# The need for a standard for the mathematical pronunciation of the natural numbers. Suggested principles of design. Implementation for English, German, French, Dutch and Danish 

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#### Abstract

Current English for 14 is fourteen but mathematically it is ten \& four. Research on number sense, counting, arithmetic and the predictive value for later mathematical abilities tends to be methodologically invalid when it doesn't measure true number sense that can develop when the numbers are pronounced in mathematical proper fashion. Researchers can correct by including proper names in the research design, but this involves some choices, and when each research design adopts a different scheme, also differently across languages, then results become incomparable. A standard would be useful, both ISO for general principles and national implementations. Research may not have the time to wait for such (inter-) national consensus. This article suggests principles of design and implementations for said languages. This can support the awareness about the need for a process towards ISO and national consensus, and in the mean time provides a baseline for research.


Keywords number sense, counting, arithmetic, mathematical ability, invalidity, design, standards, language, pronunciation, metastudy, number processing, numerical development, inversion effects, language-moderated effects, Google Translate

MeSH Terms Child, Child Development, Educational Measurement, Humans, Intelligence, Longitudinal Studies, Mathematics/education, Mathematics/methods, Mental Processes, Students

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## Introduction

First, there is the distinction between (1) a mathematical pronunciation of the natural numbers ( $0,1,2,3, \ldots$ ) and (2) the pronunciation of the natural numbers in the natural languages (English, German, ..., French). While we will use the term "natural language" those languages clearly have been subjected to changes by committees or influential authors. Thus the present discussion on a standard on mathematical pronunciation is no breach upon nature.
Subsequently we observe that the distinction between (1) and (2) hinders research on number sense, counting and arithmetic, and their predictive value for later mathematical competence. Research methods may suffer from methodological invalidity when they mistake "number sense in natural language" for "true number sense with mathematical pronunciation". Researchers can try to correct by providing pupils with mathematical names, as Ejersbo \& Misfeldt (2015) do. There is a risk that researchers implement their own interpretation of what mathematical names are, so that comparison of results becomes more and more difficult or impossible. Hence, a standard for such mathematical pronunciation will be useful, for achieving both validity and comparability.

For such a standard, we first establish the need, then propose principles of design, and then implement those principles to generate proposals for English, German, French, Dutch and Danish. It must be hoped that there will be a process towards consensus on such standards, both in ISO manner and national implementation. This article hopes to generate interest for such a process. In the mean time, researchers who are already in need of a baseline might be helped by the present suggestions.

The present issue differs principally from spelling reform. The spelling of a number ("29"), remains the same. Only its pronunciation changes. The new pronunciation will be spelled in common fashion too. This issue is not about spelling but about bilingualism and mathematical ability. A discussion in the media is by Shellenbarger (2014) in the WSJ.

## The need for a standard

Professor Fred Schuh of TU Delft in 1943 observed that the Dutch pronunciation of the numbers was awkward. While English has twenty-seven in the order of written 27, Dutch has zeven en twintig. He again discussed this in Schuh (1949) and formulated a proposal for change, focussing on the numbers above 20. The proposal reached the Dutch minister of education, see Stoffels (1952), but it was not adopted.
Researchers in Norway had observed the same problem, and the Norse parliament (Storting) adopted a change in 1950, which we see reflected in the pronunciation after 1951. ${ }^{1}$ I am not aware of an evaluation report. ${ }^{2}$ Pixner et al. (2011) observe that the Czech language allows both kinds of pronunciation, and they show that the mathematical order causes less errors than the inverted order.

Various authors look into number sense, counting and arithmetic, in which there is an interplay of language, embodiment (fingers), nonsymbolic forms (e.g. dots), symbols (Indian-Arabic numbers), and working memory. Dowker \& Roberts (2015) and Mark \& Dowker (2015) compare English, Welsh and Cantonese. Zuber et al. (2009), Moeller et al (2011), Klein et al. (2013) indicate that inversion in German slows down the learning progress w.r.t. mathematics proper. In Holland, Friso - Van den Bos (2014), XenidouDervou (2015) and Xenidou-Dervou et al. (2015) indicate the same for Dutch.

[^0]Hopefully this research generates interest amongst policy makers to adopt changes like in Norway 1950/51. However, such changes may still be limited w.r.t. a full mathematical pronunciation. Also English isn't perfect. It would be better to have ten \& one for 11 and two ten \& one for 21. Thus the challenge is larger, also for English and Norse.

Studies that compare the performances in languages suffer from the problem that they may study the obvious. Schuh (1949) didn't need modern statistics to arrive at the logical conclusion that number-names are better pronounced as they are written. The real problem lies in the policy making process, see Colignatus (2015ab).

The research on the development of number sense tends to suffer from methodological invalidity. In truth, number sense is defined with the use of mathematical pronunciation. The reason for this is that numbers themselves are defined as such. A natural language tends to be a dialect of the mathematical pronunciation. One should not take a dialect as the norm. Studies that do not allow children to develop number sense by using the mathematical names, will not observe true number sense, but "number sense in natural language". It may be admitted that one can develop statistical measures on such observations, but such a result is an awkward construct of both true number sense and confusion in language, in unclear mixture, without scientific relevance. ${ }^{3}$

The research on the development of number sense will also benefit from when researchers have deeper roots in mathematics education research (MER). The research quoted above derives mainly from the realm of (neuro-) psychology, and the problems on relevance, validity and comparability might have been observed at an earlier stage when there had been more awareness about what it actually is that pupils must learn. For a mathematician as Fred Schuh the pronunciation zeven en twintig is obviously illogical, while a neuro-psychologist may record it statistically as an "inversion", and actually think that this is how numbers are pronounced also mathematically, given that mathematicians also use such names. When (neuro-) psychologists would look deeper into MER, they must be warned that this field is not without problems of its own, however. See Colignatus (2015ab) for a longer discussion.

Relevant for research is the question whether pupils can deal with the difference between mathematical names and natural language dialect names. We see that many children can manage, see the examples of Czech, bilingual Chinese, bilingual English \& X (e.g. in Holland), and in Ejersbo \& Misfeldt (2015). The problem is not with children but in the policy making process, see Colignatus (2015a).

Thus, researchers interested in number sense, validity and relevance, will tend to follow the example by Ejersbo \& Misfeldt (2015) and include in the research design an instruction for pupils for using mathematical names. Perhaps researchers can find schools that are willing to participate in experiments with dual names, given that these aren't really much of experiments since we know that most children can deal with it. When parents are properly informed and first receive a training in the mathematical names, they might readily sign consent forms.

Colignatus (2015a) contains a chapter Marcus learns counting and arithmetic with ten. This text contains a stylized presentation for six-year olds. This is not intended for actual use in class but contains the framework for starting to think about that. There are translations for German, French, Danish and Dutch, that is: at this moment of writing the text still is in English but the numbers have been replaced by those in the Appendix below. This can also be used to instruct parents.

The real bottleneck then becomes comparability of research results. There are still questions of design. Different researchers might use different rules, and thus we would lose comparability. This establishes the need for a standard.

[^1]
## Principles of design

It is easy to suggest a "mathematical pronunciation of numbers in German", but what would that be ? When we use current zehn for 10, then there arises a problem, since the present pronunciation of 19 could be the mathematical pronunciation of 90 . This will generate great confusion, and Germans would have to check continuously whether others are using current or mathematical names. However, German might replace zehn by zig or adopt English ten or scientific deca (though two syllables).

| Number | Math in English | English | Math in German? | German | Math in German ! |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 19 | ten \& nine | nineteen | zehn \& neun | neunzehn | zig \& neun |
| 90 | nine $\operatorname{ten}$ | ninety | neun $\quad$ zehn | neunzig | neun $\mathbf{z i g}$ |

The proposed principles of design are:
(1) Pronunciation fully follows the place value system $\ldots c \times$ hundred $+b \times$ ten $+a=$ ...cba. The current convention to start with the digit with the highest place value is fine. (See Colignatus (2015a) for lesser alternatives in pronunciation and order.) Much of arithmetic can be done by proper pronunciation (e.g. $2 \times 10+4=24$ ).
(2) In writing out the pronunciation, also in educational texts, the middle dot is preferred over the hyphen since the latter may be confused with the minus-sign. ${ }^{4} \mathrm{We}$ thus say five ten \& nine for 59, where the dot is not pronounced and the order helps to decode the position (e.g. we don't have to say that 9 has place value one).
(3) There is awareness of the distinction between the process of calculation and the result given by the number. The process would be two times ten plus four and the result would be two ten \& four. On occasion two of ten and four might have the double role of both process and result. Operators might be bracketed or coloured it indicate that they are not pronounced, as in two (times) ten (plus) four. It must be tested whether young children would be served by a phase in which those operators are still pronounced also for the number result. Also elder pupils might at occasion be reminded of it. Also other names than times must be researched (e.g. the verb to of). Plus and minus however would be universal (given that "and" might not be commutative, as in he missed the train and arrived late at work).
(4) There are no exceptions in pronunciation of the digits in different place value positions. For example, German currently uses sieben in 7 and 27 and sieb in 70. A choice must be made for one name only. As a rule the shortest name is selected. For English some authors use tens as in two tens \& one, but ten is the value of the place, and must be used consistently. Multiplication can be scalar multiples ( 2 km ) or consists of making groups, and can be expressed by the word times, or find another word that expresses this better, such as grouping.
(5) A key point for the standard is that it is identified where languages can make choices. Thus, a proposal for German identifies such a choice between zig and ten. It is up to German what it selects, but the standard helps German identify the choice.
(6) If the name of 10 cannot be used as a base (e.g. German zehn and Dutch tien) then it is tried to find a close substitute already in use (e.g. zig in German and tig in Dutch), while often a clear option is to use English ten or scientific deca.
(7) The above only gives the cardinals. There are also the ordinals (first, second, third, ...) and the fractions (that abuse the ordinals, e.g. "a fifth"). The fractions are solved

[^2]by using $y x^{H}=y / x=" y$ per $x "(H=-1)$. The ordinals are solved by adopting a single extension, e.g. English "th" (one•th, two•th, three•th, ....) or Dutch "de" (een•de, twee $\cdot d e$, drie $\cdot$ de, ...). There is no linguistic morphing (Dutch tig•de doesn't become tig•ste). ${ }^{5}$ Colloquial words like English first and French premier will gradually adopt a meaning of "to begin with" rather than an ordinal number.
(8) The rule is that mathematical names are used in calculation. The national natural language is explained as a dialect of mathematics. It is an explicit educational goal to identify the national language as such a dialect.
(9) It will be useful to denote mathematical pronunciation with a label, say English-M and Deutsch-M. This now holds for numbers but this may apply to more phenomena later on, notably for the vocabulary. This suits translations too, e.g. Google Translate.
(10) These principles are targeted at becoming a consensus ISO standard. Countries define their own mathematical pronunciation based upon such a standard, and include own national improvements. For example, 7 in Dutch is consistently zeven in 7, 27 and 70, but when Dutch changes, it might opt for a single syllable zeef anyway. English might prefer thir over three, with thirteen, thirty and third then becoming ten \& thir, thir-ten and thir-th. (This choice though is not likely, because of potential confusion between thir•ten and thirteen.)

A suggestion is to have an expert meeting on this. In the mean time it still seems wise to provide this paper that identifies the issue. While the proposals in this paper may already be used in research to enhance comparability, ISO \& national standards would be needed for further use such as in official education requirements (US Common Core) and eventually national adoption also in courts of justice.

## Amendment May 142018

Colignatus (2018) provides software in Mathematica to show how it all would hear and look, taking advantage of the modern facilities for sounds and translation.

Revisiting the issue causes the following amendments.
(1) The symbol $Đ$ (capital eth) can be used as symbolic 10, and be pronounced as "deka". The number 10 is universal already, but when each language pronounces it differently, then the universal pronunciation of $\Theta=10=$ deka may help at times. For example, $\Xi^{0}, \Xi^{1}$, $\Xi^{2}, \boxplus^{3}, \ldots$ indicates the place values and does not invite to do an actual calculation.
(2) It is better to use the ampersand (\&) to separate the place value positions. This is used above but is a major revision of the earlier text of 2015 and deserves clarification.

The connectives "\&" and "." have an important role in the pronunciation and writing of the words of the numbers. They differ from the mathematical operators "plus" and "group" (multi-plus), since + and $\times$ have commutation, association and distribution.

- The ampersand (\&) is the ghost of addition, but simply "and", and not as the operator "plus" with all its properties. The ampersand should be pronounced to separate the place value positions. It is already (often) pronounced in German, Dutch and Danish, and other languages better adopt this practice too. It may take some time to get used to this but afterwards you will wonder why you never did before.
- The center dot (not pronounced) is the ghost of multiplication of the weight and the place value. It is not pure multiplication, like 5 days 2 hamburgers is not quite the same as 2 days 5 hamburgers.

[^3]Kids in kindergarten and Grade 1 live in a world of sounds. Thus it is important to also provide them with the \&-separator of the place value positions, so that they have this anchor to distinguish which from what. For adults and native speakers of English it may seem superfluous. Indeed, I myself in (2015a, footnote 10, and also the former version of this proporal for a standard) found the use of " $\alpha$ " "distractive", and proposed to use the center dot for " $\&$ " too: thus as $25=$ two ten five, without the distinction and merely as an unpronounced connective,. However, after much consideration, the empirical observation is that the \&-separator really is there. Its existence must be acknowledged instead of hidden from sight.
Namely, in natural language, putting two terms alongside, like in 2 km , means a scalar multiplication. In multiplication as grouping, kids learn to use the times-symbol, but you do not use it for 2 km , like $2 \times 1 \mathrm{~km}$. Later students will learn that 2 a is multiplication in general, also dropping the times-symbol. If they would have been trained by the pronunciation of the very numbers (and this a would be a number, in this scenario like in $a=25=$ two ten $\cdot$ five, thus without the " $\&$ ") then we create a conundrum: (1) within "a = 25 $=$ two ten five" the lack of an interfix means addition and (ii) outside of this, in $2 a$, the lack of an interfix means multiplication ? We should not create conundrums. Thus $25=$ two $\cdot$ ten \& five.

Indeed, in kindergarten and Grade 1 kids will tend to focus on the \& as an important new symbol in their universe, but this is not "distractive" but only fortunate, because it will form a stepping stone for the later learning on addition, i.e. using plus. Eventually they would tend to focus on the figures in the numbers and not the connectives.

## Implementation

The implementation of these principles of design to English, German, French, Dutch and Danish results in the proposals in the Appendix. They are also used in Marcus learns counting and arithmetic with ten in Colignatus (2015a) and its online translations.

For English, German, Dutch and Danish we skip the elaboration of the numbers 50-100 since these follow the system from 20-50.

For French, the numbers for 70-99 are fully written out however. This again shows the difficulty of international comparisons.

## Conclusions

The mathematical pronunciation of numbers is straightforward. The only bottleneck is consensus, as language tends to be social phenomenon. (It remains amazing that two people who haven't met before appear able to speak the same language.)
The principles of design are based upon the place value system, full adherence, minimal distance from current natural language, and a preference for short words. The principles allow the identification of choices to be made.

A prospective implementation is useful, firstly as an example of what it all might mean, secondly to provide researchers, who cannot wait for (inter-) national consensus to continue with their research goals, with a baseline suggestion. Both aspects would support the process towards such ISO \& national results.

## Appendix: Proposed implementations

## English

" $\& "=$ "and". The ordinals use -th, e.g. one•th, two th, three $\cdot t h, \ldots$. There is tension between current three-ten ths ( $3 / 10$ ) and mathematical three ten th ( $30 \cdot$ th), but calculation is done with mathematical name three per-ten.
zero ..... 0
one ..... 1
two ..... 2
three ..... 3
four ..... 4
five ..... 5
six ..... 6
seven ..... 7
eight ..... 8
nine ..... 9
ten ..... 10

## Ten to five•ten

| English-M |  | Current English |
| :---: | :---: | :---: |
| ten | 10 | ten |
| ten \& one | 11 | eleven |
| ten \& two | 12 | twelve |
| ten \& three | 13 | thirteen |
| ten \& four | 14 | fourteen |
| ten \& five | 15 | fifteen |
| ten \& six | 16 | sixteen |
| ten \& seven | 17 | seventeen |
| ten \& eight | 18 | eighteen |
| ten \& nine | 19 | nineteen |
| two ten | 20 | twenty |
| English-M |  | Current English |
| two ten | 20 | twenty |
| two ten \& one | 21 | twenty one |
| two $\operatorname{ten}$ \& two | 22 | twenty two |
| two ten \& three | 23 | twenty -three |
| two ten \& four | 24 | twenty four |
| two ten \& five | 25 | twenty five |
| two ten \& six | 26 | twenty six |
| two ten \& seven | 27 | twenty seven |
| two ten \& eight | 28 | twenty eight |
| two ten \& nine | 29 | twenty -nine |
| three -ten | 30 | thirty |

English-M
three $\cdot$ ten ..... 30
three $\cdot$ ten \& one ..... 31
three $\cdot$ ten \& two ..... 32
three ten \& three ..... 33
three $\cdot$ ten \& four ..... 34
three $\cdot$ ten \& five ..... 35
three $\cdot$ ten \& six ..... 36
three $\cdot$ ten \& seven ..... 37
three ten \& eight ..... 38
three ten \& nine ..... 39
four•ten ..... 40
English-M
four ten ..... 40
four ten \& one ..... 41
four ten \& two ..... 42
four $\cdot$ ten \& three ..... 43
four ten \& four ..... 44
four ten \& five ..... 45
four ten \& six ..... 46
four ten \& seven ..... 47
four ten \& eight ..... 48
four $\cdot$ ten \& nine ..... 49
five•ten ..... 50
ten 10
two ten 20
three•ten 30
four $\cdot$ ten 40
five•ten 50six•ten50
seven $\cdot$ ten ..... 70
eight•ten ..... 80
nine•ten ..... 90
ten•ten, hundred ..... 100
Numbers of ten
English-M Current English

Current English
thirty
thirty one
thirty $\cdot$ two
thirty•three
thirty•four thirty•five thirty-six thirty seven thirty-eight thirty-nine forty

Current English
forty
forty-one
forty-two
forty-three
forty-four
forty-five
forty-six
forty-seven
forty-eight
forty-nine
fifty

Current English
ten
twenty
thirty
forty
fifty
sixty
seventy
eighty
ninety
hundred

Ten to million: keep using the current language

| $10^{\wedge 1}$ | ten |
| :--- | :--- |
| $10^{\wedge 2}$ | ten $\cdot \operatorname{ten}$ |
| $10^{\wedge 3}$ | ten $\cdot \operatorname{ten} \cdot \operatorname{ten}$ |
| $10^{\wedge 4}$ | ten $\cdot \operatorname{ten} \cdot \operatorname{ten} \cdot \operatorname{ten}$ |
| $10^{\wedge} 5$ | ten $\cdot \operatorname{ten} \cdot \operatorname{ten} \cdot \operatorname{ten} \cdot \operatorname{ten}$ |
| $10^{\wedge} 6$ | ten $\cdot \operatorname{ten} \cdot \operatorname{ten} \cdot \operatorname{ten} \cdot \operatorname{ten} \cdot$ ten |

100 hundred
1,000 thousand
10,000 ten thousand
100,000 hundred•thousand
$1,000,000$ million

Current English
ten

## German

The choice of zig instead of zehn cannot be avoided because of the confusion between neunzehn (zig \& neun) and neunzig (neun•zig) if zehn were used. It remains an option to use English ten or scientific deca, but this seems unnecessary and unlikely.
" $\&$ " = "und". The choices of ein instead of eins and sieb instead of sieben are optional. Given that ein and sieb already are used, as in ein-und-siebzig, I have opted to use them universally.

The ordinals would use -te, e.g. ein $\cdot t e$, zwei•zig \& ein $\cdot t$.
null ..... 0
ein, eins ..... 1
zwei ..... 2
drei ..... 3
vier ..... 4
fünf ..... 5
sechs ..... 6
sieb, sieben ..... 7
acht ..... 8
neun ..... 9
zig, zehn ..... 10

## Zig zu fünf•zig

| Deutsch-M |  | Deutsch heute (current German) |
| :---: | :---: | :---: |
| zig | 10 | zehn |
| zig \& ein | 11 | elf |
| zig \& zwei | 12 | zwölf |
| zig \& drei | 13 | dreizehn |
| zig \& vier | 14 | vierzehn |
| zig \& fünf | 15 | fünfzehn |
| zig \& sechs | 16 | sechzehn |
| zig \& sieb | 17 | siebzehn |
| zig \& acht | 18 | achtzehn |
| zig \& neun | 19 | neunzehn |
| zwei•zig | 20 | zwanzig |
| Deutsch-M |  | Deutsch heute |
| zwei•zig | 20 | zwanzig |
| zwei•zig \& ein | 21 | ein $\cdot$ und•zwanzig |
| zwei zig \& zwei | 22 | zwei-und•zwanzig |
| zwei zig \& drei | 23 | drei•und•zwanzig |
| zwei zig \& vier | 24 | vier-und zwanzig |
| zwei $\cdot \mathrm{zig}$ \& fünf | 25 | fünf•und•zwanzig |
| zwei $\cdot$ zig \& sechs | 26 | sechs -und zw ( ${ }^{\text {anzig }}$ |
| zwei-zig \& sieb | 27 | sieben und zwanzig |
| zwei zig \& acht | 28 | acht•und zwanzig |
| zwei•zig \& neun | 29 | neun-und•zwanzig |
| drei-zig | 30 | dreißig |

## Deutsch-M

drei•zig 30
drei $\cdot$ zig \& ein $\quad 31$
drei $\cdot$ zig \& zwei 32
drei zig \& drei $\quad 33$
drei $\cdot$ zig \& vier $\quad 34$
drei $\cdot$ zig \& fünf 35
drei $\mathbf{z i g}$ \& sechs $\quad 36$
drei $\cdot$ zig \& sieb $\quad 37$
drei $\cdot$ zig \& acht 38
drei $\cdot$ zig \& neun 39
vier•zig 40
$\begin{array}{ll}\text { Deutsch-M } \\ \text { vier•zig } & 40\end{array}$
vier $\cdot$ zig \& ein 41
vier•zig \& zwei 42
vier•zig \& drei 43
vier•zig \& vier 44
vier $\cdot$ zig \& fünf 45
vier $\cdot$ zig \& sechs 46
vier•zig \& sieb 47
vier•zig \& acht 48
vier•zig \& neun 49
fünf•zig 50
The numbers of zig
Deutsch-M
zig 10
zwei•zig 20
drei•zig 30
vier•zig 40
fünf•zig 50
sechs zig 60
sieb•zig 70
acht•zig 80
neun•zig 90
zig•zig, hundert 100

## Deutsch heute

dreißig
ein-und•dreißig
zwei-und•dreißig
drei-und•dreißig
vier-und-dreißig
fünf•und•dreißig
sechs•und•dreißig
sieben $\cdot$ und•dreißig
acht-und•dreißig
neun $\cdot$ und•dreißig
vierzig

Deutsch heute
vierzig
ein•und•vierzig
zwei-und•vierzig
drei-und•vierzig
vier-und-vierzig
fünf•und•vierzig
sechs und $\cdot$ vierzig
sieben und $\cdot$ vierzig
acht•und•vierzig
neun-und•vierzig
fünfzig

Deutsch heute
zig
zwanzig
dreißig
vierzig
fünfzig
sechzig
siebzig
achtzig
neunzig
hundert

## Ten to million: keep using the current language above zig

| 10^1 | zig |
| :---: | :---: |
| 10^2 | zig•zig |
| 10^3 | zig•zig•zig |
| 10^4 | zig•zig•zig•zig |
| $10^{\wedge} 5$ | zig•zig•zig•zig•zig |
| $10^{\wedge} 6$ | zig•zig•zig•zig•zig•zig |

10
100
1,000
10,000
100,000 hundert•tausend
1,000,000 Million

## French

In French there is no problem in taking dix as the base for the numbers of ten.
The numbers of 70-100 are fully written out because of the complex French originals.
" $\&$ "= "et". The ordinals would be -ième: un •ième,deux•ième, ...

| zéro | 0 |
| :--- | ---: |
| un | 1 |
| deux | 2 |
| trois | 3 |
| quatre | 4 |
| cinq | 5 |
| six | 6 |
| sept | 7 |
| huit | 8 |
| neuf | 9 |
| dix | 10 |

Dix to cinq•dix

| Français-M |  | Français aujourd'hui |
| :---: | :---: | :---: |
| dix | 10 | dix |
| dix \& un | 11 | onze |
| dix \& deux | 12 | douze |
| dix \& trois | 13 | treize |
| dix \& quatre | 14 | quatorze |
| dix \& cinq | 15 | quinze |
| dix \& six | 16 | seize |
| dix \& sept | 17 | dix-sept |
| dix \& huit | 18 | dix•huit |
| dix \& neuf | 19 | dix-neuf |
| deux•dix | 20 | vingt |
| Français-M |  | Français aujourd'hui |
| deux•dix | 20 | vingt |
| deux $\cdot$ dix \& un | 21 | vingt et un |
| deux dix \& deux | 22 | vingt•deux |
| deux dix \& trois | 23 | vingt trois |
| deux $\cdot$ dix \& quatre | 24 | vingt quatre |
| deux•dix \& cinq | 25 | vingt $\cdot$ cinq |
| deux $\cdot$ dix \& six | 26 | vingt•six |
| deux dix \& sept | 27 | vingt•sept |
| deux $\cdot$ dix \& huit | 28 | vingt huit |
| deux dix \& neuf | 29 | vingt•neuf |
| trois•dix | 30 | trente |

Français-M
trois dix 30
trois•dix \& un 31
trois dix \& deux $\quad 32$
trois dix \& trois $\quad 33$
trois dix \& quatre $\quad 34$
trois dix \& cinq $\quad 35$
trois dix \& six $\quad 36$
trois dix \& sept $\quad 37$
trois dix \& huit 38
trois•dix \& neuf 39
quatre $\cdot$ dix 40

Français-M
quatre $\cdot$ dix 40
quatre $\cdot$ dix \& un 41
quatre $\cdot$ dix \& deux 42
quatre $\cdot$ dix \& trois $\quad 43$
quatre $\cdot$ dix \& quatre 44
quatre $\cdot$ dix \& cinq $\quad 45$
quatre $\cdot$ dix \& six $\quad 46$
quatre $\cdot$ dix \& sept 47
quatre $\cdot$ dix \& huit 48
quatre•dix \& neuf 49
cinq•dix 50

Français-M
sept•dix 70
sept•dix \& un $\quad 71$
sept•dix \& deux $\quad 72$
sept•dix \& trois $\quad 73$
sept•dix \& quatre $\quad 74$
sept•dix \& cinq $\quad 75$
sept•dix \& six $\quad 76$
sept•dix \& sept $\quad 77$
sept•dix \& huit $\quad 78$
sept•dix \& neuf $\quad 79$
huit•dix 80
huit•dix 80
huit•dix \& un 81
huit•dix \& deux 82
huit•dix \& trois 83
huit•dix \& quatre 84
huit•dix \& cinq 85
huit•dix \& six 86
huit•dix \& sept 87
huit•dix \& huit 88
huit•dix \& neuf 89
neuf•dix 90

Français aujourd'hui
trente
trente et un
trente-deux
trente-trois trente quatre
trente cinq
trente-six
trente-sept
trente-huit
trente-neuf quarante

Français aujourd'hui
quarante quarante et un quarante deux quarante trois quarante quatre quarante cinq quarante six quarante sept quarante-huit quarante-neuf cinquante

Français aujourd'hui
soixante•dix
soixante et onze
soixante•douze
soixante•treize
soixante-quatorze
soixante quinze
soixante-seize
soixante-dix•sept
soixante•dix•huit
soixante•dix•neuf
quatre $\cdot$ vingts
quatre $\cdot$ vingts quatre $\cdot$ vingt $\cdot$ un quatre $\cdot$ vingt $\cdot$ deux quatre $\cdot$ vingt $\cdot$ trois quatre vingt quatre quatre $\cdot$ vingt $\cdot$ cinq quatre vingt•six quatre $\cdot$ vingt $\cdot$ sept quatre $\cdot$ vingt $\cdot h u i t$ quatre $\cdot$ vingt $\cdot$ neuf quatre $\cdot$ vingt $\cdot$ dix

| neuf dix | 90 | quatre $\cdot$ vingt $\cdot$ dix |
| :---: | :---: | :---: |
| neuf•dix \& un | 91 | quatre vingt et onze |
| neuf.dix \& deux | 92 | quatre $\cdot$ vingt douze |
| neuf.dix \& trois | 93 | quatre vingt 'treize |
| neuf.dix \& quatre | 94 | quatre $\mathbf{v i n g t}$ quatorze |
| neuf.dix \& cinq | 95 | quatre vingt quize |
| neuf.dix \& six | 96 | quatre $\cdot$ vingt seize |
| neuf.dix \& sept | 97 | quatre $\cdot \mathrm{ving} \mathrm{t} \cdot \mathrm{dix}$-sept |
| neuf.dix \& huit | 98 | quatre $\cdot$ vingt $\cdot \mathrm{dix}$ •huit |
| neuf.dix \& neuf | 99 | quatre $\cdot \mathrm{vingt} \cdot \mathrm{dix} \cdot \mathrm{neuf}$ |
| dix•dix, cent | 100 | cent |

The numbers of dix

Français-M
dix 10
deux•dix 20
trois•dix
quatre dix
30
quindix
cinq•dix
40
six•dix
sept•dix 60
huit-dix neuf•dix 90 70
dix•dix

Français aujourd'hui
dix
vingt
trente
quarante
cinquante
soixante
soixante-dix
quatre vingts
quatre $\cdot$ vingt $\cdot$ dix
cent

Ten to million: keep using the current language

|  |  | Français aujourd'hui |  |
| :--- | :--- | ---: | :--- |
| $10^{\wedge} 1$ | dix | 10 | dix |
| $10^{\wedge} 2$ | dix•dix | 100 | cent |
| $10^{\wedge} 3$ | dix $\cdot d i x \cdot d i x$ | 1,000 | mille |
| $10^{\wedge} 4$ | dix $\cdot d i x \cdot d i x \cdot d i x$ | 10,000 | dix $\cdot$ mille |
| $10^{\wedge} 5$ | dix $\cdot d i x \cdot d i x \cdot d i x \cdot d i x$ | 100,000 | cent $\cdot$ mille |
| $10^{\wedge} 6$ | dix•dix•dix•dix•dix•dix | $1,000,000$ | million |

## Dutch

The choice of tig instead of tien cannot be avoided because of the confusion between negentien (tig \& negen) and negentig (negen-tig) if tien were used. It remains an option to use English ten, but this seems unnecessary and unlikely. " $\&$ "= "en".

Ordinals use -de: een•de, twee•de, drie•de, ..., tig•de, .....

| nul | 0 |
| :--- | :--- |
| een | 1 |
| twee | 2 |
| drie | 3 |
| vier | 4 |
| vijf | 5 |
| zes | 6 |
| zeven | 7 |
| acht | 8 |
| negen | 9 |
| tig, tien | 10 |

From ten to fifty

Nederlands-M
tig 10
tig \& een $\quad 11$
tig \& twee $\quad 12$
tig \& drie $\quad 13$
tig \& vier $\quad 14$
tig \& vijf 15
tig \& zes 16
tig \& zeven $\quad 17$
tig \& acht 18
tig \& negen 19
twee•tig 20
Nederlands-M
twee•tig 20
twee•tig \& een 21
twee•tig \& twee 22
twee•tig \& drie 23
twee•tig \& vier 24
twee•tig \& vijf 25
twee•tig \& zes 26
twee•tig \& zeven 27
twee•tig \& acht 28
twee•tig \& negen 29
drie•tig 30

Huidig Nederlands
tien
elf
twaalf
dertien
veertien
vijftien
zestien
zeventien
achttien
negentien
twintig

## Huidig Nederlands

twintig
een $\cdot$ en $\cdot t w i n t i g$
twee $\cdot$ en $\cdot$ twintig
drie $\cdot$ en $\cdot$ twintig
vier•en twintig
vijf•en twintig
zes•en twintig zeven $\cdot$ en $\cdot$ twintig acht•en $\cdot$ twintig negen $\cdot$ en $\cdot$ twintig dertig

| Nederlands-M |  | Huidig Nederlands |
| :---: | :---: | :---: |
| drie $\cdot$ tig | 30 | dertig |
| drie tig \& een | 31 | een $\cdot \mathrm{en} \cdot \mathrm{dertig}$ |
| drie -tig \& twee | 32 | twee $e n \cdot d e r t i g$ |
| drie tig \& drie | 33 | drie•en $\cdot$ dertig |
| drie-tig \& vier | 34 | vier $\cdot \mathrm{en}$-dertig |
| drie $\cdot$ tig \& vijf | 35 | vijf•en•dertig |
| drie tig \& zes | 36 | zes $\cdot \mathrm{en}$-dertig |
| drie -tig \& zeven | 37 | zeven $\cdot$ en dertig |
| drie•tig \& acht | 38 | acht $\cdot \mathrm{en}$ •dertig |
| drie $\cdot$ tig \& negen | 39 | negen $\cdot$ en $\cdot$ dertig |
| vier $\cdot$ tig | 40 | veertig |
| Nederlands-M |  | Huidig Nederlands |
| vier $\cdot$ tig | 40 | veertig |
| vier $\cdot$ tig \& een | 41 | een $\cdot$ en $\cdot v e e r t i g$ |
| vier $\cdot$ tig \& twee | 42 | twee•en veertig |
| vier tig \& drie | 43 | drie $\cdot$ en veertig |
| vier tig \& vier | 44 | vier $\cdot \mathrm{en}$-veertig |
| vier $\cdot$ tig \& vijf | 45 | vijf.en veertig |
| vier tig \& zes | 46 | zes $\cdot \mathrm{en}$-veertig |
| vier $\cdot$ tig \& zeven | 47 | zeven $\cdot \mathrm{en}$-veertig |
| vier $\cdot$ tig \& acht | 48 | acht $\cdot \mathrm{en}$-veertig |
| vier tig \& negen | 49 | negen $\cdot \mathrm{en} \cdot \mathrm{veertig}$ |
| vijf•tig | 50 | vijftig |

## The numbers of tig

## Nederlands-M

tig 10
twee•tig 20
drie•tig 30
vier•tig 40
vijf•tig 50
zes•tig 60
zeven•tig 70
acht•tig 80
negen $\cdot$ tig 90
tig•tig, honderd 100

Huidig Nederlands
tien
twintig
dertig
veertig
vijftig
zestig
zeventig
tachtig negentig honderd

Ten to million: keep using the current language above tig
Huidig Nederlands

| $10^{\wedge 1}$ | tig |
| :--- | :--- |
| $10^{\wedge} 2$ | tig•tig |
| $10^{\wedge} 3$ | tig•tig $\cdot$ tig |
| $10^{\wedge} 4$ | tig $\cdot$ tig $\cdot$ tig $\cdot$ tig |
| $10^{\wedge} 5$ | tig $\cdot$ tig $\cdot$ tig $\cdot$ tig $\cdot$ tig |
| $10^{\wedge} 6$ | tig $\cdot$ tig $\cdot$ tig $\cdot$ tig $\cdot$ tig $\cdot$ tig |

10 tien
100 honderd
1,000 duizend
10,000 tig•duizend
100,000 honderd•duizend
1,000,000 miljoen

## Danish

Danish can use current $t i$ as below, but also has the option to use English ten.
" $\& "=$ "og". For the ordinals a suggestion would be to use -de like English -th.

| nul | 0 |
| :--- | :--- |
| en | 1 |
| to | 2 |
| tre | 3 |
| fire | 4 |
| fem | 5 |
| seks | 6 |
| syv | 7 |
| otte | 8 |
| ni | 9 |
| ti | 10 |

## From ten to fifty

| Dansk-M |  | Dansk i dag |
| :--- | :--- | :--- |
| ti | 10 | ti |
| ti \& en | 11 | elleve |
| ti \& to | 12 | tolv |
| ti \& tre | 13 | tretten |
| ti \& fire | 14 | forten |
| ti \& fem | 15 | femten |
| ti \& seks | 16 | seksten |
| ti \& syv | 17 | sytten |
| ti \& otte | 18 | atten |
| ti \& ni | 19 | nitten |
| to ti | 20 | tyve |

Dansk-M
to•ti 20
to $\cdot$ ti \& en 21
to•ti\& to 22
to•ti \& tre 23
to•ti \& fire 24
to•ti \& fem 25
to•ti \& seks 26
to.ti \& syv 27
to•ti \& otte 28
to•ti \& ni 29
tre•ti 30
Danskidag
tyve
en og $\cdot$ tyve
to.og.tyve
tre.og•tyve
fire-og tyve
fem.og•tyve seks-og•tyve syv-og tyve otte•og•tyve ni.og•tyve tredive

## Dansk-M

tre•ti ..... 30
tre•ti \& en ..... 31
tre $\cdot \mathrm{ti} \&$ to ..... 32
tre $\cdot \mathrm{ti} \&$ tre ..... 33
tre $\cdot \mathrm{ti}$ \& fire ..... 34
tre ti \& fem ..... 35
tre•ti \& seks ..... 36
tre•ti \& syv ..... 37
tre•ti \& otte ..... 38
tre•ti \& ni ..... 39
fire•ti ..... 40
Dansk-M

Dansk idag
fire-ti ..... 40
fire•ti \& en ..... 41
fire ti \& to ..... 42
fire $\cdot$ ti \& tre ..... 43
fire•ti \& fire ..... 44
fire ti \& fem ..... 45
fire ti \& seks ..... 46
fire•ti \& syv ..... 47
fire ti \& otte ..... 48
fire•ti \& ni ..... 49
fem•ti ..... 50

Dansk i dag tredive en og $\cdot$ tredive to.og-tredive tre og $\cdot$ tredive fire-og•tredive fem•og•tredive seks-og•tredive syv-og•tredive otte•og•tredive ni-og tredive fyrre
fyrre
en og fyrre
to og fyrre tre•og'fyrre fire.og•fyrre fem $\cdot \mathrm{og} \cdot \mathrm{fy}$.re seks og•fyrre syv.og•fyrre otte og fyrre ni•og•fyrre halvtreds

## The numbers of $\boldsymbol{t i}$

Dansk-M
$\mathrm{ti} \quad 10$
to. ti
tre ti
fire-ti
fem•ti
seks-ti
syv•ti
otte•ti
20
30
40
50
60
80
ni-ti 90
ti•ti, hundrede 100

Dansk i dag
ti
tyve
tredive
fyrre
halvtreds
tres
halvfjerds
firs
halvfems
hundrede

## Ten to million: keep using the current language

| $10^{\wedge} 1$ | ti |
| :--- | :--- |
| $10^{\wedge} 2$ | $\mathrm{ti} \cdot \mathrm{ti}$ |
| $10^{\wedge} 3$ | $\mathrm{ti} \cdot \mathrm{ti} \cdot \mathrm{ti}$ |
| $10^{\wedge} 4$ | $\mathrm{ti} \cdot \mathrm{ti} \cdot \mathrm{ti} \cdot \mathrm{ti}$ |
| $10^{\wedge} 5$ | $\mathrm{ti} \cdot \mathrm{ti} \cdot \mathrm{ti} \cdot \mathrm{ti} \cdot \mathrm{ti}$ |
| $10^{\wedge} 6$ | $\mathrm{ti} \cdot \mathrm{ti} \cdot \mathrm{ti} \cdot \mathrm{ti} \cdot \mathrm{ti} \cdot \mathrm{ti}$ |

Dansk idag

| 10 | ti |
| ---: | :--- |
| 100 | hundrede |
| 1,000 | tusind |
| 10,000 | ti•tusind |
| 100,000 | hundrede $\cdot$ tusind |
| $1,000,000$ | million |

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PM 1. Colignatus is the name of Thomas Cool in science.
PM 2. References in footnotes need not be repeated here.

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[^0]:    ${ }^{1}$ http://blogs.transparent.com/norwegian/learning-norwegian-numbers/
    ${ }^{2}$ I have asked this question at http://www.matematikksenteret.no/

[^1]:    ${ }^{3}$ See also my weblog text https://boycottholland.wordpress.com/2015/08/29/research-on-number-sense-tends-to-be-invalid/

[^2]:    ${ }^{4}$ See the use of the minus-sign in the place value system (a chapter in Colignatus (2015a)): https://boycottholland.wordpress.com/2014/08/30/taking-a-loss/

[^3]:    ${ }^{5}$ See the importance of the ordinals for developing number sense (a chapter in Colignatus (2015a)): https://boycottholland.wordpress.com/2014/08/01/is-zero-an-ordinal-or-cardinal-number-q/

