Generalized Neutrosophic Contra-Continuity

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ABSTRACT

In this paper, the concepts of generalized neutrosophic contra-continuous function, generalized neutrosophic contra-irresolute function and strongly generalized neutrosophic contra-continuous function are introduced. Some interesting properties are also studied.

KEYWORDS: Generalized neutrosophic contra-continuity, strongly generalized neutrosophic contra-continuity, generalized neutrosophic contra-irresolute.

1 INTRODUCTION

The notion of a fuzzy set has influenced almost all branches of mathematics since its introduction by Zadeh (1965). Fuzzy sets have applications in many fields such as information theory (Smets, 1981) and control theory (Sugeno, 1985). The theory of fuzzy topological space was introduced and developed by Chang (1968) and since then various notions in classical topology have been extended to fuzzy topological space. The idea of "intuitionistic fuzzy set" was first published by Atanassov (1983) and many works by the same author and his colleagues appeared in the literature (Atanassov (1986, 1988); Atanassov and Stoeva (1983)). Later, this concept was generalized to "intuitionistic L–fuzzy sets" by Atanassov and Stoeva (1984). The concepts of "fuzzy contra-continuity" was introduced by Ekici and Kerre (2006). The concepts of generalized intuitionistic fuzzy closed set was introduced by Dhavaseelan et al. (2010) and also discussed contra-continuity (Dhavaseelan et al. (2012)). After the introduction of the concepts of neutrosophy and neutrosophic set by Smarandache (1999, 2000), the concepts of neutrosophic crisp sets and neutrosophic crisp topological spaces were introduced by Salama and Alblowi (2012).

In this paper, the concepts of generalized neutrosophic contra-continuous function, generalized neutrosophic contra-irresolute function and strongly generalized neutrosophic contra-continuous function are introduced by using the concept studied in (Dhavaseelan et al. (20xx)). Several interesting properties and characterizations are discussed. Further, interrelations among the concepts introduced are established with interesting counter examples.

2 NEUTROSOPHIC TOPOLOGY

Definition 2.1. Let T,I,F be real standard or non standard subsets of $]0^-, 1^+[$, with
$sup_T = t_{sup}, inf_T = t_{inf}$
$sup_I = i_{sup}, inf_I = i_{inf}$
$sup_F = f_{sup}, inf_F = f_{inf}$
$n - sup = t_{sup} + i_{sup} + f_{sup}$
$n - inf = t_{inf} + i_{inf} + f_{inf}$. T,I,F are neutrosophic components.

Definition 2.2. Let X be a nonempty fixed set. A neutrosophic set [NS for short] A is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$, where $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x)$ which represents the degree of membership function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) respectively of each element $x \in X$ to the set A.

Remark 2.1.

(1) A neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ can be identified to an ordered triple $\langle \mu_A, \sigma_A, \gamma_A\rangle$ in $]0^-, 1^+[$ on X.

(2) For the sake of simplicity, we shall use the symbol $A = \langle \mu_A, \sigma_A, \gamma_A\rangle$ for the neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$.

Definition 2.3. Let X be a nonempty set and the neutrosophic sets A and B in the form
$A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$, $B = \{\langle x, \mu_B(x), \sigma_B(x), \gamma_B(x)\rangle : x \in X\}$. Then

(a) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$, $\sigma_A(x) \leq \sigma_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$;

(b) $A = B$ iff $A \subseteq B$ and $B \subseteq A$;

(c) $\bar{A} = \{\langle x, \gamma_A(x), \sigma_A(x), \mu_A(x)\rangle : x \in X\}$; [Complement of A]

(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle : x \in X\}$;

(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle : x \in X\}$;

(f) $[\,]A = \{\langle x, \mu_A(x), \sigma_A(x), 1 - \mu_A(x)\rangle : x \in X\}$;

(g) $\langle\rangle A = \{\langle x, 1 - \gamma_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$.

Definition 2.4. Let $\{A_i : i \in J\}$ be an arbitrary family of neutrosophic sets in X. Then

(a) $\bigcap A_i = \{\langle x, \wedge\mu_{A_i}(x), \wedge\sigma_{A_i}(x), \vee\gamma_{A_i}(x)\rangle : x \in X\}$;

(b) $\bigcup A_i = \{\langle x, \vee\mu_{A_i}(x), \vee\sigma_{A_i}(x), \wedge\gamma_{A_i}(x)\rangle : x \in X\}$.

Since our main purpose is to construct the tools for developing neutrosophic topological spaces, we must introduce the neutrosophic sets 0_N and 1_N in X as follows:

Definition 2.5. $0_N = \{\langle x, 0, 0, 1\rangle : x \in X\}$ and $1_N = \{\langle x, 1, 1, 0\rangle : x \in X\}$.

Definition 2.6. [9] A neutrosophic topology (NT) on a nonempty set X is a family T of neutrosophic sets in X satisfying the following axioms:

(i) $0_N, 1_N \in T$,

(ii) $G_1 \cap G_2 \in T$ for any $G_1, G_2 \in T$,

(iii) $\cup G_i \in T$ for arbitrary family $\{G_i \mid i \in \Lambda\} \subseteq T$.

In this case the ordered pair (X, T) or simply X is called a neutrosophic topological space (NTS) and each neutrosophic set in T is called a neutrosophic open set (NOS). The complement $\overline{A}$ of a NOS A in X is called a neutrosophic closed set (NCS) in X.

Definition 2.7. [9] Let A be a neutrosophic set in a neutrosophic topological space X. Then

$Nint(A) = \bigcup\{G \mid G$ is a neutrosophic open set in X and $G \subseteq A\}$ is called the neutrosophic interior of A;

$Ncl(A) = \bigcap\{G \mid G$ is a neutrosophic closed set in X and $G \supseteq A\}$ is called the neutrosophic closure of A.

Definition 2.8. Let X be a nonempty set. If r, t, s are real standard or non standard subsets of $]0^-, 1^+[$ then the neutrosophic set $x_{r,t,s}$ is called a neutrosophic point(in short NP)in X given by

$$x_{r,t,s}(x_p) = \begin{cases} (r,t,s), & \text{if } x = x_p \\ (0,0,1), & \text{if } x \neq x_p \end{cases}$$

For $x_p \in X$, it is called the support of $x_{r,t,s}$, where r denotes the degree of membership value , t denotes the degree of indeterminacy and s is the degree of non-membership value of $x_{r,t,s}$.

3 GENERALIZED NEUTROSOPHIC CONTRA-CONTINUOUS FUNCTIONS

Definition 3.1. Let (X,T) and (Y,S) be any two neutrosophic topological spaces.

(i) A function $f : (X,T) \to (Y,S)$ is called neutrosophic contra-continuous if the inverse image of every neutrosophic open set in (Y,S) is a neutrosophic closed set in (X,T) .

Equivalently if the inverse image of every neutrosophic closed set in (Y,S) is a neutrosophic open set in (X,T) .

(ii) A function $f : (X,T) \to (Y,S)$ is called generalized neutrosophic contra-continuous if the inverse image of every neutrosophic open set in (Y,S) is a generalized neutrosophic closed set in (X,T) .

Equivalently if the inverse image of every neutrosophic closed set in (Y,S) is a generalized neutrosophic open set in (X,T) .

(iii) A function $f : (X,T) \to (Y,S)$ is called generalized neutrosophic contra-irresolute if the inverse image of every generalized neutrosophic closed set in (Y,S) is a generalized neutrosophic open set in (X, T).

Equivalently if the inverse image of every generalized neutrosophic open set in (Y, S) is a generalized neutrosophic closed set in (X, T).

(iv) A function $f : (X, T) \to (Y, S)$ is called strongly generalized neutrosophic contra-continuous if the inverse image of every generalized neutrosophic open set in (Y, S) is a neutrosophic closed set in (X, T).

Equivalently if the inverse image of every generalized neutrosophic closed set in (Y, S) is a neutrosophic open set in (X, T).

Proposition 3.1. Let $f : (X, T) \to (Y, S)$ be a bijective function. Then f is a generalized neutrosophic contra-continuous function if $Ncl(f(A)) \subseteq f(NGint(A))$ for every neutrosophic set A in (X, T).

Proof. Let A be a neutrosophic closed set in (Y, S). Then $Ncl(A) = A$ and $f^{-1}(A)$ is a neutrosophic set in (X, T). By hypothesis, $Ncl(f(f^{-1}(A))) \subseteq f(NGint(f^{-1}(A)))$. Since f is onto, $f(f^{-1}(A)) = A$. Therefore, $A = Ncl(A) = Ncl(f(f^{-1}(A))) \subseteq f(NGint(f^{-1}(A)))$. Now, $A \subseteq f(NGint(f^{-1}(A)))$, $f^{-1}(A) \subseteq f^{-1}(f\ (NGint(f^{-1}(A)))) = NGint(f^{-1}(A)) \subseteq f^{-1}(A)$. Hence, $f^{-1}(A)$ is a generalized neutrosophic open set in (X, T). Thus, f is a generalized neutrosophic contra-continuous function. □

Proposition 3.2. Let (X, T) and (Y, S) be any two neutrosophic topological spaces. Let $f : (X, T) \to (Y, S)$ be a function. Suppose that one of the following properties hold.

(i) $f(NGcl(A)) \subseteq Nint(f(A))$, for each neutrosophic set A in (X, T).

(ii) $NGcl(f^{-1}(B)) \subseteq f^{-1}(Nint(B))$, for each neutrosophic set B in (Y, S).

(iii) $f^{-1}(Ncl(B)) \subseteq NGint(f^{-1}(B))$, for each neutrosophic set B in (Y, S).
Then f is a generalized neutrosophic contra-continuous function.

Proof. **(i)$\Rightarrow$ (ii)** Let B be a neutrosophic set in (Y, S), then $A = f^{-1}(B)$ is a neutrosophic set in (X, T). By hypothesis, $f(NGcl(A)) \subseteq Nint(f(A))$, $f(NGcl(f^{-1}(B)) \subseteq Nint(f(f^{-1}(B))) \subseteq Nint(B)$. Now, $f(NGcl(f^{-1}(B))) \subseteq Nint(B)$. Therefore, $NGcl(f^{-1}(B)) \subseteq f^{-1}(Nint(B))$.

(ii)⇒ (iii) Let B be a neutrosophic set in (Y, S), then $f^{-1}(B)$ is a neutrosophic set in (X, T). By hypothesis, $NGcl(f^{-1}(B)) \subseteq f^{-1}(Nint(B))$. Taking complement $\overline{NGcl(f^{-1}(B))} \supseteq \overline{f^{-1}(Nint(B))}$, $NGint(\overline{f^{-1}(B)}) \supseteq f^{-1}(\overline{Nint(B)})$, $NGint(f^{-1}(\overline{B})) \supseteq f^{-1}(Ncl(\overline{B}))$.

Suppose that **(iii)** holds. Let A be a neutrosophic closed set in (Y, S). Then $Ncl(A) = A$ and $f^{-1}(A)$ is a neutrosophic set in (X, T). Now, $f^{-1}(A) = f^{-1}(Ncl(A)) \subseteq NGint(f^{-1}(A)) \subseteq f^{-1}(A)$. Therefore, $f^{-1}(A)$ is a generalized neutrosophic open set in (X, T). Thus, f is a generalized neutrosophic contra-continuous function. □

Proposition 3.3. Let (X, T) and (Y, S) be any two neutrosophic topological spaces. Let $f : (X, T) \to (Y, S)$ be a function. Suppose that one of the following properties hold.

(i) $f^{-1}(NGcl(B)) \subseteq NGint(NGcl(f^{-1}(B))$ for each neutrosophic set B in (Y, S).

(ii) $NGcl(NGint(f^{-1}(B))) \subseteq f^{-1}(NGint(B))$ for each neutrosophic set B in (Y, S).

(iii) $f(NGcl(NGint(A))) \subseteq NGint(f(A))$ for each neutrosophic set A in (X, T).

(iv) $f(NGcl(A)) \subseteq NGint(f(A))$ for each neutrosophic set A in (X, T).

Then f is a generalized neutrosophic contra-continuous function.

Proof. **(i)⇒ (ii)** Let B be a neutrosophic set in (Y, S). Then $f^{-1}(B)$ is a neutrosophic set in (X, T). By hypothesis,$f^{-1}(NGcl(B)) \subseteq NGint(\ NGcl(f^{-1}(B)))$. Taking complement $\overline{f^{-1}(NGcl(B))} \supseteq \overline{NGint(NGcl(f^{-1}(B)))}$, $f^{-1}(\overline{NGcl(B)}) \supseteq NGcl(\overline{NGcl(f^{-1}(B))})$, $f^{-1}(NGint(\overline{B})) \supseteq NGcl(NGint(\ \overline{f^{-1}(B)}))$, $f^{-1}(NGint(\overline{B})) \supseteq NGcl(NGint(f^{-1}(\overline{B})))$. Thus, $NGcl(NGint(f^{-1}(\overline{B}))) \subseteq f^{-1}(NGint(\overline{B}))$.

(ii)⇒ (iii) Let A be a neutrosophic set in (X, T). Put $B = f(A)$, then $A \subseteq f^{-1}(B)$. By hypothesis, $NGcl(NGint(A)) \subseteq$
$NGcl(NGint(f^{-1}(B))) \subseteq f^{-1}(NGint(B))$, $NGcl(NGint(A)) \subseteq f^{-1}(NGint(B))$. Therefore, $f(NGcl(\ NGint(A))) \subseteq NGint(B) = NGint(f(A))$. This means that $f(NGcl(NGint(A)))$ $\subseteq NGint(f(A))$.

(iii)⇒ (iv) Let A be any generalized neutrosophic open set of (X, T). Then $NGint(A) = A$. By hypothesis, $f(NGcl(A)) = f(NGcl(NGint(A))) \subseteq NGint(f(A))$. Thus, $f(NGcl(A)) \subseteq NGint(f(A))$.

Suppose that **(iv)** holds. Let B be a neutrosophic open set in (Y, S). Then, $f^{-1}(B) = A$ is a neutrosophic set in (X, T). By hypothesis, $f(NGcl(A)) \subseteq NGint(f(A))$. Now, $f(NGcl(A)) \subseteq NGint(f(A)) \subseteq f(A)$, $f(NGcl(A)) \subseteq f(A)$, $NGcl(A) \subseteq f^{-1}(f(A)) = A$. This means that $NGcl(A) \subseteq A$. But $A \subseteq NGcl(A)$. Hence $A = NGcl(A)$. Thus, A is a generalized neutrosophic closed set in (X, T). Hence, f is a generalized neutrosophic contra-continuous function. □

Proposition 3.4. Let (X, T) and (Y, S) be any two neutrosophic topological spaces. If $f : (X, T) \to (Y, S)$ is a neutrosophic contra-continuous function then it is a generalized neutrosophic contra-continuous function.

Proof. Let A be a neutrosophic open set in (Y, S). Since f is a neutrosophic contra-continuous function, $f^{-1}(A)$ is a neutrosophic closed set in (X, T). Every neutrosophic closed set is a generalized neutrosophic closed set. Now, $f^{-1}(A)$ is a generalized neutrosophic closed set. Hence, f is a generalized neutrosophic contra-continuous function.

The converse of Proposition 3.4., need not be true. See Example 3.1. □

Example 3.1. Let $X = \{a, b, c\}$. Define the neutrosophic sets A and B in X as follows:
$A = \langle x, (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.4}), (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.4}), (\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.4})\rangle$, and
$B = \langle x, (\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.3}), (\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.3}), (\frac{a}{0.5}, \frac{b}{0.6}, \frac{c}{0.7})\rangle$.
Then the families $T = \{0_N, 1_N, A\}$ and $S = \{0_N, 1_N, B\}$ are neutrosophic topologies on X. Thus, (X, T) and (X, S) are neutrosophic topological spaces. Define $f : (X, T) \to (X, S)$ by $f(a) = b, f(b) = a, f(c) = c$. Then f is a generalized neutrosophic contra-continuous function. Now, $f^{-1}(B)$ is not a neutrosophic closed set in (X, T) for $B \in S$. Hence, f is not a neutrosophic contra-continuous function.

Proposition 3.5. Let (X, T) and (Y, S) be any two neutrosophic topological spaces. If $f : (X, T) \to (Y, S)$ is a generalized neutrosophic contra-irresolute function then it is a generalized neutrosophic contra-continuous function.

Proof. Let A be a neutrosophic open set in (Y, S). Every neutrosophic open set is a generalized neutrosophic open set. Since f is a generalized neutrosophic contra-irresolute function, $f^{-1}(A)$ is a generalized neutrosophic closed set in (X, T). Thus, f is a generalized neutrosophic contra-continuous function.

The converse of Proposition 3.5 need not be true as shown in Example 3.2. □

Example 3.2. Let $X = \{a, b, c\}$. Define the neutrosophic sets A,B and C in X as follows: $A = \langle x, (\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.5}), (\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.5})\rangle$,
$B = \langle x, (\frac{a}{0.7}, \frac{b}{0.6}, \frac{c}{0.5}), (\frac{a}{0.7}, \frac{b}{0.6}, \frac{c}{0.5}), (\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.5})\rangle$ and
$C = \langle x, (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.5}), (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.5}), (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.5})\rangle$.
Then the families $T = \{0_N, 1_N, A, B\}$ and $S = \{0_N, 1_N, C\}$ are neutrosophic topologies on X. Thus, (X, T) and (X, S) are neutrosophic topological spaces. Define $f : (X, T) \to (X, S)$ as follows: $f(a) = b, f(b) = a, f(c) = c$. Then f is a generalized neutrosophic contra-continuous function. Let $D = \langle x, (\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.5}), (\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.5})\rangle$ be a generalized neutrosophic closed set in (X, S), $f^{-1}(D)$ is not a generalized neutrosophic open set in (X, T). Hence, f is not a generalized neutrosophic contra-irresolute function.

Proposition 3.6. Let (X, T) and (Y, S) be any two neutrosophic topological spaces. If $f : (X, T) \to (Y, S)$ is a strongly generalized neutrosophic contra-continuous function then f is a neutrosophic contra-continuous function.

Proof. Let A be a neutrosophic open set in (Y, S). Every neutrosophic open set is a generalized neutrosophic open set. Thus A is a generalized neutrosophic open set in (Y, S). Since f is a strongly generalized neutrosophic contra-continuous function, $f^{-1}(A)$ is a neutrosophic closed set in (X, T). Hence, f is a neutrosophic contra-continuous function.

The converse of Proposition 3.6 need not be true as it is shown in Example 3.3. □

Example 3.3. Let $X = \{a, b, c\}$. Define the neutrosophic sets A,B and C as follows:
$A = \langle x, (\frac{a}{0.3}, \frac{b}{0.2}, \frac{c}{0.2}), (\frac{a}{0.3}, \frac{b}{0.2}, \frac{c}{0.2}), (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.5})\rangle$,
$B = \langle x, (\frac{a}{0.1}, \frac{b}{0.1}, \frac{c}{0.1}), (\frac{a}{0.1}, \frac{b}{0.1}, \frac{c}{0.1}), (\frac{a}{0.9}, \frac{b}{0.9}, \frac{c}{0.9})\rangle$ and
$C = \langle x, (\frac{a}{0.9}, \frac{b}{0.9}, \frac{c}{0.9}), (\frac{a}{0.9}, \frac{b}{0.9}, \frac{c}{0.9}), (\frac{a}{0.1}, \frac{b}{0.1}, \frac{c}{0.1})\rangle$.
The families $T = \{0_N, 1_N, A, B\}$ and $S = \{0_N, 1_N, C\}$ are neutrosophic topologies on X. Thus, (X, T) and (X, S) are neutrosophic topological spaces. Define $f : (X, T) \to (X, S)$ as follows: $f(a) = a, f(b) = b, f(c) = b$. Then f is a neutrosophic contra-continuous function. But, for a generalized neutrosophic open set $D = \langle x, (\frac{a}{0.9}, \frac{b}{0.99}, \frac{c}{0.9}), (\frac{a}{0.9}, \frac{b}{0.99}, \frac{c}{0.9}),$ in (X, S), $f^{-1}(D)$ is not a neutrosophic closed set in (X, T). Hence, f is not a strongly generalized neutrosophic contra-continuous function.

Proposition 3.7. Let (X, T) and (Y, S) be any two neutrosophic topological spaces. If $f : (X, T) \to (Y, S)$ is a strongly generalized neutrosophic contra-continuous function then f is a generalized neutrosophic contra-continuous function.

Proof. Let A be a neutrosophic open set in (Y,S). Every neutrosophic open set is a generalized neutrosophic open set. Therefore A is a generalized neutrosophic open set in (Y,S). Since f is a strongly generalized neutrosophic contra-continuous function, $f^{-1}(A)$ is a neutrosophic closed set in (X,T). Every neutrosophic closed set is a generalized neutrosophic closed set. Hence, f is a generalized neutrosophic contra-continuous function.

The converse of Proposition 3.7 need not be true. See Example 3.4. □

Example 3.4. Let $X=\{a,b,c\}$. Define the neutrosophic sets A,B and C as follows:
$A=\langle x,(\frac{a}{0.4},\frac{b}{0.5},\frac{c}{0.5}),(\frac{a}{0.4},\frac{b}{0.5},\frac{c}{0.5}),(\frac{a}{0.5},\frac{b}{0.5},\frac{c}{0.5})\rangle$,
$B=\langle x,(\frac{a}{0.6},\frac{b}{0.7},\frac{c}{0.5}),(\frac{a}{0.6},\frac{b}{0.7},\frac{c}{0.5}),(\frac{a}{0.4},\frac{b}{0.3},\frac{c}{0.5})\rangle$ and
$C=\langle x,(\frac{a}{0.4},\frac{b}{0.4},\frac{c}{0.5}),(\frac{a}{0.4},\frac{b}{0.4},\frac{c}{0.5}),(\frac{a}{0.4},\frac{b}{0.4},\frac{c}{0.5})\rangle$.
The families $T=\{0_N,1_N,A,B\}$ and $S=\{0_N,1_N,C\}$ are neutrosophic topologies on X. Thus, (X,T) and (X,S) are neutrosophic topological spaces. Define $f:(X,T)\to(X,S)$ as follows: $f(a)=c, f(b)=c, f(c)=c$. Then f is a generalized neutrosophic contra-continuous function. Let $D=\langle x,(\frac{a}{0.4},\frac{b}{0.5},\frac{c}{0.5}),(\frac{a}{0.4},\frac{b}{0.5},\frac{c}{0.5}),(\frac{a}{0.4},\frac{b}{0.4},\frac{c}{0.3})\rangle$ be a generalized neutrosophic open set in (X,S), then $f^{-1}(D)$ is not a neutrosophic closed set in (X,T). Hence, f is not a strongly generalized neutrosophic contra-continuous function.

Proposition 3.8. Let (X,T) and (Y,S) be any two neutrosophic topological spaces. If $f:(X,T)\to(Y,S)$ is a strongly generalized neutrosophic contra-continuous function, then f is a generalized neutrosophic contra-irresolute function.

Proof. Let A be a generalized neutrosophic open set in (Y,S). Since f is a strongly generalized neutrosophic contra-continuous function, $f^{-1}(A)$ is a neutrosophic closed set in (X,T). Every neutrosophic closed set is a generalized neutrosophic closed set. Now, $f^{-1}(A)$ is a generalized neutrosophic closed set in (X,T). Hence, f is a generalized neutrosophic contra-irresolute function.

The converse of Proposition 3.8 need not be true as it is shown in Example 3.5. □

Example 3.5. Let $X=\{a,b,c\}$. Define the neutrosophic sets A,B and C as follows:
$A=\langle x,(\frac{a}{0.4},\frac{b}{0.4},\frac{c}{0.5}),(\frac{a}{0.4},\frac{b}{0.4},\frac{c}{0.5}),(\frac{a}{0.5},\frac{b}{0.5},\frac{c}{0.5})\rangle$,
$B=\langle x,(\frac{a}{0.7},\frac{b}{0.6},\frac{c}{0.5}),(\frac{a}{0.7},\frac{b}{0.6},\frac{c}{0.5}),(\frac{a}{0.3},\frac{b}{0.4},\frac{c}{0.5})\rangle$ and
$C=\langle x,(\frac{a}{0.4},\frac{b}{0.5},\frac{c}{0.5}),(\frac{a}{0.4},\frac{b}{0.5},\frac{c}{0.5}),(\frac{a}{0.4},\frac{b}{0.5},\frac{c}{0.5})\rangle$.
The families $T=\{0_N,1_N,A,B\}$ and $S=\{0_N,1_N,C\}$ are neutrosophic topologies on

X. Thus, (X,T) and (X,S) are neutrosophic topological spaces. Define $f:(X,T)\to(X,S)$ as follows: $f(a)=b, f(b)=a, f(c)=c$. Then f is a generalized neutrosophic contra-irresolute function. But, for a generalized neutrosophic closed set $D=\langle x,(\frac{a}{0.3},\frac{b}{0.4},\frac{c}{0.5}),(\frac{a}{0.3},\frac{b}{0.4},\frac{c}{0.5}),(\frac{a}{0.5},\frac{b}{0.5},\frac{c}{0.5})\rangle$ in (X,S). $f^{-1}(D)$ is not a neutrosophic open set in (X,T). Hence, f is not a strongly generalized neutrosophic contra-continuous function.

Proposition 3.9. Let (X,T),(Y,S) and (Z,R) be any three neutrosophic topological spaces. Let $f:(X,T)\to(Y,S)$ and $g:(Y,S)\to(Z,R)$ be functions. If f is a generalized neutrosophic contra-irresolute function and g is a generalized neutrosophic contra-continuous function, then $g\circ f$ is a generalized neutrosophic continuous function.

Proof. Let A be a neutrosophic open set in (Z,R). Since g is a generalized neutrosophic contra-continuous function, $g^{-1}(A)$ is a generalized neutrosophic closed set in (Y,S). Since f is a generalized neutrosophic contra-irresolute function, $f^{-1}(g^{-1}(A))$ is a generalized neutrosophic open set in (X,T). Hence, $g\circ f$ is a generalized neutrosophic continuous function. □

Proposition 3.10. Let (X,T),(Y,S) and (Z,R) be any three neutrosophic topological spaces. Let $f:(X,T)\to(Y,S)$ and $g:(Y,S)\to(Z,R)$ be functions. If f is a generalized neutrosophic contra-irresolute function and g is a generalized neutrosophic continuous function, then $g\circ f$ is a generalized neutrosophic contra-continuous function.

Proof. Let A be a neutrosophic open set in (Z,R). Since g is a generalized neutrosophic continuous function, $g^{-1}(A)$ is a generalized neutrosophic open set in (Y,S). Since f is a generalized neutrosophic contra-irresolute function, $f^{-1}(g^{-1}(A))$ is a generalized neutrosophic closed set in (X,T). Hence, $g\circ f$ is a generalized neutrosophic contra-continuous function. □

Proposition 3.11. Let (X,T) ,(Y,S) and (Z,R) be any three neutrosophic topological spaces. Let $f:(X,T)\to(Y,S)$ and $g:(Y,S)\to(Z,R)$ be functions. If f is a generalized neutrosophic irresolute function and g is a generalized neutrosophic contra-continuous function, then $g\circ f$ is a generalized neutrosophic contra-continuous function.

Proof. Let A be a neutrosophic open set in (Z,R). Since g is a generalized neutrosophic contra-continuous function, $g^{-1}(A)$ is a generalized neutrosophic closed set in (Y,S). Since f is a generalized neutrosophic irresolute function, $f^{-1}(g^{-1}(A))$ is a generalized neutrosophic closed set in (X,T). Hence, $g \circ f$ is a generalized neutrosophic contra-continuous function. $\square$

Proposition 3.12. Let (X,T) ,(Y,S) and (Z,R) be any three neutrosophic topological spaces. Let $f:(X,T) \to (Y,S)$ and $g:(Y,S) \to (Z,R)$ be functions. If f is a strongly generalized neutrosophic contra-continuous function and g is a generalized neutrosophic contra-continuous function, then $g \circ f$ is a neutrosophic continuous function.

Proof. Let A be a neutrosophic open set in (Z,R). Since g is a generalized neutrosophic contra-continuous function, $g^{-1}(A)$ is a generalized neutrosophic closed set in (Y,S). Since f is a strongly generalized neutrosophic contra-continuous function, $f^{-1}(g^{-1}(A))$ is a neutrosophic open set in (X,T). Hence, $g \circ f$ is a neutrosophic continuous function. $\square$

Proposition 3.13. Let (X,T) ,(Y,S) and (Z,R) be any three neutrosophic topological spaces. Let $f:(X,T) \to (Y,S)$ and $g:(Y,S) \to (Z,R)$ be functions. If f is a strongly generalized neutrosophic contra-continuous function and g is a generalized neutrosophic continuous function, then $g \circ f$ is a neutrosophic contra-continuous function.

Proof. Let A be a neutrosophic open set in (Z,R). Since g is a generalized neutrosophic continuous function, $g^{-1}(A)$ is a generalized neutrosophic open set in (Y,S). Since f is a strongly generalized neutrosophic contra-continuous function, $f^{-1}(g^{-1}(A))$ is a neutrosophic closed set in (X,T). Hence, $g \circ f$ is a neutrosophic contra-continuous function. $\square$

Proposition 3.14. Let (X,T) ,(Y,S) and (Z,R) be any three neutrosophic topological spaces. Let $f:(X,T) \to (Y,S)$ and $g:(Y,S) \to (Z,R)$ be functions. If f is a strongly generalized neutrosophic continuous function and g is generalized neutrosophic contra-continuous function, then $g \circ f$ is a neutrosophic contra-continuous function.

Proof. Let A be a neutrosophic open set in (Z,R). Since g is a generalized neutrosophic contra-continuous function, $g^{-1}(A)$ is a generalized neutrosophic closed set in (Y,S). Since f is a strongly generalized neutrosophic continuous function, $f^{-1}(g^{-1}(A))$ is a neutrosophic closed set in (X,T). Hence, $g \circ f$ is a neutrosophic contra-continuous function. $\square$

Proposition 3.15. Let (X,T) ,(Y,S) and (Z,R) be any three neutrosophic topological spaces. Let $f:(X,T) \to (Y,S)$ and $g:(Y,S) \to (Z,R)$ be functions and (Y,S)

be a neutrosophic $T_{\frac{1}{2}}$ space if f and g are generalized neutrosophic contra-continuous functions, then $g \circ f$ is a generalized neutrosophic continuous function.

Proof. Let A be a neutrosophic open set in (Z, R). Since g is a generalized neutrosophic contra-continuous function, $g^{-1}(A)$ is a generalized neutrosophic closed set in (Y, S). Since (Y, S) is a neutrosophic $T_{\frac{1}{2}}$ space, $g^{-1}(A)$ is a neutrosophic closed set in (Y, S). Since f is a generalized neutrosophic contra-continuous function, $f^{-1}(g^{-1}(A))$ is a generalized neutrosophic open set in (X, T). Hence, $g \circ f$ is a generalized neutrosophic continuous function.

The Proposition 3.15., need not be true, if (Y, S) is not a neutrosophic $T_{\frac{1}{2}}$ as shown in Example 3.6. $\square$

Example 3.6. Let $X = \{a, b, c\}$. Define the neutrosophic sets A,B,C and D as follows:
$A = \langle x, (\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.4}), (\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.4}), (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.5})\rangle$,
$B = \langle x, (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.6}), (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.6}), (\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.3})\rangle$,
$C = \langle x, (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.4}), (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.4}), (\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.4})\rangle$ and
$D = \langle x, (\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.3}), (\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.3}), (\frac{a}{0.5}, \frac{b}{0.6}, \frac{c}{0.7})\rangle$.
Observe that the families $T = \{0_N, 1_N, A, B\}$,$S = \{0_N, 1_N, C\}$ and $R = \{0_N, 1_N, D\}$ are neutrosophic topologies on X. Thus, (X, T) ,(X, S) and (X, R) are neutrosophic topological spaces. Define $f : (X, T) \to (X, S)$ by $f(a) = a, f(b) = b, f(c) = b$ and $g : (X, S) \to (X, R)$ by $g(a) = b, g(b) = a, g(c) = c$. Then f and g are generalized neutrosophic contra-continuous functions. Let D be a neutrosophic open set in (X, R). $f^{-1}(g^{-1}(D))$ is not a generalized neutrosophic open set in (X, T). Therefore, $g \circ f$ is not a generalized neutrosophic continuous function. Further (X, S) is not neutrosophic $T_{\frac{1}{2}}$.

Proposition 3.16. Let (X, T),(Y, S) and (Z, R) be any three neutrosophic topological spaces. Let $f : (X, T) \to (Y, S)$ and $g : (Y, S) \to (Z, R)$ be functions and (Y, S) be neutrosophic $T_{\frac{1}{2}}$. If f is a neutrosophic contra-continuous function and g is a generalized neutrosophic contra-irresolute function, then $g \circ f$ is a strongly generalized neutrosophic continuous function.

Proof. Let A be a generalized neutrosophic open in (Z, R). Since g is a generalized neutrosophic contra-irresolute function, $g^{-1}(A)$ is a generalized neutrosophic closed set in (Y, S). Since (Y, S) is a neutrosophic $T_{\frac{1}{2}}$ space, $g^{-1}(A)$ is a neutrosophic closed set in (Y, S). Since f is a neutrosophic contra-continuous function, $f^{-1}(g^{-1}(A))$ is a neutrosophic open set in (X, T). Hence, $g \circ f$ is a strongly generalized neutrosophic continuous function.

If (Y, S) is not a neutrosophic $T_{\frac{1}{2}}$ space, then Proposition 3.16 need not be true as it is shown in Example 3.7. $\square$

Example 3.7. Let $X = \{a, b, c\}$. Define the neutrosophic sets A,B,C and D as follows:
$A = \langle x, (\frac{a}{0.9}, \frac{b}{0.9}, \frac{c}{0.9}), (\frac{a}{0.9}, \frac{b}{0.9}, \frac{c}{0.9}), (\frac{a}{0.1}, \frac{b}{0.1}, \frac{c}{0.1})\rangle$,
$B = \langle x, (\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.4}), (\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.4}), (\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.5})\rangle$,
$C = \langle x, (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.5}), (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.5}), (\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.5})\rangle$ and
$D = \langle x, (\frac{a}{0.4}, \frac{b}{0.2}, \frac{c}{0.3}), (\frac{a}{0.4}, \frac{b}{0.2}, \frac{c}{0.3}), (\frac{a}{0.6}, \frac{b}{0.8}, \frac{c}{0.7})\rangle$.
The families $T = \{0_N, 1_N, A, B\}$, $S = \{0_N, 1_N, C\}$ and $R = \{0_N, 1_N, D\}$ are neutrosophic topologies on X. Thus, (X, T), (X, S) and (X, R) are neutrosophic topological spaces. Define $f : (X, T) \to (X, S)$ as $f(a) = a, f(b) = a, f(c) = b$ and $g : (X, S) \to (X, R)$ by $g(a) = c, g(b) = a, g(c) = b$. Then f is neutrosophic contra-continuous function and g is a generalized neutrosophic contra-irresolute function. But, for the generalized neutrosophic open set D in (X, R), $f^{-1}(g^{-1}(D))$ is not a neutrosophic open set in (X, T). Hence $g \circ f$ is not a strongly generalized neutrosophic continuous function. Moreover, (X, S) is not a neutrosophic $T_{\frac{1}{2}}$ space.

Proposition 3.17. Let (X, T) and (Y, S) be any two neutrosophic topological spaces. For a function $f : (X, T) \to (Y, S)$, the following statements are equivalent:

(i) f is a generalized neutrosophic contra-continuous function;

(ii) For each neutrosophic point $x_{r,t,s}$ of X and for each neutrosophic closed set B of (Y, S) containing $f(x_{r,t,s})$, there exists a generalized neutrosophic open set A of (X, T) containing $x_{r,t,s}$, such that $A \subseteq f^{-1}(B)$;

(iii) For each neutrosophic point $x_{r,t,s}$ of X and for each neutrosophic closed set B of (Y, S) containing $f(x_{r,t,s})$, there exists a generalized neutrosophic open set A of (X, T) containing $x_{r,t,s}$, such that $f(A) \subseteq B$.

Proof. **(i)⇒ (ii)** Let f be a generalized neutrosophic contra-continuous function. Let B be a neutrosophic closed set in (Y, S) and $x_{r,t,s}$ a neutrosophic point of X such that $f(x_{r,t,s}) \in B$. Then $x_{r,t,s} \in f^{-1}(B) = NGint(f^{-1}(B))$. Let $A = NGint(f^{-1}(B))$, then A is a generalized neutrosophic open set and $A = NGint(f^{-1}(B)) \subseteq f^{-1}(B)$. This implies that $A \subseteq f^{-1}(B)$.

(ii)⇒ (iii) Let B be a neutrosophic closed set in (Y, S) and let $x_{r,t,s}$ be a neutrosophic point in X, such that $f(x_{r,t,s}) \in B$. Then $x_{r,t,s} \in f^{-1}(B)$. By hypothesis, $f^{-1}(B)$ is a generalized neutrosophic open set in (X, T) and $A \subseteq f^{-1}(B)$. This implies that $f(A) \subseteq f(f^{-1}(B)) \subseteq B$. Thus, $f(A) \subseteq B$

(iii)⇒ (i) Let B be a neutrosophic closed set in (Y, S) and let $x_{r,t,s}$ be a neutrosophic point in X, such that $f(x_{r,t,s}) \in B$. Then $x_{r,t,s} \in f^{-1}(B)$. By hypothesis, there exists a generalized neutrosophic open set A of (X, T), such that $x_{r,t,s} \in A$ and $f(A) \subseteq B$. This implies, $x_{r,t,s} \in A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(B)$. Since A is generalized neutrosophic open, $A = NGint(A) \subseteq NGint(f^{-1}(B))$. Therefore, $x_{r,t,s} \in NGint(f^{-1}(B))$, $f^{-1}(B) = \bigcup_{x_{r,t,s} \in f^{-1}(B)}(x_{r,t,s}) \subseteq NGint(f^{-1}(B) \subseteq f^{-1}(B)$. Hence, $f^{-1}(B)$ is a generalized neutrosophic open set in (X, T). Thus, f is generalized neutrosophic contra-continuous function.

□

Proposition 3.18. Let (X, T) and (Y, S) be any two neutrosophic topological spaces. Let $f : (X, T) \to (Y, S)$ be any function. If the graph $g : X \to X \times Y$ of f is a generalized neutrosophic contra-continuous function, then f is also a generalized neutrosophic contra-continuous function.

Proof. Let A be a neutrosophic open set in (Y, S). By definition $f^{-1}(A) = 1_N \bigcap f^{-1}(A) = g^{-1}(1_N \times A)$. Since g is a generalized neutrosophic contra-continuous function, $g^{-1}(1_N \times A)$ is a generalized neutrosophic closed set in (X, T). Now, $f^{-1}(A)$ is a generalized neutrosophic closed set in (X, T). Thus, f is a generalized neutrosophic contra-continuous function. □

Proposition 3.19. Let (X, T) and (Y, S) be any two neutrosophic topological spaces. Let $f : (X, T) \to (Y, S)$ be any function. If the graph $g : X \to X \times Y$ of f is a strongly generalized neutrosophic contra-continuous function, then f is also a strongly generalized neutrosophic contra-continuous function.

Proof. Let A be a generalized neutrosophic open set in (Y, S). By definition $f^{-1}(A) = 1_N \bigcap f^{-1}(A) = g^{-1}(1_N \times A)$. Since g is strongly generalized neutrosophic contra-continuous, $g^{-1}(1_N \times A)$ is a neutrosophic closed set in (X, T). Now, $f^{-1}(A)$ is a neutrosophic closed set in (X, T). Thus, f is a strongly generalized neutrosophic contra-continuous function. □

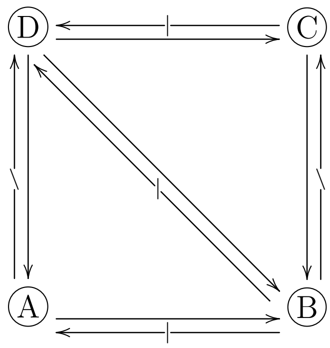
Proposition 3.20. Let (X, T) and (Y, S) be any two neutrosophic topological spaces. Let $f : (X, T) \to (Y, S)$ be any function. If the graph $g : X \to X \times Y$ of f is a generalized neutrosophic contra-irresolute function, then f is also a generalized neutrosophic contra-irresolute function.

Proof. Let A be a generalized neutrosophic open set in (Y, S). By definition $f^{-1}(A) = 1_N \bigcap f^{-1}(A) = g^{-1}(1_N \times A)$. Since g is a generalized neutrosophic contra-irresolute function, $g^{-1}(1_N \times A)$ is a generalized neutrosophic closed set in (X, T). Now, $f^{-1}(A)$ is a generalized neutrosophic closed set in (X, T). Thus, f is a generalized neutrosophic contra-irresolute function. $\square$

4 INTERRELATION

From the above results proved, we have a diagram of implications as shown below.

In the diagram A, B, C and D denote a neutrosophic contra-continuous function, generalized neutrosophic contra-continuous function, generalized neutrosophic contra-irresolute function and strongly generalized neutrosophic contra-continuous function respectively.



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