Neutrosophic Resolvable and Neutrosophic Irresolvable Spaces

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ABSTRACT

In this paper, the concepts of neutrosophic resolvable, neutrosophic irresolvable, neutrosophic open hereditarily irresolvable spaces and maximally neutrosophic irresolvable spaces are introduced. Also we study several properties of the neutrosophic open hereditarily irresolvable spaces besides giving characterization of these spaces by means of somewhat neutrosophic continuous functions and somewhat neutrosophic open functions.

KEYWORDS: Neutrosophic resolvable, neutrosophic irresolvable, neutrosophic submaximal, neutrosophic open hereditarily irresolvable space, somewhat neutrosophic continuous and somewhat neutrosophic open functions.

1 INTRODUCTION

Zadeh (1965) introduced the important and useful concept of a fuzzy set which has invaded almost all branches of mathematics. The theory of fuzzy topological spaces was introduced and developed by Chang (1968) and since then various notions in classical topology have been extended to fuzzy topological spaces. The idea of "intuitionistic fuzzy set" was first published by Atanasov (1983) and some research works appeared in the literature (Atanassov (1986, 1988); Atanassov and Stoeva (1983)). Smarandache introduced the concepts of neutrosophy and neutrosophic set (Smarandache, (1999, 2002)). The concepts of neutrosophic crisp sets and neutrosophic crisp topological spaces were introduced by Salama and Alblowi (2012). The concept of fuzzy resolvable and fuzzy irresolvable spaces were introduced by G. Thangaraj and G.

Balasubramanian (2009). The concepts of resolvability and irresolvability in intuitionistic fuzzy topological spaces were introduced by Dhavaseelan et al. (2011).

In this paper, the concepts of neutrosophic resolvable, neutrosophic irresolvable, neutrosophic open hereditarily irresolvable spaces and maximally neutrosophic irresolvable spaces are introduced. Further, we study several interesting properties of the neutrosophic open hereditarily irresolvable spaces and present characterizations of these spaces by means of somewhat neutrosophic continuous functions and somewhat neutrosophic open functions. Some basic properties and related examples are given.

2 PRELIMINARIES

Definition 2.1. *(Smarandache, (1999, 2002)) Let T, I, F be real standard or non standard subsets of* $]0^-, 1^+[$, *with* $sup_T = t_{sup}, inf_T = t_{inf}$
$sup_I = i_{sup}, inf_I = i_{inf}$
$sup_F = f_{sup}, inf_F = f_{inf}$
$n - sup = t_{sup} + i_{sup} + f_{sup}$
$n - inf = t_{inf} + i_{inf} + f_{inf}$. *T, I, F are neutrosophic components.*

Definition 2.2. *(Smarandache, (1999, 2002)) Let X be a nonempty fixed set. A neutrosophic set A is an object having the form* $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$, *where* $\mu_A(x), \sigma_A(x)$ *and* $\gamma_A(x)$ *represents the degree of membership function (i.e.,* $\mu_A(x)$*), the degree of indeterminacy (namely* $\sigma_A(x)$*) and the degree of nonmembership (i.e.,* $\gamma_A(x)$*) of each element* $x \in X$ *to the set A, respectively.*

Remark 2.1. *(Smarandache, (1999, 2002))*

(1) *A neutrosophic set* $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ *can be identified to an ordered triple* $\langle \mu_A, \sigma_A, \gamma_A\rangle$ *in* $]0^-, 1^+[$ *on X.*

(2) *For the sake of simplicity, we shall use the symbol* $A = \langle \mu_A, \sigma_A, \gamma_A\rangle$ *for the neutrosophic set* $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$.

Definition 2.3. *(Salama and Alblowi (2012)) Let X be a nonempty set and the neutrosophic sets A and B be in the form*
$A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$, $B = \{\langle x, \mu_B(x), \sigma_B(x), \gamma_B(x)\rangle : x \in X\}$. *Then*

(a) $A \subseteq B$ *iff* $\mu_A(x) \leq \mu_B(x)$, $\sigma_A(x) \leq \sigma_B(x)$ *and* $\gamma_A(x) \geq \gamma_B(x)$ *for all* $x \in X$;

(b) $A = B$ *iff* $A \subseteq B$ *and* $B \subseteq A$;

(c) $\bar{A} = \{\langle x, \gamma_A(x), \sigma_A(x), \mu_A(x)\rangle : x \in X\}$; *[Complement of A]*

(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle : x \in X\}$;

(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle : x \in X\}$;

(f) $[\ ]A = \{\langle x, \mu_A(x), \sigma_A(x), 1 - \mu_A(x)\rangle : x \in X\}$;

(g) $\langle\rangle A = \{\langle x, 1 - \gamma_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$.

Definition 2.4. *(Salama and Alblowi (2012)) Let* $\{A_i : i \in J\}$ *be an arbitrary family of neutrosophic sets in X. Then*

(a) $\bigcap A_i = \{\langle x, \wedge\mu_{A_i}(x), \wedge\sigma_{A_i}(x), \vee\gamma_{A_i}(x)\rangle : x \in X\}$.

(b) $\bigcup A_i = \{\langle x, \vee\mu_{A_i}(x), \vee\sigma_{A_i}(x), \wedge\gamma_{A_i}(x)\rangle : x \in X\}$.

Definition 2.5. *(Salama and Alblowi (2012))* $0_N = \{\langle x, 0, 0, 1\rangle : x \in X\}$ *and* $1_N = \{\langle x, 1, 1, 0\rangle : x \in X\}$.

Definition 2.6. *(Dhavaseelan and S. Jafari (20xx)) A neutrosophic topology (NT) on a nonempty set* X *is a family* T *of neutrosophic sets in* X *satisfying the following axioms:*

(i) $0_N, 1_N \in T$,

(ii) $G_1 \cap G_2 \in T$ *for any* $G_1, G_2 \in T$,

(iii) $\cup G_i \in T$ *for arbitrary family* $\{G_i \mid i \in \Lambda\} \subseteq T$.

In this case, the ordered pair (X, T) *or simply* X *is called a neutrosophic topological space and each neutrosophic set in* T *is called a neutrosophic open set. The complement* $\overline{A}$ *of a neutrosophic open set* A *in* X *is called a neutrosophic closed set in* X.

Definition 2.7. [8] Let A be a neutrosophic set in a neutrosophic topological space X. Then
$Nint(A) = \bigcup\{G \mid G$ is a neutrosophic open set in X and $G \subseteq A\}$ is called the neutrosophic interior of A;
$Ncl(A) = \bigcap\{G \mid G$ is a neutrosophic closed set in X and $G \supseteq A\}$ is called the neutrosophic closure of A.

Definition 2.8. [7] An intuitionistic fuzzy topological space (X, T) is called intuitionistic fuzzy resolvable if there exists an intuitionistic fuzzy dense set A in (X, T) such that $IFcl(\overline{A}) = 1_\sim$. Otherwise (X, T) is called intuitionistic fuzzy irresolvable.

3 NEUTROSOPHIC RESOLVABLE AND NEUTROSOPHIC IRRESOLVABLE

Definition 3.1. *A neutrosophic set* A *in neutrosophic topological space* (X, T) *is called neutrosophic dense if there exists no neutrosophic closed set* B *in* (X, T) *such that* $A \subset B \subset 1_N$

Definition 3.2. *A neutrosophic topological space* (X, T) *is called neutrosophic resolvable if there exists a neutrosophic dense set* A *in* (X, T) *such that* $Ncl(\overline{A}) = 1_N$. *Otherwise* (X, T) *is called neutrosophic irresolvable.*

Example 3.1. Let $X = \{a, b, c\}$. Define the neutrosophic sets A, B and C as follows.

$$A = \langle x, (\tfrac{a}{0.6}, \tfrac{b}{0.6}, \tfrac{c}{0.5}), (\tfrac{a}{0.6}, \tfrac{b}{0.6}, \tfrac{c}{0.5}), (\tfrac{a}{0.3}, \tfrac{b}{0.3}, \tfrac{c}{0.5})\rangle,$$

$$B = \langle x, (\tfrac{a}{0.4}, \tfrac{b}{0.4}, \tfrac{c}{0.5}), (\tfrac{a}{0.4}, \tfrac{b}{0.4}, \tfrac{c}{0.5}), (\tfrac{a}{0.5}, \tfrac{b}{0.5}, \tfrac{c}{0.4})\rangle,$$

and

$$C = \langle x, (\tfrac{a}{0.3}, \tfrac{b}{0.3}, \tfrac{c}{0.4}), (\tfrac{a}{0.3}, \tfrac{b}{0.3}, \tfrac{c}{0.4}), (\tfrac{a}{0.7}, \tfrac{b}{0.7}, \tfrac{c}{0.6})\rangle.$$

Observe that $T = \{0_N, 1_N, A\}$ is a neutrosophic topology on X. Thus (X, T) is a neutrosophic topological space. Now $Nint(B) = 0_N, Nint(C) = 0_N, Nint(\overline{B}) = 0_N, Nint(\overline{C}) = A, Ncl(B) = 1_N, Ncl(C) = 1_N, Ncl(\overline{B}) = 1_N$ and $Ncl(\overline{C}) = \overline{A}$. Hence there exists a neutrosophic dense set B in (X, T) such that $Ncl(\overline{B}) = 1_N$. Therefore the neutrosophic topological space (X, T) is called a neutrosophic resolvable.

Example 3.2. Let $X = \{a, b, c\}$. Define the neutrosophic sets A, B and C as follows.

$$A = \langle x, (\tfrac{a}{0.6}, \tfrac{b}{0.5}, \tfrac{c}{0.5}), (\tfrac{a}{0.6}, \tfrac{b}{0.5}, \tfrac{c}{0.5}), (\tfrac{a}{0.4}, \tfrac{b}{0.5}, \tfrac{c}{0.5})\rangle,$$

$$B = \langle x, (\tfrac{a}{0.7}, \tfrac{b}{0.8}, \tfrac{c}{0.6}), (\tfrac{a}{0.7}, \tfrac{b}{0.8}, \tfrac{c}{0.6}), (\tfrac{a}{0.3}, \tfrac{b}{0.1}, \tfrac{c}{0.3})\rangle,$$

and

$$C = \langle x, (\tfrac{a}{0.6}, \tfrac{b}{0.5}, \tfrac{c}{0.5}), (\tfrac{a}{0.6}, \tfrac{b}{0.5}, \tfrac{c}{0.5}), (\tfrac{a}{0.4}, \tfrac{b}{0.4}, \tfrac{c}{0.4})\rangle.$$

It can be seen that $T = \{0_N, 1_N, A\}$ is a neutrosophic topology on X. Thus (X, T) is a neutrosophic topological space. Now $Nint(B) = A, Nint(C) = A, Ncl(B) = 1_N, Ncl(C) = 1_N$ and $Ncl(B) = 1_N$. Thus B and C are neutrosophic dense set in (X, T) such that $Ncl(\overline{B}) = \overline{A}$ and $Ncl(\overline{C}) = \overline{A}$. Hence the neutrosophic topological space (X, T) is called a neutrosophic irresolvable.

Proposition 3.1. A neutrosophic topological space (X, T) is a neutrosophic resolvable space iff (X, T) has a pair of neutrosophic dense set A_1 and A_2 such that $A_1 \subseteq \overline{A_2}$.

Proof. Let (X, T) be a neutrosophic topological space and (X, T) a neutrosophic resolvable space. Suppose that for all neutrosophic dense sets A_i and A_j, we have $A_i \nsubseteq \overline{A_j}$. Then $A_i \supset \overline{A_j}$. Then $Ncl(A_i) \supset Ncl(\overline{A_j})$ which implies that $1_N \supset Ncl(\overline{A_j})$. Then $Ncl(\overline{A_j}) \neq 1_N$. Also $A_j \supset \overline{A_i}$, then $Ncl(A_j) \supset Ncl(\overline{A_i})$ which implies that $1_N \supset Ncl(\overline{A_i})$. Therefore $Ncl(\overline{A_i}) \neq 1_N$. Hence $Ncl(A_i) = 1_N$, but $Ncl(\overline{A_i}) \neq 1_N$ for all neutrosophic set A_i in (X, T) which is a contradiction. Hence (X, T) has a pair of neutrosophic dense set A_1 and A_2 such that $A_1 \subseteq \overline{A_2}$.

Conversely, suppose that the neutrosophic topological space (X, T) has a pair of neutrosophic dense set A_1 and A_2 such that $A_1 \subseteq \overline{A_2}$. Suppose that (X, T) is a neutrosophic irresolvable space. Then for all neutrosophic dense sets A_1 and A_2 in (X, T), we have $Ncl(\overline{A_1}) \neq 1_N$. Then $Ncl(\overline{A_2}) \neq 1_N$ implies that there exists a neutrosophic closed set B in (X, T) such that $\overline{A_2} \subset B \subset 1_N$.Then $A_1 \subseteq \overline{A_2} \subset B \subset 1_N$ implies that $A_1 \subset B \subset 1_N$. But this is a contradiction. Hence (X, T) is a neutrosophic resolvable space. □

Proposition 3.2. If (X, T) is neutrosophic irresolvable iff $Nint(A) \neq 0_N$ for all neutrosophic dense set A in (X, T).

Proof. Since (X, T) is a neutrosophic irresolvable space for all neutrosophic dense set A in (X, T), $Ncl(\overline{A}) \neq 1_N$. Then $\overline{Nint(A)} \neq 1_N$ which implies $Nint(A) \neq 0_N$.

Conversely $Nint(A) \neq 0_N$, for all neutrosophic dense set A in (X, T). Suppose that (X, T) is neutrosophic resolvable. Then there exists a neutrosophic dense set A in (X, T) such that $Ncl(\overline{A}) = 1_N$. This implies that $\overline{Nint(A)} = 1_N$ which again implies $Nint(A) = 0_N$. But this is a contradiction. Hence (X, T) is neutrosophic irresolvable space. □

Definition 3.3. *A neutrosophic topological space (X, T) is called a neutrosophic submaximal space if for each neutrosophic set A in (X, T), $Ncl(A) = 1_N$.*

Proposition 3.3. If the neutrosophic topological space (X, T) is neutrosophic submaximal, then (X, T) is neutrosophic irresolvable.

Proof. Let (X, T) be a neutrosophic submaximal space. Assume that (X, T) is a neutrosophic resolvable space. Let A be a neutrosophic dense set in (X, T). Then $Ncl(\overline{A}) = 1_N$. Hence $\overline{Nint(A)} = 1_N$ which implies that $Nint(A) = 0_N$. Then $A \notin T$. This is a contradiction. Hence (X, T) is neutrosophic irresolvable space.

The converse of Proposition 3.3 is not true. See Example 3.2. □

Definition 3.4. *A neutrosophic topological space (X, T) is called a maximal neutrosophic irresolvable space if (X, T) is neutrosophic irresolvable and every neutrosophic dense set A of (X, T) is neutrosophic open.*

Example 3.3. Let $X = \{a, b, c\}$. Define the neutrosophic sets A,B,$A \cap B$ and $A \cup B$ as follows.

$$A = \langle x, (\tfrac{a}{0.5}, \tfrac{b}{0.4}, \tfrac{c}{0.5}), (\tfrac{a}{0.5}, \tfrac{b}{0.4}, \tfrac{c}{0.5}), (\tfrac{a}{0.4}, \tfrac{b}{0.4}, \tfrac{c}{0.4})\rangle,$$

$$B = \langle x, (\tfrac{a}{0.4}, \tfrac{b}{0.5}, \tfrac{c}{0.5}), (\tfrac{a}{0.4}, \tfrac{b}{0.5}, \tfrac{c}{0.5}), (\tfrac{a}{0.5}, \tfrac{b}{0.5}, \tfrac{c}{0.5})\rangle,$$

$$A \cap B = \langle x, (\tfrac{a}{0.4}, \tfrac{b}{0.4}, \tfrac{c}{0.5}), (\tfrac{a}{0.4}, \tfrac{b}{0.4}, \tfrac{c}{0.5}), (\tfrac{a}{0.5}, \tfrac{b}{0.5}, \tfrac{c}{0.5})\rangle,$$

and

$$A \cup B = \langle x, (\tfrac{a}{0.5}, \tfrac{b}{0.5}, \tfrac{c}{0.5}), (\tfrac{a}{0.5}, \tfrac{b}{0.5}, \tfrac{c}{0.5}), (\tfrac{a}{0.4}, \tfrac{b}{0.4}, \tfrac{c}{0.4})\rangle.$$

It is obvious that $T = \{0_N, 1_N, A, B, A \cap B, A \cup B\}$ is a neutrosophic topology on X. Thus (X, T) is a neutrosophic topological space. Now $Nint(\overline{A}) = 0_N$, $Nint(\overline{B}) = \bigcup\{0_N, B, A \cap B\} = B$, $Nint(\overline{A \cup B}) = 0_N$, $Nint(\overline{A \cap B}) = \bigcup\{0_N, B, A \cap B\} = B$ and $Ncl(A) = 1_N$,$Ncl(B) = \overline{B}$, $Ncl(A \cup B) = 1_N$, $Ncl(A \cap B) = \overline{B}$, $Ncl(\overline{A \cup B}) = \bigcap\{1_N, \overline{A \cup B}, \overline{B}, \overline{A \cap B}\} = \overline{A \cup B}$, $Ncl(\overline{A}) = \bigcap\{1_N, \overline{A}, \overline{A \cap B}\} = \overline{A}$, $Ncl(0_N) \neq 1_N$. Hence (X, T) is a neutrosophic irresolvable and every neutrosophic dense set of (X, T) is neutrosophic open. Therefore, (X, T) is a maximally neutrosophic irresolvable space.

4 NEUTROSOPHIC OPEN HEREDITARILY IRRESOLVABLE

Definition 4.1. *(X, T) is said to be neutrosophic open hereditarily irresolvable if $Nint(Ncl(A)) \neq 0_N$ and $Nint(A) \neq 0_N$, for any neutrosophic set A in (X, T).*

Example 4.1. Let $X = \{a, b, c\}$. Define the neutrosophic sets A_1, A_2 and A_3 as follows.

$$A_1 = \langle x, (\tfrac{a}{0.4}, \tfrac{b}{0.4}, \tfrac{c}{0.4}), (\tfrac{a}{0.4}, \tfrac{b}{0.4}, \tfrac{c}{0.4}), (\tfrac{a}{0.5}, \tfrac{b}{0.5}, \tfrac{c}{0.5})\rangle,$$

$$A_2 = \langle x, (\tfrac{a}{0.6}, \tfrac{b}{0.5}, \tfrac{c}{0.4}), (\tfrac{a}{0.6}, \tfrac{b}{0.5}, \tfrac{c}{0.4}), (\tfrac{a}{0.4}, \tfrac{b}{0.5}, \tfrac{c}{0.4})\rangle,$$

and

$$A_3 = \langle x, (\tfrac{a}{0.4}, \tfrac{b}{0.4}, \tfrac{c}{0.5}), (\tfrac{a}{0.4}, \tfrac{b}{0.4}, \tfrac{c}{0.5}), (\tfrac{a}{0.4}, \tfrac{b}{0.4}, \tfrac{c}{0.5})\rangle.$$

Clearly $T = \{0_N, 1_N, A_1, A_2\}$ is a neutrosophic topology on X. Thus (X,T) is a neutrosophic topological space. Now $Ncl(A_1) = \overline{A_1}; Ncl(A_2) = 1_N$ and $Nint(A_3) = A_1$. Also $Nint(Ncl(A_1)) = Nint(\overline{A_1}) = \overline{A_1} \neq 0_N$ and $Nint(A_1) = A_1 \neq 0_N$, $Nint(Ncl(A_2)) = Nint(1_N) = 1_N \neq 0_N$ and $Nint(A_2) = A_2 \neq 0_N$, $Nint(Ncl(A_3)) = Nint(\overline{A_1}) = \overline{A_1} \neq 0_N$ and $Nint(A_3) = A_1 \neq 0_N$ and $Nint(Ncl(\overline{A_3})) = Nint(\overline{A_1}) = \overline{A_1} \neq 0_N$ and $Nint(\overline{A_3}) = A_1 \neq 0_N$. Hence if $Nint(Ncl(A)) \neq 0_N$, then $Nint(A) \neq 0_N$ for any non zero neutrosophic set A in (X,T). Thus, (X,T) is a neutrosophic open hereditarily irresolvable space.

Proposition 4.1. Let (X,T) be a neutrosophic topological space. If (X,T) is neutrosophic open hereditarily irresolvable, then (X,T) is neutrosophic irresolvable

Proof. Let A be a neutrosophic dense set in (X,T). Then $Ncl(A) = 1_N$ which implies that $Nint(Ncl(A)) = 1_N \neq 0_N$. Since (X,T) is neutrosophic open hereditarily irresolvable, we have $Nint(A) \neq 0_N$. Therefore by Proposition 3.2 $Nint(A) \neq 0_N$ for all neutrosophic dense set in (X,T) implies that (X,T) is neutrosophic irresolvable.

The converse of Proposition 4.1 is not true. See Example 4.2 □

Example 4.2. Let $X = \{a,b,c\}$. Define the neutrosophic sets A, B and C as follows.

$$A = \langle x, (\tfrac{a}{0.3}, \tfrac{b}{0.3}, \tfrac{c}{0.4}), (\tfrac{a}{0.3}, \tfrac{b}{0.3}, \tfrac{c}{0.4}), (\tfrac{a}{0.5}, \tfrac{b}{0.5}, \tfrac{c}{0.5})\rangle,$$

$$B = \langle x, (\tfrac{a}{0.4}, \tfrac{b}{0.5}, \tfrac{c}{0.4}), (\tfrac{a}{0.4}, \tfrac{b}{0.5}, \tfrac{c}{0.4}), (\tfrac{a}{0.4}, \tfrac{b}{0.4}, \tfrac{c}{0.4})\rangle,$$

and

$$C = \langle x, (\tfrac{a}{0.4}, \tfrac{b}{0.4}, \tfrac{c}{0.4}), (\tfrac{a}{0.4}, \tfrac{b}{0.4}, \tfrac{c}{0.4}), (\tfrac{a}{0.3}, \tfrac{b}{0.3}, \tfrac{c}{0.3})\rangle.$$

It is obvious that $T = \{0_N, 1_N, A, B\}$ is a neutrosophic topology on X. Thus (X,T) is a neutrosophic topological space. Now C and 1_N are neutrosophic dense sets in (X,T). Then $Nint(C) = A \neq 0_N$ and $Nint(1_N) \neq 0_N$. Hence (X,T) is a neutrosophic irresolvable. But $Nint(Ncl(\overline{C})) = Nint(\overline{A}) = A \neq 0_N$ and $Nint(\overline{C}) = 0_N$. Therefore, (X,T) is not a neutrosophic open hereditarily irresolvable space.

Proposition 4.2. Let (X,T) be a neutrosophic open hereditarily irresolvable. Then $Nint(A) \nsubseteq \overline{Nint(B)}$ for any two neutrosophic dense sets A and B in (X,T).

Proof. Let A and B be any two neutrosophic dense sets in (X,T). Then $Ncl(A) = 1_N$ and $Ncl(B) = 1_N$ implies that $Nint(Ncl(A)) \neq 0_N$ and $Nint(Ncl(B)) \neq 0_N$. Since (X,T) is neutrosophic open hereditarily irresolvable, $Nint(A) \neq 0_N$ and $Nint(B) \neq 0_N$. Hence by Proposition 3.1, $A \nsubseteq \overline{B}$. Therefore $Nint(A) \subseteq A \nsubseteq \overline{B} \subseteq \overline{Nint(B)}$. Hence we have $Nint(A) \subseteq \overline{Nint(B)}$ for any two neutrosophic dense sets A and B in (X,T). □

Proposition 4.3. Let (X,T) be a neutrosophic topological space. If (X,T) is neutrosophic open hereditarily irresolvable, then $Nint(A) = 0_N$ for any nonzero neutrosophic dense set A in (X,T) which implies that $Nint(Ncl(A)) = 0_N$.

Proof. Let A be a neutrosophic set in (X,T) such that $Nint(A) = 0_N$. We claim that $Nint(Ncl(A)) = 0_N$. Suppose that $Nint(Ncl(A)) = 0_N$. Since (X,T) is neutrosophic open hereditarily irresolvable, we have $Nint(A) \neq 0_N$ which is a contradiction to $Nint(A) = 0_N$. Hence $Nint(Ncl(A)) = 0_N$. □

Proposition 4.4. Let (X, T) be a neutrosophic topological space. If (X, T) is neutrosophic open hereditarily irresolvable, then $Ncl(A) = 1_N$ for any nonzero neutrosophic dense set A in (X, T) which implies that $Ncl(Nint(A)) = 0_N$.

Proof. Let A be a neutrosophic set in (X, T) such that $Ncl(A) = 1_N$. Then we have $\overline{Ncl(A)} = 0_N$ which implies that $Nint(\overline{A}) = 0_N$. Since (X, T) is neutrosophic open hereditarily irresolvable by Proposition 4.3. We have $Nint(Ncl(\overline{A})) = 0_N$. Therefore $\overline{Ncl(Nint(A))} = 0_N$ implies that $Ncl(Nint(A)) = 1_N$. □

5 SOMEWHAT NEUTROSOPHIC CONTINUOUS AND SOMEWHAT NEUTROSOPHIC OPEN

Definition 5.1. *Let (X, T) and (Y, S) be any two neutrosophic topological spaces. A function $f : (X, T) \to (Y, S)$ is called somewhat neutrosophic continuous if for $A \in S$ and $f^{-1}(A) \neq 0_N$, there exists a $B \in T$ such that $B \neq 0_N$ and $B \subseteq f^{-1}(A)$.*

Definition 5.2. *Let (X, T) and (Y, S) be any two neutrosophic topological spaces. A function $f : (X, T) \to (Y, S)$ is called somewhat neutrosophic open if for $A \in T$ and $A \neq 0_N$, there exists a $B \in S$ such that $B \neq 0_N$ and $B \subseteq f(A)$.*

Proposition 5.1. Let (X, T) and (Y, S) be any two neutrosophic topological spaces. If the function $f : (X, T) \to (Y, S)$ is somewhat neutrosophic continuous and injective. If $Nint(A) = 0_N$ for any nonzero neutrosophic set A in (X, T), then $Nint(f(A)) = 0_N$ in (Y, S).

Proof. Let A be a nonzero neutrosophic set in (X, T) such that $Nint(A) = 0_N$. Now we prove that $Nint(f(A)) = 0_N$. Suppose that $Nint(f(A)) \neq 0_N$ in (Y, S). Then there exists a nonzero neutrosophic set B in (Y, S) such that $B \subseteq f(A)$. Thus, we have $f^{-1}(B) \subseteq f^{-1}(f(A))$. Since f is somewhat neutrosophic continuous, there exists a $C \in T$ such that $C \neq 0_N$ and $C \subseteq f^{-1}(B)$. Hence $C \subseteq f^{-1}(B) \subseteq A$ which implies that $Nint(A) \neq 0_N$. This is a contradiction. Hence $Nint(f(A)) = 0_N$ in (Y, S). □

Proposition 5.2. Let (X, T) and (Y, S) be any two neutrosophic topological spaces. If the function $f : (X, T) \to (Y, S)$ is somewhat neutrosophic continuous, injective and $Nint(Ncl(A)) = 0_N$ for any nonzero neutrosophic set A in (X, T), then $Nint(Ncl(f(A))) = 0_N$ in (Y, S).

Proof. Let A be a nonzero neutrosophic set in (X, T) such that $Nint(Ncl(A)) = 0_N$. We claim that $Nint(Ncl(f(A))) = 0_N$ in (Y, S). Suppose that $Nint(Ncl(f(A))) \neq 0_N$ in (Y, S). Then $Ncl(f(A)) \neq 0_N$ and $\overline{Ncl(f(A))} \neq 0_N$. Now $\overline{Ncl(f(A))} \neq 0_N \in S$. Since f is somewhat neutrosophic continuous, there exists a $B \in T$, such that $B \neq 0_N$ and $B \subseteq f^{-1}(\overline{Ncl(f(A))})$. Observe that $B \subseteq \overline{f^{-1}(Ncl(f(A)))}$ which implies that $f^{-1}(Ncl(f(A))) \subseteq \overline{B}$. Since f is injective, thus $A \subseteq f^{-1}(f(A) \subseteq f^{-1}(Ncl(f(A))) \subseteq \overline{B}$ which implies that $A \subseteq \overline{B}$. Therefore $B \subseteq \overline{A}$. This implies that $Nint(\overline{A}) \neq 0_N$. Let $Nint(\overline{A}) = C \neq 0_N$. Then we have $Ncl(Nint(\overline{A})) = Ncl(C) \neq 1_N$ which implies that $Nint(Ncl(A)) \neq 0_N$. But this is a contradiction. Hence $Nint(Ncl(f(A))) = 0_N$ in (Y, S). □

Proposition 5.3. Let (X,T) and (Y,S) be any two neutrosophic topological spaces. If the function $f : (X,T) \to (Y,S)$ is somewhat neutrosophic open and $Nint(A) = 0_N$ for any nonzero neutrosophic set A in (Y,S), then $Nint(f^{-1}(A)) = 0_N$ in (X,T).

Proof. Let A be a nonzero neutrosophic set in (Y,S) such that $Nint(A) = 0_N$. We claim that $Nint(f^{-1}(A)) = 0_N$ in (X,T). Suppose that $Nint(f^{-1}(A)) \neq 0_N$ in (X,T). Then there exists a nonzero neutrosophic open set B in (X,T) such that $B \subseteq f^{-1}(A)$. Thus, we have $f(B) \subseteq f(f^{-1}(A)) \subseteq A$. This implies that $f(B) \subseteq A$. Since f is somewhat neutrosophic open, there exists a $C \in S$ such that $C \neq 0_N$ and $C \subseteq f(B)$. Therefore $C \subseteq f(B) \subseteq A$ which implies that $C \subseteq A$. Hence $Nint(A) \neq 0_N$ which is a contradiction. Hence $Nint(f^{-1}(A)) = 0_N$ in (X,T). □

Proposition 5.4. Let (X,T) and (Y,S) be any two neutrosophic topological spaces. Let (X,T) be a neutrosophic open hereditarily irresolvable space. If $f : (X,T) \to (Y,S)$ is somewhat neutrosophic open, somewhat neutrosophic continuous and a bijective function, then (Y,S) is a neutrosophic open hereditarily space.

Proof. Let A be a nonzero neutrosophic set in (Y,S) such that $Nint(A) = 0_N$. Now $Nint(A) = 0_N$ and f is somewhat neutrosophic open which implies $Nint(f^{-1}(A)) = 0_N$ in (X,T) by Proposition 5.3. Since (X,T) is a neutrosophic open hereditarily irresolvable space, we have $Nint(Ncl(f^{-1}(A))) = 0_N$ in (X,T) by Proposition 4.3. Since $Nint(Ncl(f^{-1}(A))) = 0_N$ and f is somewhat neutrosophic continuous by Proposition 5.2, we have that $Nint(Ncl(f(f^{-1}(A)))) = 0_N$. Since f is onto, thus $NintNcl(A) = 0_N$. Hence by Proposition 4.3. (Y,S) is a neutrosophic open hereditarily irresolvable space. □

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