{(T-\gamma)^3}{6} - \frac{\alpha(T-\gamma)^{\beta+3}}{(\beta+2)(\beta+3)}\right)\right] \qquad \_\_\_\_\_\_\_(12)$$

4) The shortage cost per cycle

$$SC = c_s\left[\int_{0}^{\mu} Q(t)\,dt + \int_{\mu}^{t_1} Q(t)\,dt\right]$$

$$\Rightarrow SC = -c_s A\left[\frac{1}{b}\left(1-\frac{e^{b\mu}}{b}\right) - e^{b\mu}\frac{(\gamma-\mu)^2}{2} + \frac{1}{b}\left(1-e^{b\mu}\right)(\gamma-\mu)\right] \qquad \_\_\_\_\_\_\_(13)$$

5) The purchase cost

$$PC = c_p(S+S_2)$$

$$\Rightarrow PC = c_p\left\{Ae^{b\mu}\left[(T-\mu) + \frac{\alpha}{\beta+1}(T-\gamma)^{\beta+1}\right] - \frac{A}{b}\left(1-e^{b\mu}\right)\right\} \qquad \_\_\_\_\_\_\_(14)$$

Hence the total cost per unit time is given by

$$TC = \frac{1}{T}(OC + IHC + DC + SC + PC)$$

$$\Rightarrow TC = \frac{1}{T}\left\{\begin{array}{l} A + C_d\left[S - Ae^{b\mu}(T-\gamma)\right] + \\ Ae^{b\mu}\left[(r\gamma+h)\left(\frac{(T-\gamma)^2}{2} + \frac{\alpha\beta(T-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}\right) + r\left(\frac{(T-\gamma)^3}{6} - \frac{\alpha(T-\gamma)^{\beta+3}}{(\beta+2)(\beta+3)}\right)\right] \\ -c_s A\left[\frac{1}{b}\left(1-\frac{e^{b\mu}}{b}\right) - e^{b\mu}\frac{(\gamma-\mu)^2}{2} + \frac{1}{b}\left(1-e^{b\mu}\right)(\gamma-\mu)\right] \\ +c_p\left\{Ae^{b\mu}\left[(T-\mu) + \frac{\alpha}{\beta+1}(T-\gamma)^{\beta+1}\right] - \frac{A}{b}\left(1-e^{b\mu}\right)\right\} \end{array}\right\} \qquad \_\_\_\_\_\_\_(15)$$

μ* and T* are the optimum values of μ and T respectively, which minimize the cost function TC and they are the solutions of the equations $\frac{\partial TC}{\partial \mu} = 0, \quad \frac{\partial TC}{\partial T} = 0,$ satisfying the sufficient condition H > 0, and $\frac{\partial^2 TC}{\partial \mu^2} = 0$ at μ* and T* where

$$H = \begin{vmatrix} \frac{\partial^2 TC}{\partial \mu^2} & \frac{\partial^2 TC}{\partial \mu \partial T} \\ & \frac{\partial^2 TC}{\partial T^2} \end{vmatrix}$$ is Hessian determinant. _______(16)

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