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ON RECONCILING ATMOSPHERIC, LSND, AND SOLAR NEUTRINO-OSCILLATION DATA

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The L/E -flatness of the e -like events observed in the recent atmospheric-neutrino data from super-Kamiokande (SuperK) is interpreted to reflect a new symmetry of the neutrino-oscillation mixing matrix. From that we obtain an analytical set of constraints yielding a class of mixing matrices of the property to simultaneously fit both the SuperK and the LSND data. The resulting mass squared difference relevant for the LSND experiment is found as 0.3 eV^2 . The discussed symmetry, e.g., carries the nature that expectation values of masses for ν_μ and ν_τ are identical. These considerations are purely data dictated. A different framework is then applied to the solar neutrino problem. It is argued that a single sterile neutrino is an unlikely candidate to accommodate the data from the four solar neutrino experiments. A scenario is discussed which violates CPT symmetry, and favors the $\nu_e\text{-}\bar{\nu}_e$ system to belong to the “self”–“anti-self” charge conjugate construct in the $(1/2, 0) \oplus (0, 1/2)$ representation space, where the needed helicity flipping amplitudes are preferred, rather than the usual Dirac, or Majorana, constructs. In the presented framework the emerging SuperK data on solar neutrino flux is reconciled with the Homestake, GALLEX, and SAGE experiments. This happens because the former detects not only the solar ν_e but also, at a lower cross-section, the oscillated solar $\bar{\nu}_e$; while the latter are sensitive only to the oscillation-diminished solar ν_e flux. A direct observation of solar $\bar{\nu}_e$ by SNO will confirm our scenario. Finally, we consider the possibility for flavor-dependent gravitational couplings of neutrinos as emerging out of the noncommutativity of the quantum operators associated with the measurements of energy and flavor.

1. Introduction

“Is it worth to search for the particle–antiparticle mass differences in sectors other than $K^0\bar{K}^0$?” L. B. Okun.¹

With the preliminary results of Kamiokande and the LSND experiments presented a few years ago,^{2,3} and now both groups presenting more definitive evidence for

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neutrino oscillations,^{4,5} the original Pontecorvo suggestion⁶ that the long-standing solar neutrino anomaly⁷ may be pointing towards the repeat of the K system in neutrino oscillations seems confirmed.

Evidence for neutrino oscillations provide empirical support for flavor eigenstates of neutrinos not to be mass eigenstates. Instead, these flavor eigenstates are suggested to be a linear superposition of some underlying mass eigenstates

$$|\nu_\ell\rangle = \sum_j U_{\ell j} |m_j\rangle, \quad (1)$$

where the flavor index $\ell = e, \mu, \tau$ and mass index $j = 1, 2, 3$. The $|\nu_\ell\rangle$ and $|m_j\rangle$ are flavor and mass eigenstates respectively, while $U_{\ell j}$ are elements of a 3×3 unitary matrix to be determined from data.

Now, the question arises, why, after the $K^0 \bar{K}^0$ system, has Nature chosen to create another physical system whose elements are linear superposition of different mass eigenstates? May this be the expression of something fundamentally new? We here argue that this fundamentally “new something” may be the violation of CPT symmetry, and the related violation of the principle of equivalence. Specifically, in this scenario the mass eigenstates underlying the ν_e and $\bar{\nu}_e$ may carry slightly different masses.

These opening remarks are followed, in Sec. 2, by an introduction to the SuperK data on the e -like events and the questions raised by them. Section 3 is devoted to obtaining a set of analytical constraints that the L/E -flatness of the e -like events imposes. Section 4 contains an analytic and numeric study of the constraint. This study yields a class of mixing matrices U . This class of matrices is investigated in light of the existing data to yield a subclass that fits all existing data except the data on the solar neutrino deficit. The solar neutrino deficit problem is then investigated separately in Secs. 5–7 where considerations enumerated in the abstract are established. Section 8 closes this review with a few concluding remarks. Unless otherwise stated, we follow the notation of Ref. 8.

The reader whose interest is purely in the reconciliation of the SuperK data with the LSND experiment need only read Secs. 2–4. These sections establish that, in the absence of CP-violation, the SuperK observed L/E -flatness of the e -like events essentially determines the neutrino mixing matrix U .

Sections 5–7 carry the flavor of the “oral tradition”. There, we suggest experimentally verifiable implications of the emerging data on the solar neutrino flux. These sections argue that the apparent incompatibility between SuperK solar neutrino data and Homestake chlorine experiment, GALLEX and SAGE may be pointing towards a possible new era in physics where CPT symmetry and the principle of equivalence are only approximately valid.

Thus, this review consists of two themes: (a) the L/E -flatness of the e -like events in the SuperK data severely restricts the class of allowed neutrino mixing matrices; (b) one needs to be very careful and open minded while looking at the apparent and emerging incompatibilities between various solar neutrino experiments.

2. Peculiarities of the Super-Kamiokande Data on the e -Like Events

One of the noteworthy results of the recent SuperK data on atmospheric neutrinos is the L/E (i.e. zenith angle) dependence of the following ratios:

$$\mathcal{R}_e \equiv \frac{\text{experimentally observed } e\text{-like events}}{\text{theoretically expected } e\text{-like events (without } \nu \text{ oscillations)}}, \quad (2)$$

$$\mathcal{R}_\mu \equiv \frac{\text{experimentally observed } \mu\text{-like events}}{\text{theoretically expected } \mu\text{-like events (without } \nu \text{ oscillations)}}. \quad (3)$$

A remarkable feature of the SuperK data is that, while the second ratio \mathcal{R}_μ reveals a significant zenith angle dependence, \mathcal{R}_e is, within experimental errors, L/E -independent and consistent with unity.⁵ To be more precise, slight deviations from the cited flatness and unity cannot be ruled out. However, to gain first order insights into the observed evidence for neutrino oscillations we shall assume the flatness and take it to be identical to unity.

Interpreting the zenith angle dependence of \mathcal{R}_μ and \mathcal{R}_e as evidences for neutrino flavor oscillations, the SuperK collaboration has tentatively concluded the maximal $\nu_\mu \leftrightarrow \nu_\tau$ (or ν_s) mixing by taking the neutrino mixing matrix to be of the form given below

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\Theta & s_\Theta \\ 0 & -s_\Theta & c_\Theta \end{pmatrix}, \quad (4)$$

with $c_\Theta = \cos(\Theta)$ and $s_\Theta = \sin(\Theta)$, respectively.

The interpretation presented by SuperK has the disadvantage of ignoring the results from the LSND experiment, where the neutrino source is apparently best understood, and which has reported a direct evidence for neutrino oscillations in two channels.^{3,4,a}

The independence of \mathcal{R}_e on the L/E parameter together with its closeness with unity either implies, as SuperK inferred, that the mixing matrix is given by Eq. (4), or, that there is some underlying new symmetry hidden in the full 3×3 space spanned by the three neutrino flavors.

By means of the scenario envisaged by SuperK, the E -dependence of the solar neutrino deficit along with the atmospheric neutrino anomaly could perhaps be explained (see Sec. 5 for the cautionary tone of this remark). Still, the mixing matrix given by Eq. (4) with a string of zeros seems too accidental. In the zenith

^aThe “exclusion region” presented recently by KARMEN experiment⁹ appears to cover most of the LSND allowed parameter space. However, their exclusion curve lies well outside the detector’s sensitivity and emerges from zero events observed for an expected background of 2.8 events. I thank LSND collaboration for this observation.

angle independence of \mathcal{R}_e we suspect a hint on some symmetry hidden in the mixing matrix, otherwise the flatness at $\mathcal{R}_e = 1$ would appear to be much too accidental.^b

From that point of view we are going to study below the surprising independence of the $\mathcal{R}_e (\simeq 1)$ ratio on the L/E parameter. To simplify the mathematical structure of the analysis and for the sake of transparency of the physical insights, we henceforth use systematically the unit value for \mathcal{R}_e . Accommodations can be considered later for any relatively small variations (with L/E) in \mathcal{R}_e that may emerge as more data become available. The purpose at present is to roughly obtain the structure of all mixing matrices compatible with the restriction $\mathcal{R}_e = 1$ (considered as an equation) for all the relevant L/E 's and use them to reveal compatibility between the various neutrino oscillation experiments.

In other words, in a full three-flavor neutrino oscillation basis, the independence of \mathcal{R}_e on the L/E parameter either indicates that: (a) the neutrino-mixing matrix be (4), thus effectively reducing the mixing matrix to a 2×2 space, or (b) for it to be “natural” the L/E -independence of \mathcal{R}_e must arise from some very specific underlying class of mixing matrices. If we discover this underlying class of matrices, we may be able to unearth some yet-unknown symmetry hidden in neutrino oscillations. At the same time this may allow us to consider an alternative scenario which allows the relevant atmospheric-neutrino-anomaly $\Delta m^2 \simeq 0.6 \times 10^{-2} \text{ eV}^2$ and the LSND-relevant $\Delta m^2 \simeq 0.3 \text{ eV}^2$ to explain all terrestrial experiments. The E -dependence of the solar neutrino deficit would then require either a sterile neutrino, or other possibilities as discussed in Secs. 5 and 6.

It is to be noted that for the atmospheric neutrino anomaly, the LSND-relevant mass squared difference may play a significant role for the data point corresponding to $L \sim 20 \text{ km}$, and then that same mass squared difference makes a constant contribution for $L \gg 20 \text{ km}$. This happens because L/E for LSND, and that for the $L \sim 20 \text{ km}$ bin, carry similar L/E values. The precise contributions from the two mass squared differences are determined, on a bin-by-bin basis, by the various oscillation amplitudes determined by the mixing matrix.

3. Constraints on the Neutrino-Oscillation Matrix as Implied by the L/E -Flatness of the e -Like Events

In this section a general procedure for obtaining 3×3 mixing matrices following from $\mathcal{R}_e = 1$ condition is developed.

^bOne may think of looking at the water level in a vessel. If one sees no change in the water level, one may assume no inflow, or outflow, of water. Ordinarily, such flows would have to cancel each other remarkably to yield a constant water level. However, there is an exception. That case belongs to the situation when there is a “water pump” connecting the two flows. The zenith angle independence of $\mathcal{R}_e (\simeq 1)$ poses a similar situation. The neutrino mixing matrix (that acts as a counterpart of the water pump), however, contains more than a single inflow and outflow channel. Therefore, for the experimentally observed situation the possibility that the neutrino mixing matrix carries certain symmetries that cancel the inflow and outflow channels rises.

At values of $t = 0$, corresponding to the “top of the terrestrial atmosphere”, we will assume N_e and N_μ to be the respective numbers of e - and μ -type neutrinos. Then within the detector, i.e. at a distance $L = t$ away from the “top”, the number of e -type neutrinos will be

$$N'_e = N_e P(\nu_e \rightarrow \nu_e) + N_\mu P(\nu_\mu \rightarrow \nu_e). \tag{5}$$

Here, P stands for the oscillation probability over the distance L at the relevant energy and in the indicated channel.^c

Now, in assuming, for concreteness, $N_\mu/N_e = 2$ for the relevant energy range, one easily realizes that the constancy of $\mathcal{R}_e = 1$ over the range of L/E relevant to SuperK implies the following relation between the $\nu_e \rightarrow \nu_e$, and $\nu_\mu \rightarrow \nu_e$ oscillation probabilities^d:

$$P(\nu_e \rightarrow \nu_e) + 2P(\nu_\mu \rightarrow \nu_e) = 1. \tag{6}$$

A further constraint on the neutrino oscillation probabilities is obtained from the unitarity condition on the 3×3 neutrino mixing matrix

$$U = \begin{pmatrix} c_\theta c_\beta & s_\theta c_\beta & s_\beta \\ -c_\theta s_\beta s_\psi e^{i\delta} - s_\theta c_\psi & c_\theta c_\psi - e^{i\delta} s_\theta s_\beta s_\psi & c_\beta s_\psi e^{i\delta} \\ -c_\theta s_\beta c_\psi + s_\theta s_\psi e^{-i\delta} & -s_\theta s_\beta c_\psi - c_\theta s_\psi e^{-i\delta} & c_\beta c_\psi \end{pmatrix}, \tag{7}$$

where $c_\beta = \cos(\beta)$, $s_\beta = \sin(\beta)$, etc. as

$$P(\nu_e \rightarrow \nu_e) + P(\nu_e \rightarrow \nu_\mu) + P(\nu_e \rightarrow \nu_\tau) = 1. \tag{8}$$

Allowing for a possible CP-violation in the neutrino sector, Eqs. (6) and (8) then yield:

$$2P(\nu_\mu \rightarrow \nu_e) - P(\nu_e \rightarrow \nu_\mu) - P(\nu_e \rightarrow \nu_\tau) = 0. \tag{9}$$

Recall now the general expression for the neutrino oscillation probabilities⁸

$$P(\nu_\ell \rightarrow \nu_{\ell'}) = \delta_{\ell\ell'} - 4U_{\ell'1}U_{\ell 1}^*U_{\ell'2}U_{\ell 2}^* \sin^2(\varphi_{21}^0) - 4U_{\ell'1}U_{\ell 1}^*U_{\ell'3}U_{\ell 3}^* \sin^2(\varphi_{31}^0) - 4U_{\ell'2}U_{\ell 2}^*U_{\ell'3}U_{\ell 3}^* \sin^2(\varphi_{32}^0), \tag{10}$$

with the kinematic phase being defined as^e

$$\varphi_{j\ell}^0 = 2\pi \frac{L}{\lambda_{j\ell}^{\text{osc}}}. \tag{11}$$

^cIt will be assumed that a relation that parallels Eq. (5) is valid for antineutrinos.

^dIf more precise data were to establish the L/E independent value of \mathcal{R}_e to be different from unity, then such a change can be immediately accommodated with minimal changes in the presented arguments.

^eNote that the kinematic phases can be modified for some dynamical reasons.^{10,11}

The definition of the oscillation length, $\lambda_{j^i}^{\text{osc}}$, is now standard and reads

$$\lambda_{j^i}^{\text{osc}} = \frac{2\pi}{\alpha} \frac{E}{\Delta m_{j^i}^2}. \tag{12}$$

The kinematic phase may also be written as: $\varphi_{j^i}^0 = 1.27\Delta m_{j^i}^2 \times (L/E)$. Here, $\alpha = \bar{\alpha}/2$; $\bar{\alpha} = 2.54$ is the usual factor that arises from expressing E in MeV, L in meters, and $\Delta m_{j^i}^2$ in eV^2 . E refers to neutrino kinetic energy, $\sqrt{\mathbf{p}^2 + m^2}$, and L is the distance between the creation region and the detection region for the neutrino oscillation event. The six neutrino oscillation parameters are the two mass squared differences, the three mixing angles, and a CP-violating phase angle δ .

The L/E independence of $\mathcal{R}_e = 1$, as translated into Eq. (9), now becomes a set of three coupled equations:

$$2U_{\mu 1}^* U_{\mu 2} U_{e 1} U_{e 2}^* - U_{e 1}^* U_{e 2} U_{\mu 1} U_{\mu 2}^* - U_{e 1}^* U_{e 2} U_{\tau 1} U_{\tau 2}^* = 0, \tag{13}$$

$$2U_{\mu 1}^* U_{\mu 3} U_{e 1} U_{e 3}^* - U_{e 1}^* U_{e 3} U_{\mu 1} U_{\mu 3}^* - U_{e 1}^* U_{e 3} U_{\tau 1} U_{\tau 3}^* = 0, \tag{14}$$

$$2U_{\mu 2}^* U_{\mu 3} U_{e 2} U_{e 3}^* - U_{e 2}^* U_{e 3} U_{\mu 2} U_{\mu 3}^* - U_{e 2}^* U_{e 3} U_{\tau 2} U_{\tau 3}^* = 0. \tag{15}$$

We thus have *three* transcendental equations in *four* parameters of the mixing matrix U . These are the constraints on the neutrino-oscillation matrices as implied by the L/E -flatness of the e -like events.

Rigorously speaking, the considerations presented above should be in terms of⁸

$$\mathcal{P}(\nu_\ell \rightarrow \nu_{\ell'}) = \int_{E_{\text{min}}}^{E_{\text{max}}} dE f_\ell(E) P(\nu_\ell \rightarrow \nu_{\ell'}), \tag{16}$$

rather than in terms of $P(\nu_\ell \rightarrow \nu_{\ell'})$. In the last equation, $f_\ell(E)$ is the neutrino flux as normalized to unity, while E_{min} and E_{max} refer to the minimum and maximum neutrino energies as determined by the combined system of the beam and the detector. However, for the existing precision of the data such a detailed analysis seems unwarranted.

4. Analytical and Numerical Solutions for Neutrino-Oscillation Matrices

Using the program-package for analytical calculations “MacSYMA”, the left-hand sides of Eqs. (13)–(15) can be simplified towards expression containing only trigonometric functions of a certain form. A dramatic simplification occurs on setting $\psi = \theta = \pi/4$. One is then led to the following set of equations:

$$[e^{-i\delta}(e^{i\delta} - 1)(e^{i\delta} + 1)]c_\beta^2 s_\beta = 0, \quad e^{i\delta} c_\beta^2 s_\beta = 0, \quad e^{i\delta} c_\beta^2 s_\beta = 0. \tag{17}$$

Thus the first set of mixing angles $\{\theta = \pi/4, \beta = 0, \psi = \pi/4\}$ provides one possible neutrino oscillation matrix.^f It reads:

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & e^{i\delta} \\ e^{-i\delta}/\sqrt{2} & -e^{-i\delta}/\sqrt{2} & 1 \end{pmatrix}. \quad (18)$$

A second analytically obtained solution is found as:

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 \\ -e^{i\delta}/\sqrt{2} & 1 & e^{i\delta}/\sqrt{2} \\ -1/\sqrt{2} & -e^{-i\delta} & 1/\sqrt{2} \end{pmatrix}. \quad (19)$$

Finally, a set of physically irrelevant solutions has been found to exist too. Those solutions have the generic form in which each of the three rows, at mutually non-intersecting positions, carries a +1, or a -1, and two zeros in the remaining positions.

Apart from the analytical solutions presented above, we also find numerical similar solutions. As an example, two typical numeric solutions (with the CP-violating phase angle δ being set to zero) are given below as

$$U \simeq \begin{pmatrix} 0.88 & 0.48 & 0.00 \\ -0.34 & 0.62 & 0.71 \\ 0.34 & -0.62 & 0.71 \end{pmatrix}, \quad U \simeq \begin{pmatrix} 0.65 & 0.76 & 0.00 \\ -0.54 & 0.46 & 0.071 \\ 0.54 & -0.46 & 0.071 \end{pmatrix}. \quad (20)$$

These solutions establish the existence of mixing matrices in the full (3×3) -dimensional neutrino flavor space so as to satisfy the SuperK observed L/E independence of $\mathcal{R}_e (= 1)$.^g However, none of these solutions satisfies the other existing data and constraints. Indeed, assume a vanishing CP-violating phase $\delta = 0$ in agreement with the LSND's decay-in-flight results.⁴ Consider then, for concreteness, the mixing matrix in Eq. (19). Since U_{e2} identically vanishes, the LSND relevant expression for the neutrino oscillation probability simplifies to

$$P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = -4U_{e1}U_{\mu 1}U_{e3}U_{\mu 3} \sin^2(\varphi_{31}^0) \quad (21)$$

$$= 0.5 \sin^2 \left(\frac{1.27 \times \Delta m_{31}^2 \times 30}{45} \right), \quad (22)$$

where for the LSND's decay-at-rest data we have set $\langle L \rangle \approx 30$ m, and $\langle E \rangle \approx 45$ MeV. Setting LSND observed $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \simeq 0.3 \times 10^{-2}$, we obtain $\Delta m_{31}^2 \simeq 0.1$ eV². One can further calculate the probability for the disappearance of

^fThe set of angles $\{\theta = \pi/4, \beta = \pi/2, \psi = \pi/4\}$, which also solves Eq. (17), does not result in a full 3×3 mixing in the neutrino flavor space.

^gNote, how different U of Eqs. (18)–(20) are from the SuperK's U given by Eq. (4).

the electron antineutrino (as in the reactor disappearance experiments) to obtain

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - 4U_{e1}U_{e1}U_{e3}U_{e3} \sin^2 \left(\frac{1.27 \times \Delta m_{31}^2 \times L}{E} \right) \tag{23}$$

$$= 1 - 1.0 \sin^2 \left(\frac{1.27 \times \Delta m_{31}^2 \times L}{E} \right). \tag{24}$$

Taking $E \simeq 5$ MeV as the typical reactor $\bar{\nu}_e$ energy and setting $L = 50$ m and $L = 100$ m for two representative settings of the detector in the reactor $\bar{\nu}_e$ disappearance experiments, we obtain $P(\nu_e \rightarrow \nu_e; L = 50 \text{ m}) = 1-0.91$, $P(\bar{\nu}_e \rightarrow \bar{\nu}_e; L = 100 \text{ m}) = 1-0.32$. These findings are clearly in strong disagreement with the known experimental results.¹²

Finally, we also find numerical solutions of the type^h

$$U \simeq \begin{pmatrix} 0.99 & 0.00 & 0.10 \\ -0.07 & 0.71 & 0.70 \\ -0.07 & -0.71 & 0.70 \end{pmatrix}. \tag{25}$$

These mixing matrices have the property that the expectation values of the masses associated with the ν_μ and ν_τ neutrinos now appear identical:

$$\langle m(\nu_\mu) \rangle = \langle m(\nu_\tau) \rangle. \tag{26}$$

Here $\langle m(\nu_\ell) \rangle \equiv U_{\ell 1}^2 m_1 + U_{\ell 2}^2 m_2 + U_{\ell 3}^2 m_3$.

Further, in using the mixing matrix in Eq. (25) in combination with the LSND result, $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \simeq 0.3 \times 10^{-2}$, one arrives at

$$P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = 0.3 \times 10^{-2} = 0.02 \times \sin^2 \left(\frac{1.27 \times \Delta m_{31}^2 \times 30}{45} \right). \tag{27}$$

This yields, $\Delta m_{31}^2 = 0.2 \text{ eV}^2$. However, a more detailed calculation that uses the LSND- $\bar{\nu}_\mu$ spectral function $f_{\bar{\nu}_\mu}$ and a cut-off at 20 MeV yields a (see Ref. 8 for the notational details) $\mathcal{O}'(30 \text{ m}, \Delta m_{31}^2, E_{\min} = 20.0 \text{ MeV}) = 0.15$. Figure 2 of Ref. 8 graphs $\mathcal{O}'(30 \text{ m}, \Delta m_{31}^2, E_{\min} = 20.0 \text{ MeV})$ as a function of Δm_{31}^2 .

^hThe essential difference with the above quoted solutions is in the order-of-magnitude difference in the amplitude of oscillations carried by each of the sine squared terms in the expression for oscillation probability given by Eq. (10). Another related matrix is:

$$U \simeq \begin{pmatrix} 0.99 & 0.10 & 0.00 \\ -0.07 & 0.70 & 0.70 \\ 0.07 & -0.70 & 0.70 \end{pmatrix}.$$

Also, now note the similarities and differences between these solutions and that presented by SuperK — cf. footnote g.

For \mathcal{O}' (30 m, Δm_{31}^2 , $E_{\min} = 20.0$ MeV) below about 0.4, this function is single valued, and yields a unique solution. This solution is

$$\Delta m_{31}^2 = 0.3 \text{ eV}^2. \tag{28}$$

Taking this value for Δm_{31}^2 we find $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ for the reactor disappearance experiments and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu)$ for ν_μ disappearance experiments¹³ read (assuming $L \ll \lambda_{21}^{\text{osc}}$):

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - 0.04 \sin^2 \left(\frac{1.27 \times 0.3 \times L}{E} \right), \tag{29}$$

$$P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu) = 1 - 0.998 \sin^2 \left(\frac{1.27 \times 0.3 \times L}{E} \right). \tag{30}$$

Thus, while a detailed global analysis of existing neutrino oscillation data is beyond the scope of this review, the calculations from above indicate that matrices of the type just enumerated, (up to minor higher order corrections) can accommodate all existing data from the reactor-disappearance experiments,¹² the ν_μ disappearance experiment,¹³ the LSND and KARMEN experiments,^{3,9} and the atmospheric neutrino data from SuperK.⁵

A cautionary remark: A comparison of Eqs. (27) and (29) shows, for example, that a naive use of the two-dimensional $[\Delta m^2, \sin^2(2\Theta)]$ exclusion plots can be quite misleading. Both the two-parameter description of the $[\Delta m^2, \sin^2(2\Theta)]$ formalism, and the very specific mixing matrix under consideration here for the five-parameter formalism, we obtain very similar expressions for the neutrino-oscillation probabilities. The amplitude of oscillation in the $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ channel and the $\bar{\nu}_e \rightarrow \bar{\nu}_e$ channel are the same for the two-parameter formalism, while for five-parameter case, and for the specific choice of the mixing matrix, they are different by a factor of 2. The general situation is often even more different.

The subject of the solar neutrino deficit is taken up next.

5. Solar Neutrino Deficit and the Problem with a Sterile Neutrino Solution

Within the framework presented here, the mixing matrix in (25) yields an energy-independent solar neutrino deficit of 0.98. Compared to the data this number is too large by a factor of two to three. Furthermore, the experiments rule out an energy-independent solar neutrino deficit.¹⁴ The Data/SSM-prediction ($\equiv \delta$)ⁱ is now known to be as follows: $\delta = 0.33 \pm 0.029$ (Homestake), 0.474 ± 0.020 (SuperK), 0.52 ± 0.06 (SAGE), 0.60 ± 0.06 (GALLEX). The Homestake chlorine experiment is the oldest running experiment with an energy threshold of about 0.8 MeV. GALLEX and SAGE carry an energy threshold of about 0.2 MeV. SuperK, which has a higher

ⁱSSM = Standard Solar Model, see Table 1 of Ref. 14.

energy threshold of 6.5 MeV, has the advantage in that it is able to study the solar neutrino deficit as a function of energy. *The SuperK preliminary data carries a higher δ for higher neutrino energies, with δ becoming constant towards lower energies.*

With three underlying mass eigenstates one finds only two independent mass squared differences in the standard neutrino oscillation phenomenology. Therefore, one cannot accommodate the energy dependence of the three length scales implied by the existing data. Under these circumstances one invokes a sterile neutrino to accommodate all existing data on neutrino oscillations.

For the vacuum-oscillation solution, the SuperK solar neutrino data suggests an oscillation length of the order of an astronomical unit for neutrino energies around 10 MeV. Thus, at lower energies of about 1 MeV the oscillation length becomes about 1/10 of an astronomical unit and the deficit becomes energy-independent (because of the “energy averaging” in this energy region), and the SuperK data yields¹⁵ $\delta \simeq 0.375 \pm 0.025$ to be compared with the δ 's for the Homestake, GALLEX and SAGE experiments. Thus the solar neutrino data from SuperK comes in conflict with the other experiments.

One may then consider an oscillation length that fits the Homestake, GALLEX, and SAGE experiments. But with such an oscillation length it would be difficult to fit the energy dependence of the SuperK's data on solar neutrinos.

Thus a single sterile neutrino is an unlikely candidate to accommodate the data from the four solar neutrino experiments.

Therefore, the scenario within the standard neutrino oscillation framework as discussed above leaves the energy dependence of the solar neutrino anomaly to be accommodated in some different manner. One possibility is that one of the underlying mass eigenstates is nonrelativistic. This possibility was recently discussed by the present author with Goldman in Ref. 16. Two other possibilities are: (a) violation of CPT symmetry in the neutrino sector, and/or (b) violation of the principle of equivalence. These are now discussed here in Secs. 6 and 7.

6. Solar Neutrino Deficit and the Possibility for CPT Symmetry Violation

In the standard model the neutrinos and antineutrinos are CP-conjugated partners. Therefore, the CPT symmetry requires ν_ℓ and $\bar{\nu}_\ell$ to have identical masses. More precisely, the CPT symmetry requires identical expectation values for the $\bar{\nu}_e$ and ν_e masses: $\langle m(\bar{\nu}_e) \rangle = \langle m(\nu_e) \rangle$. Moreover, these neutrinos are built from the eigen-spinors of the charge operator in the $(1/2, 0) \oplus (0, 1/2)$ representation space. Both the Dirac fields and the Majorana fields are expanded in terms of the standard Dirac spinors $\{u_\sigma(p^\mu), v_\sigma(p^\mu)\}$. As a result, and for relativistic neutrinos, in both cases the helicity-flipping oscillations appear strongly suppressed. This result is mainly due to the orthogonality of the four spinors $\{u_\sigma(p^\mu), v_\sigma(p^\mu)\}$.

In order to allow for significant helicity-flipping oscillations one may consider self/anti-self charge conjugate eigenspinors in the $(1/2, 0) + (0, 1/2)$ representation space. In this construct the spinors are not orthogonal in the helicity index, but bi-orthogonal.¹⁸

Then referring to Eqs. (36a) and (36b) of Ref. 18, and exploiting the identities given by Eqs. (48a) and (48b) there, we immediately obtain

$$\text{CP } \lambda^S(p^\mu) = \rho^A(p'^\mu), \quad (31)$$

where p'^μ is the parity transformed p^μ , while S , and A refer to self- and anti-self C conjugacy of the $(1/2, 0) + (0, 1/2)$ representation C -eigenspinors $\{\lambda^S(p^\mu), \rho^A(p'^\mu)\}$.

The “neutrino” and “antineutrino” spinors can, therefore, be identified with the set $\{\lambda^S(p^\mu), \rho^A(p'^\mu)\}$. This raises the important possibility that because of the *bi-orthogonal* nature of these spinors in the helicity index (see Table 1 of Ref. 18) helicity flipping oscillations are the ones that are not suppressed. However, this framework is still in its infancy and an interacting theory has yet to be developed. One of the nice features of this construct is that parity violation is deeply embedded in the construct as shown by Dvoeglazov.¹⁹ Nevertheless, it is to be noted that helicity-flipping transitions can also occur in the context of violation of the principle of equivalence and astrophysical magnetic fields.²⁰ In what follows we shall assume that helicity flipping transitions are allowed at significant level regardless of their physical origin.

There exist various reasons to believe that gravitation plays significant role in the “smallness” of the neutrino masses via physics of some unification scale. If that is the case, then the assumption of locality must be abandoned with the consequence that CPT symmetry is no longer on a firm theoretical foundation.^{21–23} Within the framework outlined above, a CPT-violating splitting of $\Delta m^2 \sim 10^{-10}$ to 10^{-11} eV^2 between the mass eigenstates underlying ν_e and $\bar{\nu}_e$ can provide $\nu_e \leftrightarrow \bar{\nu}_e$ oscillations.

In this scenario, for example, one may choose a Δm^2 to fit the Homestake–GALLEX–SAGE data. The reconciliation with the SuperK occurs because SuperK detects not only the solar ν_e but also the oscillated solar $\bar{\nu}_e$, whereas the three other experiments are sensitive only to the oscillation-reduced flux of solar ν_e . SuperK still sees the solar neutrino “deficit” in the solar ν_e – $\bar{\nu}_e$ flux because the detection cross-section for the $\bar{\nu}_e$ is lower than that for the ν_e .

We finally present some speculative considerations on the possibility that the principle of equivalence may be violated. We are encouraged in these speculations by a recently discovered incompleteness of the general relativistic description of gravitation. The incompleteness argument was presented in a recent paper.²⁴ Further, Halprin, Leung, and Pantaleone showed that the atmospheric neutrino data as well as the data on the solar neutrinos point toward the same level of violation of the principle of equivalence.

7. Solar Neutrino Deficit and Violation of the Principle of Equivalence

Consider a flavor eigenstate state $|\nu_\ell\rangle$ given by Eq. (1). Let us further assume for the sake of simplicity that each of the mass eigenstate is also an energy eigenstate,

$$|\nu_\ell\rangle = \sum_j U_{\ell j} |E_j\rangle, \quad \ell = e, \mu, \tau, \tag{32}$$

and perform a measurement of its energy. This measurement projects the flavor eigenstate, with probability $U_{\ell j}^* U_{\ell j}$ (no sum on ℓ , or j), into an energy eigenstate:

$$\mathcal{M}_{\mathcal{H}}|\nu_\ell\rangle \mapsto |E_j\rangle, \quad j = 1, 2, 3. \tag{33}$$

Next, consider the same flavor eigenstate as before. This time a flavor measurement is performed (without allowing the flavor state to evolve into other flavors). Let the corresponding operator be $\mathcal{M}_{\mathcal{F}}$. The result of measurement is (with unit probability)^j

$$\mathcal{M}_{\mathcal{F}}|\nu_\ell\rangle \mapsto |\nu_\ell\rangle, \quad \ell = e, \mu, \tau. \tag{34}$$

As a consequence, the measurements $\mathcal{M}_{\mathcal{H}}$ and $\mathcal{M}_{\mathcal{F}}$ do not commute while acting upon the space spanned by the flavor eigenstates of neutrinos, $\{\nu_e, \nu_\mu, \nu_\tau, \bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau\}$,

$$[\mathcal{M}_{\mathcal{H}}, \mathcal{M}_{\mathcal{F}}] \neq 0. \tag{35}$$

On the other hand, for the flavor states that are mass eigenstates, $\{e^\pm, \mu^\pm, \tau^\pm\}$, one obtains

$$[\mathcal{M}_{\mathcal{H}}, \mathcal{M}_{\mathcal{F}}] = 0, \tag{36}$$

because then the flavor eigenstates and energy eigenstates are identical.

Now the energy eigenstates considered above are simultaneously created by the action of gravitation also on the foundation that gravity is known to couple to matter via the energy momentum tensor. On the other hand, the flavor states are the eigenstates created in the electroweak interactions. In this sense, Eq. (35) tells us that, in some quantum mechanical contexts, gravitation and electroweak interactions have to interfere at some more fundamental level. A gravitational measurement that projects a system into an energy eigenstate destroys the coherence of different mass eigenstates of an electroweak-measurement produced flavor eigenstate. Noncommutativity expressed by Eq. (35) indicates that flavor and gravitation are somehow coupled to each other, and are associated with mutually incompatible observables.

^jAlternately, we could allow an evolution of the flavor eigenstate. In that case flavor measurement projects into one of the three flavor states with the probability given by Eq. (10)

$$\mathcal{M}_{\mathcal{F}}|\nu_\ell\rangle \mapsto |\nu_{\ell'}\rangle, \quad \ell' = e, \mu, \tau.$$

In other words, in the absence of any theory that incorporates gravitation and electroweak interactions on an equal footing, it is reasonable to assume that gravitational interactions of test particles are intimately connected with the energy measurement operator. For states that satisfy Eq. (36) gravitation is decoupled from flavor and one may, therefore, assert universal gravitational coupling for all flavors. Whereas, states for which one obtains Eq. (35), as is the case for neutrinos, gravitation is not decoupled from flavor, and, one may, therefore, not assume universal gravitational coupling for all flavors. This then suggests that weak-interaction flavor eigenstates of neutrinos may violate principle of equivalence, and different neutrino flavors may couple differently to gravity. Such considerations may underlie a conjecture of Gasperini where he postulated nonuniversal coupling of neutrino flavors to gravity, and has recently been considered as a serious candidate to explain the solar neutrino anomaly.^{25–27} In fact, as already noted, Halprin, Leung and Pantaleone showed that the atmospheric neutrino data as well as the data on the solar neutrinos point toward the same level of violation of the principle of equivalence.

Finally, we note the two scenarios discussed here are likely to be deeply intertwined. The proposed mass squared difference $\Delta m^2 \sim 10^{-10}$ to 10^{-11} eV² for the $\nu_e - \bar{\nu}_e$ system is perhaps only confined to electron neutrinos (and further conjectured to be of the type introduced in Ref. 18). The muonic and tauonic neutrinos need not belong to the construct involving $\{\lambda^S(p^\mu), \rho^A(p^\mu)\}$. This difference may underlay the specific form of the mixing matrix U . The U as given in Eq. (25) has a dominant block diagonal form consisting of a 1×1 matrix, and another 2×2 matrix, embedded in a 3×3 matrix. This dominant block diagonal form thus separates not only the $\{\nu_e, \bar{\nu}_e\}$ and $\{\nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau\}$ but in the process approximately factorizes the $\{\lambda^S(p^\mu), \rho^A(p^\mu)\}$ and $\{u(p^\mu), v(p^\mu)\}$ degrees of freedom in the $(1/2, 0) \oplus (0, 1/2)$ representation space. In fact all spinors refer to a specific mass, and for that reason it may be more appropriate to associate the spinorial properties just indicated with the underlying mass eigenstates rather than the flavor eigenstates.

8. Concluding Remarks

The L/E -flatness of the e -like events, observed in the recent atmospheric-neutrino data from SuperK, yields a severe constraint on the neutrino mixing matrix U . This constraint is expressed by Eqs. (13)–(15). When combined with other existing experiments, Eqs. (13)–(15) imply the results given by Eqs. (25) and (28) for the neutrino mixing matrix and the LSND relevant mass squared difference respectively. The obtained mixing matrix is such that it yields identical expectation values for the masses of ν_μ and ν_τ neutrinos by inducing a yet-to-be-understood symmetry in neutrino oscillations. These results are collected together for easy reference:

$$U \simeq \begin{pmatrix} 0.99 & 0.00 & 0.10 \\ -0.07 & 0.71 & 0.70 \\ -0.07 & -0.71 & 0.70 \end{pmatrix}, \quad \Delta m_{\text{LSND}}^2 = 0.3 \text{ eV}^2, \quad (37)$$

and

$$\langle \mathbf{m}(\nu_\mu) \rangle = \langle \mathbf{m}(\nu_\tau) \rangle. \tag{38}$$

We considered the emerging experimental and theoretical situation on the solar neutrino deficit and suggested that as experimental results become more secure, we may be forced into seriously contemplating violations of CPT and/or the principle of equivalence.

Whether the presented “solar antineutrinos as a solution to the solar neutrino problem” is exercised by Nature shall be known in the near future by observations of the Sudbury Neutrino Observatory (SNO).²⁸ Similarly, the results from LSND, KARMEN, SuperK, and other neutrino oscillation experiments shall also settle the issue as to whether or not neutrino oscillations require three independent mass squared differences.

Now, to close, we return again to the question asked in the opening section of this review, why, after the $K^0\bar{K}^0$ system has Nature chosen to create another physical system whose elements are linear superposition of different mass eigenstates? We asked, if this may be an expression of something fundamentally new? We here argued that this fundamentally “new something” may be the violation of CPT symmetry, and the related violation of the principle of equivalence. Specifically, in this scenario the mass eigenstates underlying ν_e and $\bar{\nu}_e$ may carry slightly different masses.

However, how this happens we do not know. Specifically, it may happen, that the ν_ℓ and $\bar{\nu}_\ell$ mixing matrices may be slightly different. This slight difference then results in the different expectation values for masses of ν_ℓ and $\bar{\nu}_\ell$ *without* requiring the masses of the underlying mass eigenstates to be different. This, however, does not lead to the required mass squared difference for the underlying mass eigenstates. Or, it could be that underlying mass eigenstates for ν_ℓ and $\bar{\nu}_\ell$ carry slightly different masses. We assumed the latter to be the case.

Another exciting possibility is that the mixing matrices for the ν_ℓ and $\bar{\nu}_\ell$ are slightly different, and at the same time the underlying mass eigenstates for the ν_ℓ and $\bar{\nu}_\ell$ also differ slightly, *but* with the constraint that expectation values for the masses of ν_ℓ and $\bar{\nu}_\ell$ are *identical*. If this were to be the case, the CPT symmetry will effectively remain unbroken — at least, partly, in so far as CPT symmetry requires identical particle–antiparticle masses — at the level of weak interactions and at the same time provide the needed mass squared difference. Such a scenario would provide an intriguing parallel to the result expressed by Eq. (38),

$$\langle m(\bar{\nu}_e) \rangle = \langle m(\nu_e) \rangle. \tag{39}$$

If in the CPT-violating framework envisaged here, \bar{U} is to represent the mixing matrix for the $\bar{\nu}_\ell$, and U represents the same for ν_ℓ , with \bar{m}_j and m_j representing the masses of the underlying mass eigenstates, following relations can be assumed

to parallel Eq. (38):

$$\langle m(\bar{\nu}_\ell) \rangle \equiv \sum_j \bar{U}_{\ell j}^2 \bar{m}_j = \sum_j U_{\ell j}^2 m_j \equiv \langle m(\nu_\ell) \rangle, \quad \ell = e, \mu, \tau. \quad (40)$$

Thus, while some of our remarks still remain speculative, the experiments on neutrino oscillations open without doubt a window into a new physics. The emerging experimental situation hints at a possible violation of the CPT symmetry and the principle of equivalence. In support to the theory, it should be noted, that an *ab initio* recent investigation of “flavor oscillation clocks”¹¹ has revealed an inherent incompleteness of the general-relativistic description of gravitation,²⁴ an incompleteness that may carry significant implications for the neutrino-oscillation governed physics in astrophysical environments.^{11,27} In addition, as Paul Langacker remarked at Wein ’98, the SuperK results have also tested quantum coherence for length scales up to 1.3×10^4 km in the atmospheric neutrino data, and to the length scales of one astronomical unit in the solar neutrino experiment.

“But, irrespective of all these theoretical considerations, one has to follow the advice of Galileo and measure everything that can be measured.”

L. B. Okun.¹

Acknowledgments

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