# Evanescent dark energy and dark matter

Maurice H.P.M. van Putten*
(Dated: May 9, 2018)

Cosmological dark energy and dark matter are identified with evanescent waves at super-horizon scales. They enter the visible universe across the cosmological horizon $\mathcal{H}$ below a fundamental frequency $\omega_0 = \sqrt{1-q}H$ at a Hubble parameter $H$ and deceleration parameter $q$. The cosmological vacuum hereby acquires a stress-energy tensor with nonzero trace, $T_{ab} = \left[(1-q)\pi^-_{ab} + q\pi^+_{ab}\right]\rho_c$ of radiation ($\pi^+_{ab}$) and dark component of evanescent waves ($\pi^-_{ab}$), where $\rho_c$ denotes the closure energy density, $\mathrm{tr}\,\pi^+_{ab} = 0$ and $\mathrm{tr}\,\pi^-_{ab} = 2$. It defines fast expansion with $H'(0) = H_0(3\omega_m - 1) \simeq 0$, $Q(0) \simeq 2.75 \pm 0.15$, $Q(z) = dq(z)/dz$. Nonlinear model regression applied to $H(z)$ over an intermediate range of redshift $z$ identifies a tension-free estimate $H_0 = 74.9 \pm 2.6\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ consistent with Riess et al. (2016) in surveys of the Local Universe and a recent estimate produced by GW170817.

*Introduction.* Modern cosmology reveals a dark energy highlighting potentially new physics at the lowest energy scales in the Universe, that appears in a tension in measurements of the Hubble parameter higher than that expected from ΛCDM [1]. This anomaly appears about the de Sitter scale of acceleration $a_{dS} = cH$, where $c$ is the velocity of light and $H$ is the Hubble parameter. By Einstein's equivalence principle [2], it represents weak gravitation far below the scales conventionally encountered as Newtonian gravity in the Solar system [3]. While Einstein's theory of gravitation gives a highly successful extension to strong gravity, the limits of which include black holes and gravitational waves from mergers [4], taking it to weak gravity in the face of $a_{dS}$ and the cosmological horizon $\mathcal{H}$ at the Hubble radius $R_H = c/H$ is highly uncertain if the vacuum is entangled [5,6] with the cosmological background of our Friedmann-Robertson-Walker (FRW) Universe [7].

On the largest scales, our three-flat Universe features a cosmological horizon $\mathcal{H}$ at the Hubble radius $R_H = c/H$ (Fig. 1), where $c$ is the velocity of light and $H = \dot{a}/a$ is the Hubble parameter in the FRW line-element in $(t, r, \vartheta, \varphi)$,

$$ds^2 = -dt^2 + a(t)^2(dr^2 + r^2 d\vartheta^2 + r^2 \sin^2\vartheta d\varphi^2) \quad (1)$$

with deceleration parameter $q = -\ddot{a}a/\dot{a}^2$, $q(z) = -1 + (1+z)H^{-1}(z)H'(z)$ for $H(z)$ as a function of redshift $z$. Evolving (1) by general relativity on a classical vacuum, we infer a dark energy density $\Lambda/8\pi > 0$ from the observed sign [8]

$$q_{GR} = \frac{1}{2}\Omega_m - \Omega_\Lambda < 0. \quad (2)$$

In the standard frame work of a constant Λ and cold dark matter (ΛCMD), we asymptotically evolve towards a de Sitter state ($q = -1$) in the distant future ($z = -1$), as $H$ descends to a finite value $\sqrt{\Lambda/3}$. This highlights a tight relation between dark energy and $\mathcal{H}$ at the Hubble radius $R_H = c/H$, where $c$ is the velocity of light.

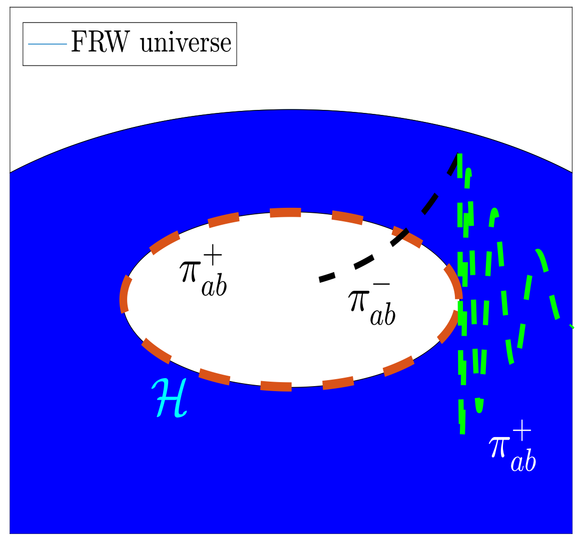


FIG. 1. FRW cosmologies feature a cosmological horizon $\mathcal{H}$, defined in the geometric optics limit as a causal separation surface bounding our visible universe at given cosmic time $t$. Since $\mathcal{H}$ is compact, it is dispersive, due to a finite eigenfrequency $\omega_0$ defined by the equations of geodesic separation. Below $\omega_0$, super-horizon scale waves are evanescent, and tunnel through $\mathcal{H}$ into our visible universe. Evanescent waves have stress-energy ($\pi^-_{ab}$) with nonzero trace, different from radiation ($\pi^+_{ab}$) on sub-horizon scales, whose trace is zero. By evanescence, our vacuum is entangled with the cosmological background.

*$H_0$ tension.* $H_0 = H(0)$ measured from surveys of the Local Universe [9] increasingly indicate a value higher than $H_0$ obtained from ΛCDM analysis of the Cosmic Microwave Background (CMB) [6]. While ?CMB provides a powerful framework to CMB power spectra, its applicability to low-z evolution of $H(z)$ is challenged by potential dynamical evolution of Λ and associated evolution in the cosmological horizon.

As an apparent horizon surface, H is defined in the geometric optics limit as a causal separation surface, originally introduced by Penrose as an outermost trapped surface in black hole formation by gravitational collapse

* Physics and Astronomy, Sejong University, 143-747 Seoul; mvp@sejong.ac.kr

[10]. In the face of particles described as waves in quantum field theory, Hawking radiation demonstrates that $\mathcal{H}$ is dispersive, allowing radiation to leak out by pair creation, effectively photons at super-horizon scales propagating back out to the celestial sphere [11]. However, this picture does not address the converse, how wave mechanics - in quantum mechanics or field theory - gives rise to spacetime curvature as described by general relativity (e.g. [12]). In particular, the vacuum of general relativity is classical, and does not account for the Hawking temperature of black holes or the Gibbons-Hawking temperature of $\mathcal{H}$ [13]. Perhaps the most striking discrepancy is that universal coupling to energy in classical general relativity runs counter to the UV-divergent bare cosmological constant $\Lambda_0$ [14].

And yet, spacetime - of black holes and matter alike - is sure to satisfy the third law of thermodynamics. It leads to the Bekenstein-Hawking entropy $S_H = (1/4)k_B N$ of black hole event horizons that, by unitarity, must lead to information retrieval over the time scale of evaporation in detailed Hawking radiation in photon emission one-by-one back to the celestial sphere [5,15]. Here, $k_B$ is Boltzmann's constant and $N = A_H/l_p^2$ denotes black hole surface area measured in Planck sized units $l_p^2$, $l_p = \sqrt{G\hbar/c^3}$, where $G$ is Newton's constant. The fact that $S$ scales with horizon area rather than black hole volume is significant. As information, though inaccessible during the lifetime of the black hole, it suggests a holographic principle, wherein the dimension of phase space is set by area of a boundary and entanglement entropy encodes a distribution of mass-energy within [5,6]. The vacuum within wave fronts, generally expanding at non-extremal entanglement, hereby acquires a stress-energy tensor with nonzero trace. *This outcome should be the same for the cosmological vacuum associated with $\mathcal{H}$.* In its present form, however, holography leaves open the detailed mechanism of inducing off-shell vacua from entanglement information.

*Cosmological vacuum.* In this Report, we derive the stress-energy tensor of the cosmological vacuum from evanescent waves across the cosmological horizon as an apparent horizon surface, resulting from tension between dispersive wave mechanics and $\mathcal{H}$ as a classical, compact surface in the geometric optics approximation. Explicit solutions for $H(z)$ in three-flat Friedmann-Robertson-Walker (FRW) cosmologies enable a direct confrontation with current observational data.

The dispersive nature of $\mathcal{H}$ arises from entanglement between the visible universe within and the universe unseen outside. This is described by transition amplitudes defined by the propagator of a particle

$$\langle B|A\rangle = e^{iS}, \tag{3}$$

between spacetime positions $A$ and $B$ as a function of the action $S = [\varphi]_A^B\,\xi(A,B)$, where $[\varphi]_A^B = \varphi(B) - \varphi(A)$ is the jump in Compton phase between $A$ and $B$ and $\xi(A,B) = \{1, i\}$ represents the local causal structure defined by light cones in the geometric optics approximation. For time-like separations, $\langle B, A\rangle$ is the Feynman phase factor $e^{iS}$, whereas for spacelike separations, $e^{-S}$ gives an exponentially small tunneling amplitude *of evanescent waves entering the light cone* [16].

By unitarity of the propagator, the probabilities $P_\pm$ of finding a particle outside or inside satisfy $P_- + P_+ \equiv 1$. As an identity, the exponentially small tunneling probability has an associated information $I_1$ on the light cone that encodes the presence of a particle inside [17]. For a particle of mass $m$ at the centre of a spherical wave front of radius $r$, $I_1 = 2\pi\varphi_C$, where $\varphi_C = kr$, where $k = 2\pi/\lambda_C$ is the wave number of the Compton wave length $\lambda_C = m\hbar/c$, where $\hbar$ is Planck's constant.

Entanglement entropy $I_1$ is a two-point correlation, visible as the Einstein area $A_E$ in strong gravitational lensing in the appearance of double images of point sources. For a lens of mass $m$ half-way between the source and the observer, Einstein ring in the source plane observed at a distance $2r$ has an area $A_E = \pi\left(2r\theta_E\right)^2 = 4I_1 l_p^2$. Generally, two images are observed at angles $\theta_{1,2}$, $\theta_1\theta_2 = \theta_E^2$, where $\theta_1$ is inside and $\theta_2$ is outside the Einstein ring seen at the Einstein angle $\theta_E = \sqrt{2R_s/r}$, $R_S = 2Gm/c^2$ [18]. The sphere of radius $r$ about $m$ hereby leaves a remaining phase space (measured by area) $A' = A - A_E = A\left(1 - \theta_E^2\right)$. Taken to a minimum at $r = R_S$, the result is a black hole event horizon for which $A_E$ saturates $A_H = 4\pi R_S^2$. In the limit $A_E = A_H$, the wave front is trapped with vanishing expansion by extremal entanglement entropy. For $r > R_S$, $A_E$ from $I_1$ leads to wave front regression

$$r\frac{dI_1}{dt} = I_1 - k_B^{-1} S_H \geq 0 \tag{4}$$

by wave regression at group velocity $dr/dt = 1 - A_E/A = 1 - k_B I_1/S_H$ (a perturbation of Huygen's principle by $\theta_E^2$) of null-geodesics in the Schwarzschild line-element in $(t, r, \vartheta, \varphi)$,

$$ds^2 = -(1-\theta_E^2)dt^2 + \frac{dr^2}{1-\theta_E^2} + r^2 d\vartheta^2 + r^2\sin^2\vartheta d\varphi^2 \tag{5}$$

wherein $m$ within appears in Gauss' law

$$aA = 4\pi G m, \tag{6}$$

from the associated surface gravity $a$.

*Vacuum stress-energy tensor.* Here, we show that the cosmological horizon induces dark energy and mass from dispersion by tunneling, described by a stress-energy tensor with nonzero trace derived from a fundamental frequency $\omega_0 = \sqrt{1-q}H$ defined by the equations of geodesic separation and surface gravity implied by the Gauss-Bonnet theorem [19]. Applied to a null-vector $k^b = (k^t, 0, k^\theta, 0)$ using local eigentime $\tau$ as an affine parameter, an azimuthal separation $u = r\delta\varphi$ satisfies

$$\ddot{u} + \omega^2 u = 0 \tag{7}$$

with $\omega^2 = R_{\vartheta\varphi\vartheta}{}^{\varphi} r^{-2} = 2m/r^3$, where the right hand side of the second is specific to (5). In spherical symmetry,

$\omega$ is the fundamental eigenfrequency of the wave front as a compact surface. According to (7), $\omega^2$ with twice the Gauss curvature of the wave front at a fixed time $\tau$, giving a dynamical surface gravity [20], here of the form

$$a = \frac{1}{2}\omega^2 r, \tag{8}$$

where $A$, $V$ refer to the area and volume of the wave front. As $a$ measures Gauss-curvature, (8) represents the Gauss-Bonnet theorem, inferring it from from internal geometry, here measured by geodesic separation.

By the above, super-horizon scale evanescent waves give rise to a stress-energy of the cosmological vacuum from time-like, off-shell behaviour $\mathcal{H}$ as a compact apparent horizon surface. It represents a mass-energy ccording to the quantum mechanical definition as a time rate-of-change of total phase, $d\Phi/dt = mc^2/\hbar$. By (8) and Gauss' law (6), it has a dark energy density

$$e = (1-q)\rho_c, \tag{9}$$

satisfying $\rho_T V_H = a_H A_H/4\pi$, where $V_H = (4\pi/3)R_H^3$ is the Hubble volume associated with the Hubble area $A_H = 4\pi R_H^2$ and $\rho_c = 3c^2H^2/8\pi G$ is the closure energy density with corresponding mass density $\rho_c/c^2 \simeq 10^{-29}\mathrm{g\,cm^{-3}}$ at present.

On the scale of Planck sized surface elements, $\omega_0$ is a fundamental mode by which super-horizon scale electromagnetic and gravitational radiation become dispersive, by an effective mass $\omega_0^2$, satisfying the dispersion relation

$$\omega = \sqrt{c^2k^2 + \omega_0^2} \tag{10}$$

for frequencies $\omega$ at wave numbers $k$. (As a wave front, $\mathcal{H}$ itself is hereby chromatic.) The cosmological vacuum assumes a dynamical dark energy

$$\Lambda = \omega_0^2, \tag{11}$$

given the common coupling of the electromagnetic vector field $A_a$ and the Riemann-Cartan connections $\omega_{a\mu\nu}$ to the Ricci tensor $R_{ab}$ [21]. A three-flat cosmology hereby acquires dark energy and matter densities $\Omega_\Lambda = (1-q)/3$, $\Omega_m = (2+q)/3$, the latter accompanied by a pressure $q = 3\Omega_p$ in satisfying the Friedmann energy and momentum equations. Crucially, this gives

$$q = 2q_{GR}, \tag{12}$$

showing a cosmological vacuum with nonzero trace stress-energy tensor

$$T_{ab} = (\rho_m + p)u_a u_b + p g_{ab} + \Lambda g_{ab}, \tag{13}$$

where $\rho_m = (1/3)(2+q)\rho_c$ with $u^b = (\partial_t)^b$.

Originating in evanescent wave propagation through $\mathcal{H}$, we identify $T_{ab}$ with on- and off-shell radiation tensors $\pi^\pm_{ab}$ of sub- and super-horizon scale waves, satisfying $\mathrm{tr}\,\pi^+_{ab} = 0$, $\mathrm{tr}\,\pi^-_{ab} = -2$ for the equations of state $p = \pm(\gamma-1)e$, $\gamma = 4/3$. Positive and negative pressures explicitly derive from (10) in the form $ck = \pm\sqrt{\omega^2 - \omega_0^2}$ from $\omega > \omega_0$ and, respectively, $\omega < \omega_0$, whereby the trace of the stress-energy tensor of radiation and evanescent waves is zero, respectively, nonzero. Thus, we write

**Table. 1.** Canonical cosmological states.

| $q$ | TYPE | $\rho_c^{-1}T_{ab}$ |
|---|---|---|
| 1 | Radiation dominated | $\pi^+_{ab}$ |
| $\frac{1}{2}$ | Matter dominated | $u_a u_b$ |
| 0 | Zero Hubble flow | $\pi^-_{ab}$ |
| $-1$ | de Sitter | $-g_{ab}$ |

$$T_{ab} = \rho_c\left[(1-q)\pi^-_{ab} + q\,\pi^+_{ab}\right]. \tag{14}$$

It supports the four canonical states listed in Table 1, where $u^b = (\partial_t)^b$.

*Hubble parameter* $H_0$. An explicit solution

$$H(z) = H_0\frac{\sqrt{1+\omega_m(6z+12z^2+12z^3+6z^4+(6/5)z^5)}}{1+z} \tag{15}$$

from the FRW equations of motion with (14) shows a relatively fast evolution in the Hubble parameter $H(z)$ around redshift zero: $H'(0) = H_0(3\omega_m - 1) \simeq 0$, distinct from $H'(0) = \frac{3}{2}\omega_m \simeq 0.5$ in $\Lambda$CDM [19]. Nonlinear regression over $H_0 = H(0)$ and present-day matter content $\omega_m = \Omega_m(0)$ against data $H(z)$ over an intermediate range of redshifts $z$ (see [22]) obtains

$$H_0 = 74.9 \pm 2.6\,\mathrm{km\,s^{-1}Mpc^{-1}} \tag{16}$$

with $\omega_m = 0.2719 \pm 0.028$, in tight agreement with $H_0 = 74.4 \pm 4.9\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ obtained by a model-independent cubic polynomial fit, $H(z) = H_0(1 + (1 + q_0)z + \frac{1}{2}(Q_0 + q_0(1+q_0))z^2 + b_3z^3 + \cdots)$ to the same data.[19] This result agrees with $H_0$ from surveys of the local Universe [18]. In contrast, analysis of the same data for $H(z)$ in $\Lambda$CDM gives $H_0 = 66.8 \pm 1.9\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, $\omega_m = 0.3330 \pm 0.040$. These results can be conveniently presented in a $qQ$-diagram (Fig. 2), where $q(z) = -1 + (1+z)H^{-1}(z)H'(z)$, $Q(z) = dq(z)/dz$ [23]. Models (14) and $\Lambda$CDM are distinct with gap between $Q_0 \gtrsim 2.5$ for the first and $Q_0 \lesssim 1$ for the second. Results for (14) and aforementioned cubic fit give consistent results $Q_0 = 2.37 \pm 0.073$ and, respectively, $Q_0 = 2.49 \pm 0.55$. Table 2 lists model fits to analytic solutions $H(z)$ parameterized by $(H_0, \omega_m)$ for (14), $\Lambda$CDM with $H(z) = H_0\sqrt{1-\omega_m+\omega_m(1+z)^3}$ and a recent proposal for scale-free cosmology with $H(z) = H_0\left(\omega_m(1+z)^{9/4} + (1-\omega_m)(1+z)^{3/4}\right)^{2/3}$ [24], along with model-independent analysis based on cubic and quartic polynomials.

At super-horizon scales, radiation is found to leak into the visible universe as evanescent waves, giving a nonzero trace to the cosmological vacuum observed as a combination of a cosmological distribution of dark energy and dark matter. This theory recovers the four canonical states of cosmological evolution (Table 1) and produces a Hubble parameter $H_0$ from cosmological data $H(z)$ in agreement with $H_0$ measured in local surveys (Table 2).

**Table 2.** Results for $(H_0, q_0, Q_0, \omega_m)$ with $1\sigma$ uncertainties by nonlinear model regression applied to polynomials and models and current data on $H_0$. $H_0$ is expressed in units of $\mathrm{km\,s^{-1}Mpc^{-1}}$.

| Model | $H_0$ | $q_0$ | $Q_0$ | $\omega_m$ | $h'(0)$ |
|---|---|---|---|---|---|
| Cubic | $74.4 \pm 4.9$ | $-1.17 \pm 0.34$ | $2.49 \pm 0.55$ | - | -0.17 |
| Quartic | $74.5 \pm 7.3$ | $-1.18 \pm 0.67$ | $2.54 \pm 1.99$ | | -0.18 |
| $\Lambda = \omega_0^2$ | $74.9 \pm 2.6$ | $-1.18 \pm 0.084$ | $2.75 \pm 0.15$ | $0.2719 \pm 0.028$ | -0.18 |
| $\Lambda$CDM | $66.8 \pm 1.9$ | $-0.50 \pm 0.060$ | $1.00 \pm 0.060$ | $0.3330 \pm 0.040$ | 0.50 |
| Scale-invariant | $62.0 \pm 1.6$ | $-0.16 \pm 0.056$ | $0.33 \pm 0.02$ | $0.3404 \pm 0.056$ | 0.84 |
| Observations | | Reference | | | |
| Local surveys | 73.06±1.76 | Anderson & Riess (2017) | | | |
| GW180817 | $75.5\pm\frac{11.6}{9.6}$ | Guidorzi et al. (2017) | | | |
| CMB | 66.93±0.62 | Planck Collaboration et al. (2016) | | | |

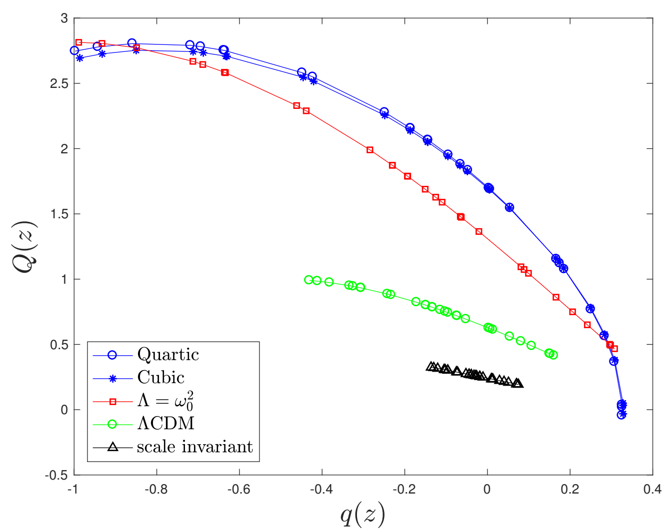


FIG. 2. $qQ$-diagram showing results of nonlinear model regression of model-independent polynomials and analytic solutions $H(z)$ for $\Lambda = \omega_0^2$, $\Lambda$CDM and scale-invariant cosmologies. The graphs shown are restricted to redshifts $z_{min} \leq z_k \lesssim 1$ within the radius of convergence of the Taylor series expansions about $z = 0$ of these three models, where $z_k$ are the ordinates with $z_{min} = 0.07$ in recently tabulated data [22].

In the $qQ$-diagram, the results are consistent with those obtained from model-independent analysis using polynomial fits, while $\Lambda$CDM is ruled out by $2.7\,\sigma$. The predicted relatively large value of $Q_0$ is characteristic for $H'(0) \simeq 0$ indicates an unstable de Sitter limit ($q = -1$), driven by entanglement of our visible universe with the global FRW spacetime.

*Acknowledgements.* This research is supported in part by the National Research Foundation of Korea (No. 2015R1D1A1A01059793 and 2016R1A5A1013277)

### References and Notes


[1] Famae, G., & McGaugh, S.S., 2012, Liv. Rev. Rel., 1; Freedman, W.L., 2017, Nat. Astron., 1, 0121

[2] Feynman, R.P., Moringo, F.B., & Wagner, W.G., 2003, *Feynman Lectures on Gravitation*, Westview Press, §7.1-2

[3] Will, C.M., 2014, Liv. Rev. Rel, 17, 1

[4] Abbott et al. 2017, Phys. Rev. Lett., 119, 161101; Guidorzi, C., Margutti, R., Brout, D., Scoling, D., Fong, W., et al., 2017, arXiv:1710.06426

[5] 't Hooft, G. 1993, arXiv:9310026; Susskind, L. 1995, JMP, 36, 6377;'t Hooft, G. 2015, IJMPD, 24, 1543001

[6] Jacobson, T., 1995, Phys. Rev. Lett., 75, 1260; E. P. Verlinde, 2011, JHEP 029; Jacobson, T., 2011, Phys. Rev. Lett., 116, 201101

[7] Planck Collaboration, Ade, P. A. R., Aghanim, N., et al. 2016, A&A, 594, A13

[8] Riess, A., et al., 1998, ApJ, 116, 1009; Perlmutter, S., et al., 1999, ApJ, 517, 565

[9] Riess, A.G., Macri, L.M., Hoffmann, S.L., Scolnic, D., Stefano, C., et al., 2016, ApJ, 826, 56; Anderson, R.I., & Riess, A.G., 2017, arXiv:1712.01065v1; Guidorzi, C., Margutti, R., Brout, D., et al., 2017, arXiv:1710.06426v2

[10] Penrose, R., 1965, Phys. Rev. Lett., 14, 57; Brewin, L., 1988, Phys. Rev. D, 38, 3020; York, J.W., 1989, in Frontiers in Numerical Relativity, ed. C.R. Evans, L.S. Finn & D.W. Hobill (Cambridge University Press); Wald, R.M., & Iyer, V., 1991, Phys. Rev. D, 44, R3719; Cook, G.B., & Abrahams, A.M., 1992, Phys. Rev. D, 46, 702; Cook, G.B., 2000, Liv. Rev. Rel., 3, 5; Thornburg, J., 2007, Liv. Rev. Rel., 10, 7

[11] Hawking, S., 1975, Commun. Math. Phys. 43, 199; Wald, R.M., Commun. Math. Phys., 1975, 45

[12] Copeland, E.J., Myers, R.C., & Polchinski, J., 2004, JHEP, 6, 13

[13] Gibbons, G. W., & Hawking, S. W. 1977, PhRvD, 15, 2738

[14] Tong, D., 2007, *Quantum Field Theory*, Univ. Cambridge Part III Mathematical Tripos

[15] Bekenstein, J.D., 1973, Phys. Rev. D 7, 2333; 't Hooft, G., 1996, IJMP-A, 11, 4623; Stephens, C.R., 't Hooft, G., Whiting, B.F., 1994, Class. Quant. Grav., 11, 621; Bekenstein, J.D., & Mukhanov, V.F., 1995, Phys. Lett. B, 360, 7; van Putten, M.H.P.M., 2015, arXiv:1506.08075

[16] Feynman, R.P., 1995, *The Theory of Fundamental Processes* (Avalon Publishing), Ch. 4, 18

[17] van Putten, M.H.P.M., 2015, IJMP-D, 24, 1550024. $I_1$ derives from information *intensity* $i_1 = \Delta\varphi_c/8\pi r$ in spin-2 geodesic deviations, analogous to radiation intensity in ray tracing of spin-1 electromagnetic radiation from $m$. For $m$ at the centre of a cube of side $l = 2r$ and area $A = 6l^2$, $i_1 = (1/4\pi)I_1 = (1/8\pi)kl_p^2/s$ derives from $I_1 = 12\Delta\varphi_C$ encoding $m$ by $2\Delta\varphi_C$ relative to six faces based on unitarity of its propagator. $I_1$ obtains by integration over $4\pi$ and the area of a sphere of radius $r$. Holographic phase space over a surface $A$ is hereby $\{\Omega_x | x\,\epsilon\, A\}$, regressed by $A_E$ visible in gravitational lensing.

[18] Einstein, A., 1936, Science, 84, 506

[19] van Putten, M.H.P.M., 2017, ApJ, 837, 22; van Putten, M.H.P.M., 2017, ApJ, 848, 28

[20] Kodama, H., 1980, Progr. Theor. Phys., 63, 1217; Hayward, S.A., 1998, Class. Quant. Grav., 15, 3147; Hayward, S.A., Mukohyama, S., & Ashworth, M.C., 1999, Phys. Lett. A, 256, 347; Bak, D., & Rey, S.-J., 2000, Class. Quant. Grav., 2000, L83; Cai, R.-G., & Kim, S.P., 2005, JHEP, 2, 50

[21] Wald, R.M., 1984, *General Relativity* (University of Chicago Press); van Putten, M.H.P.M., & Eardley, D.M., 1996, Phys. Rev. D., 53, 3056

[22] Farooq, O., Madiyar, F.R., Crandall, S., & Ratra, B., 2017, *ApJ*, **835**, 26

[23] Model results are $Q_0 = (9/2)\omega_m(1-\omega_m)$ for ΛCDM, $Q_0 = 3\omega_m(5-6\omega_m)$ for $\Lambda = \omega_0^2$, and $Q_0 = (3/2)\omega_m(1-\omega_m)$ for scale-invariant cosmologies recently proposed [24]

[24] Maeder, A., 2017, ApJ, 849, 194; Jesus, J.F., 2017, arXiv:1712.00697