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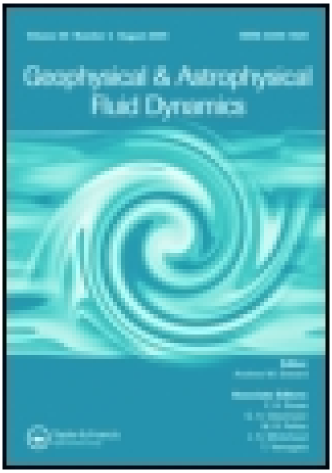

Geophysical & Astrophysical Fluid Dynamics
Publication details, including instructions for authors and subscription information:
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Numerical simulations of stellar convective dynamos III. At the base of the convection zone

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Published online: 18 Aug 2006.

To cite this article: Gary A. Glatzmaier (1985) Numerical simulations of stellar convective dynamos III. At the base of the convection zone, Geophysical & Astrophysical Fluid Dynamics, 31:1-2, 137-150, DOI: 10.1080/03091928508219267

To link to this article: http://dx.doi.org/10.1080/03091928508219267

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Geophys. Astrophys. Fluid Dynamics, 1985, Vol. 31, pp. 137–150
0309-1929/85/3102-0137 $18.50/0
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Numerical Simulations of Stellar Convective Dynamos III. At the Base of the Convection Zone

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(*Received August* 14, 1984)

We describe numerical simulations of giant-cell solar convection and magnetic field generation. Nonlinear, three-dimensional, time-dependent solutions of the anelastic magnetohydrodynamic equations are presented for a stratified, rotating, spherical shell of ionized gas. The velocity, magnetic field, and thermodynamic variables are solved simultaneously and self-consistently with full nonlinear feedback. Convection, driven in the outer part of this shell by a superadiabatic gradient, penetrates into the inner, subadiabatic part. Previous dynamic dynamo simulations have demonstrated that, when the dynamo operates in the convection zone, the magnetic fields propagate away from the equator in the opposite direction inferred from the solar butterfly diagram. Our simulations suggest that the solar dynamo may be operating at the base of the convection zone in the transition region between the stable interior and the turbulent convective region. There our simulated angular velocity decreases with depth, as it does in the convection zone; but the simulated helicity has the opposite sign compared to its convection zone value. As a result, our simulated magnetic fields in this transition region initially propagated toward the equator. However, due to our limited numerical resolution of the small amplitude helical fluid motions in this dense, stable region, only the initial phase propagation could be simulated, not a complete magnetic cycle.

1. INTRODUCTION

Considerable effort has been made in recent years to understand how the interaction of rotation and convection drives a stellar dynamo. The physical explanation of the Sun's 22-year magnetic cycle (Hale 1924) has been the object of both kinematic and dynamic models. Kinematic dynamo models (Soward and Roberts 1977; Moffatt 1978; Parker 1979; Stix 1981; Krause and Rädler 1981), which solve for the magnetic field after prescribing a parameterized velocity field, have been relatively successful in simulating the solar magnetic cycle. Dynamic dynamo models for a Boussinesq fluid (Gilman and Miller 1981; Gilman 1983) and for an anelastic gas (Glatzmaier 1984 (Paper I), 1985 (Paper II)), which simultaneously solve for the velocity, magnetic, and thermodynamic fields with full nonlinear feedback, have numerically simulated dynamos in rotating convection zones; however, the simulated magnetic fields propagate away from the equator (contrary to what is inferred from the solar butterfly diagram) with periods shorter than 22 years. According to stellar dynamo theory, originally proposed by Parker (1955), the direction of magnetic field propagation and the cycle period depend on helicity and differential rotation. In both types of models, the average helicity in the convection zone is negative in the northern hemisphere and positive in the southern hemisphere. To fit the solar butterfly diagram, the prescribed angular velocity in kinematic models increases with depth. However, the simulated angular velocity in dynamic models decreases with depth whenever a surface equatorial acceleration, like that observed on the Sun, is obtained. One might argue that, since the simulated magnetic fields propagate in the wrong direction in the dynamic models, the simulated differential rotation is not correct. However, recent analysis of the frequency splitting of solar oscillations (Duvall and Harvey 1984; Duvall *et al.* 1984) suggests that angular velocity decreases with depth through most of the solar convection zone and through the stable region down to about 45% of the solar radius.

If the solar convection zone is so turbulent that the vast majority of the magnetic flux is concentrated into small-scale intermittent tubes (Galloway and Weiss 1981) as observed on the solar surface (Stenflo 1976), dynamo models of large-scale continuous magnetic fields would be inappropriate for the convection zone. It has been suggested (Schüssler 1980) that the solar cycle may be driven by a "flux tube dynamo"; however, the physical mechanisms for such a dynamo have not been established. In addition, the global characteristics of the solar butterfly diagram and Hale's polarity law (Hale 1924) suggest that the solar cycle is the product of global magnetic fields. If magnetic flux is shredded by turbulent convection and

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magnetic instabilities and expelled from the convective region by magnetic buoyancy and topological pumping, the solar dynamo may be operating with global fields in the less turbulent transition region at the base of the convection zone (Drobyshevski and Yuferev 1974; Parker 1975; Spiegel and Weiss 1980; Golub *et al.* 1981; Galloway and Weiss 1981; Spruit and van Ballegooijen 1982; van Ballegooijen 1982; Arter 1983) where differential rotation and helicity may have the right signs to produce magnetic fields that propagate toward the equator (Durney 1976; Schmidt and Stix 1983; Glatzmaier 1985).

In this paper we study the feasibility of this latter hypothesis by numerically simulating a dynamo with the velocity and magnetic fields coupled (via the Lorentz force, Joule heating, and magnetic induction) at the base of the convection zone and in the stable region below. However, we numerically decouple them in the turbulent convective region since we can not resolve, with our global model, the small-scale flux tubes we assume exist in this part of the Sun. In reality, after the magnetic field is concentrated between cells it will not be affected by the full helicity of the convection and therefore will be decoupled to some extent from the motion (Gilman and Miller 1981, Galloway and Weiss 1981). We include the unstable convective region in our model in order to simulate convective overshooting which maintains the required differential rotation and helicity in the stable region below. The model, numerical method, and solutions are described in Papers I and II. Modifications made to the model for this study are discussed in Section 2. We describe our simulated differential rotation and helicity in Section 3 and the generation and propagation of our simulated magnetic field in Section 4.

2. THE MODEL

As described in Paper I, we model a spherical shell of ionized gas constrained by the solar luminosity, gravity, composition, and average rotation rate. As in Paper II, the top and bottom boundaries have been set at 93% and 46% of the solar radius, respectively. There are seven pressure scale-heights across this shell with the outer two-thirds (in radius) superadiabatic and the inner third subadiabatic. Although most one-dimensional solar models predict a rapid transition from a slightly superadiabatic convection zone to a very subadiabatic radiative interior, we have specified only a slightly subadiabatic stable region below our convection zone in order to avoid high numerical resolution requirements. As a result, our convective motions penetrate further into the stable region than they probably do on the Sun. (The solar abundances of lithium and beryllium place a constraint on the depth of convective motions in the Sun (Vauclair *et al.* 1978).) In this respect, our numerical solutions represent only the qualitative features of a dynamo seated at the base of the convection zone.

We solve the anelastic magnetohydrodynamic equations for the velocity, magnetic, and thermodynamic fields in three dimensions and time. The anelastic approximation filters out short time-scale, small amplitude acoustic waves while retaining the effects of a large density stratification. The nonlinear effects of the unresolved scales are parameterized via viscous, thermal, and magnetic eddy diffusion using subgrid-scale eddy diffusivities (Paper II).

Our dependent variables are expanded in spherical harmonics, $Y_l^m(\theta, \phi)$, and Chebyshev polynomials, $T_n(r)$, with $0 \leq |m| \leq l \leq 31$ and $0 \leq n \leq 32$. A second-order semi-implicit time-integration scheme is employed; and at each time-step nonlinear terms are computed in physical space while spatial derivatives are computed analytically in spectral space. In order to conserve computer time, we have imposed symmetry with respect to the equator such that the latitudinal component of velocity and the radial and longitudinal components of the magnetic field vanish in the equatorial plane. This was the preferred symmetry manifested in our full spherical shell solutions.

A modification we have made for this study is the decoupling of the velocity and magnetic fields in the turbulent convective region in order to simulate a dynamo seated at the base of the convection zone. We do this by forcing the magnetic field to match to a time-dependent, non-prescribed, potential field midway between the top and bottom boundaries of the shell. That is, instead of applying this boundary condition at the top of the shell as was done in Papers I and II, we apply it in the middle of the shell and solve the magnetic induction equation only in the inner half of the shell. Therefore, since the potential field in the outer part of the shell is curl-free, there is no Lorentz force or Joule heating in this region. Note, however, that the velocity and thermodynamic variables are fully simulated throughout the entire shell.

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The resulting potential magnetic field in the outer half of the shell certainly is not a realistic representation of the assumed small-scale flux-tube structure there; however, we wish to study the evolution of the magnetic field in the inner half of the shell, not in the turbulent convective region above. This drastic modification was required, with the present model and affordable numerical resolution, to simulate a dynamo seated at the base of the convection zone.

3. DIFFERENTIAL ROTATION AND HELICITY

The two major ingredients for a cyclic dynamo model (Parker 1955) are differential rotation, i.e., the variation in radius and latitude of angular velocity $<v_\phi/r\sin\theta>$, and helicity, i.e., the dot product of velocity and vorticity $\langle \mathbf{v}\cdot\nabla\times\mathbf{v}\rangle$. (The brackets represent averages in longitude and the variables have their usual meanings.) Differential rotation generates toroidal magnetic fields from poloidal magnetic fields; helicity generates poloidal fields from toroidal fields. We describe in Paper II how the effects of rotation, spherical geometry, and density stratification cause angular momentum to be transported in latitude toward the equator and in radius toward the surface. Although the profile and time dependence of the differential rotation is also determined by the Coriolis forces resulting from the meridional circulation and the viscous and Lorentz forces which try to maintain solid body rotation, our simulated differential rotation is maintained primarily by the convergence of angular momentum flux, i.e., the nonlinear Reynolds stress of the explicitly resolved, large-scale motions which are strongly influenced by rotation, spherical geometry, and density stratification.

A typical profile of our simulated differential rotation (Figure 2a of Paper II) illustrates how angular velocity (relative to the rotating frame) peaks in the equatorial region at the surface and decreases with depth below the surface. The latitudinal variation of the surface rotation rate agrees, to within about 10%, with Doppler measurements of the solar surface differential rotation (Howard *et al.* 1983). The radial variation agrees with a recent analysis (Duvall *et al.* 1984) of the rotational frequency splitting of solar oscillations (Duvall and Harvey 1984) which suggests that angular velocity in the equatorial region decreases by about 15% from 93% to 45% of the solar radius.

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Our simulated differential rotation is quite similar to that obtained from previous nonlinear Boussinesq simulations (Gilman 1983).

The other important ingredient for a cyclic dynamo is helicity. We describe in Paper II how the effects of rotation, spherical geometry, and density stratification cause helicity in the turbulent convective region to be negative, left-handed, in the northern hemisphere and to be positive, right-handed, in southern hemisphere. One contribution to helicity is due to Coriolis forces resulting from the expansion of rising fluid and the contraction of sinking fluid. However, in the inner part of the shell the relative amount of expansion and contraction due to the density stratification is small because there the density scale-height is large. In addition, in this region rising fluid tends to converge horizontally as it begins its ascent and sinking fluid tends to diverge as it terminates its descent. Due to this latter effect, Coriolis forces in this region produce positive helicity in the northern hemisphere and negative helicity in the southern hemisphere. However, our simulated helicity in this region is typically between two and three orders of magnitude smaller than what it is in the outer part of the shell because, in the inner part, convective velocities are small due to the large mass density and the stable entropy gradient. A typical profile of our longitudinally averaged helicity is illustrated in Figure 2b of Paper II.

A somewhat similar helicity configuration has been simulated by Yoshimura (1972) under quite different physical and numerical conditions. He solves a linear equation of motion with a single spherical harmonic for a thin shell representing the outer 10% of the solar radius with no density stratification and in the limit of slow rotation. His resulting helicity in the northern hemisphere is negative (positive) in the outer (inner) regions of his shell with approximately the same amplitude in both regions.

4. MAGNETIC FIELD GENERATION AND PROPAGATION

Previous simulations of a dynamic dynamo in the solar convection zone have produced magnetic fields that propagate too fast and in the wrong direction (Gilman 1983; Glatzmaier 1985). Here we describe the maintenance and evolution of the axisymmetric part of our simulated magnetic field at the base of the convection zone. The profile of the magnetic field, after several thousand time-steps, is

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illustrated in Figure 1 where the toroidal part, i.e., the longitudinal component $\langle B_\phi \rangle$, is represented by contours and the poloidal part, i.e., the radial and colatitudinal components $\langle B_r \hat{\mathbf{r}} + B_\theta \hat{\boldsymbol{\theta}} \rangle$, is represented by lines of force. Instead of beginning with a very small-scale, randomly structured seed magnetic field (Papers I and II), for this study we have initialized the magnetic field with relatively smooth functions.

According to stellar dynamo theory, differential rotation shears the poloidal field generating a new toroidal field and helical fluid motions twist the toroidal field generating a new poloidal field while eddy diffusion continually works to destroy both fields. Since our simulated angular velocity increases with distance from the rotation axis (Figure 2a of Paper II), it shears the outward directed poloidal field at low latitude (Figure 1b, in the northern hemisphere) producing a toroidal field contribution in the direction of rotation (into the paper). Likewise, the rotational shearing of the inward directed poloidal field at high latitude (Figure 1b, in the northern hemisphere)

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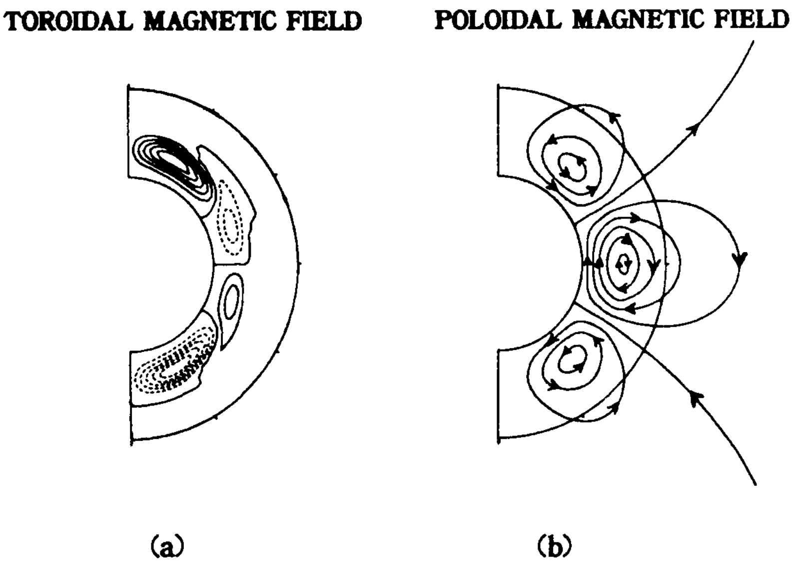

FIGURE 1 (a) Solid (broken) contours in the meridian plane represent the toroidal magnetic field directed into (out of) the paper. (b) Lines of force represent the poloidal magnetic field.

produces a toroidal field contribution in the opposite direction of rotation (out of the paper). The opposite toroidal field contributions are generated in the southern hemisphere. Consequently, the toroidal fields (Figure 1a) are being destroyed on their poleward sides and enhanced on their equatorward sides, except near the equatorial plane. This is illustrated in Figure 2a where the contribution to the time derivative of the toroidal field due to rotational shearing of the poloidal field,

$$\frac{\partial}{\partial t}\langle B_\phi\rangle = r\sin\theta\left[\langle B_r\rangle\frac{\partial}{\partial r}\left\langle\frac{v_\phi}{r\sin\theta}\right\rangle + \frac{\langle B_\theta\rangle}{r}\frac{\partial}{\partial\theta}\left\langle\frac{v_\phi}{r\sin\theta}\right\rangle\right] + \ldots,$$

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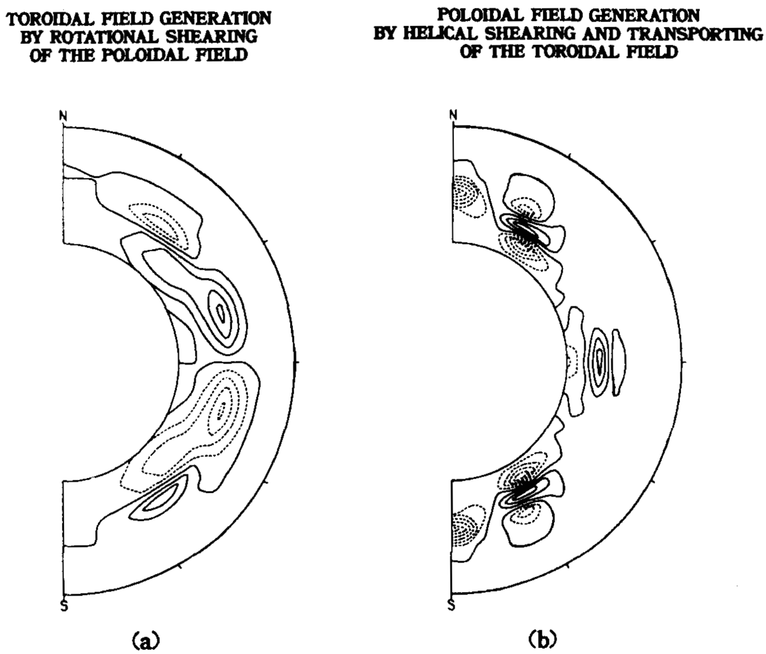

FIGURE 2 (a) Solid (broken) contours in the meridian plane represent the contribution directed into (out of) the paper to the toroidal magnetic field made by the rotational shearing of the poloidal magnetic field. (b) Solid (broken) contours represent the positive (negative) contribution to the poloidal magnetic field energy made by the shear and transport processes operating on the twisted toroidal magnetic field.

is plotted in a meridian plane for the same time step depicted in Figure 1. It predicts the toroidal field phase propagation toward the equator at the base of the convection zone. Note that twisting the nonaxisymmetric magnetic fields by the nonaxisymmetric motions also contributes to the axisymmetric toroidal magnetic field; however, this contribution, which tends to oppose the above effect, is about five times smaller than the effect of the differential rotation illustrated in Figure 2a.

The propagation of the axisymmetric part of our toroidal field in the inner part of the shell is illustrated in Figure 3. The five profiles span a simulated time of four years in approximately equal increments. It is difficult to estimate what the period of the magnetic cycle would be when only a fraction has been simulated; however, if the fields would continue to propagate at the present rate, the period would be relatively close to the observed solar period.

The poloidal magnetic field is maintained against diffusion by helical fluid motions that twist the toroidal field. In the northern hemisphere, right-handed helical fluid motions at the base of the convection zone (Figure 2b of Paper II) twist toroidal field lines into left-handed lines of force; and vice versa in the southern hemisphere. (This is illustrated schematically in Paper II for the opposite helicity

TOROIDAL MAGNETIC FIELD

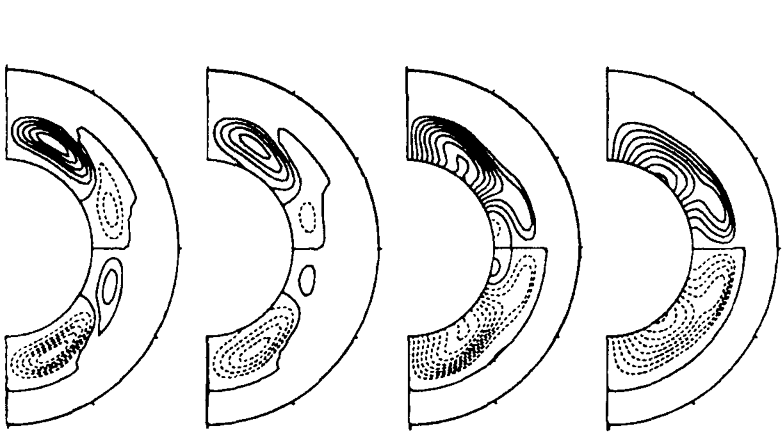

FIGURE 3 A sequence (left to right) of toroidal magnetic field profiles (as described in Fig. 1a) spanning four years of simulated time.

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in the outer part of the shell.) One can envision the left-handed (right-handed) helical lines of force in the northern (southern) hemisphere by adding the axisymmetric toroidal and poloidal fields in Figure 1. The contribution made to the time derivative of the energy in the axisymmetric poloidal magnetic field by the shear and transport processes operating on the twisted toroidal magnetic field,

$$\frac{1}{8\pi}\frac{\partial}{\partial t}\langle B_r^2+B_\theta^2\rangle=\frac{1}{4\pi r}\left[\frac{\langle B_r\rangle}{\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\langle v_r B_\theta-v_\theta B_r\rangle)\right.$$

$$\left.+\langle B_\theta\rangle\frac{\partial}{\partial r}(r\langle v_\theta B_r-v_r B_\theta\rangle)\right]+\ldots,$$

is plotted in Figure 2b. This plot illustrates how at least the centers of the poloidal loops (Figure 1b) are maintained (solid contours in Figure 2b) by twisting the toroidal field (Figure 1a).

Since the poloidal field is generated by twisting the toroidal field and is destroyed by eddy diffusion, it should propagate toward the equator following the toroidal field (Figure 3). However, although our simulated poloidal field initially propagated a small amount toward the equator, it eventually stopped. This was probably due to our inadequate numerical resolution of the shear and transport processes in the inner part of the shell. The problem is that we need to simulate the convective motions in the outer part of the shell in order to simulate their penetration into the inner part where they maintain the desired differential rotation and helicity. However, it is difficult to maintain sufficient radial resolution of the small amplitude motions in the dense, subadiabatic, inner part of the shell while also resolving the large amplitude motions in the less dense, superadiabatic region above.

5. SUMMARY

We have simulated a dynamic dynamo at the base of the solar convection zone. However, since we are unable to numerically resolve, with our global model, the thin magnetic flux tubes that presumably thread the turbulent convective region, we make no attempt to do so. Instead, we match our simulated magnetic field to

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a potential field in the convective region. That is, we solve for the giant-cell convection in the outer, superadiabatic part of the shell and the resulting convective overshooting into the inner, subadiabatic part of the shell; but the magnetic field is solved dynamically with full nonlinear feedback only in the inner part of the shell.

In this region our simulated angular velocity decreases with depth due to the transport of angular momentum; and our simulated helicity is positive in the northern hemisphere and negative in the southern hemisphere due to the horizontal divergence (convergence) of sinking (rising) fluid near the bottom of the overshooting convective cells. In addition, the amplitude of the helicity in this inner part of the shell is much smaller than in the outer part due to the large mass density and stable entropy gradient in the inner part. As a result, toroidal magnetic fields, which are generated by the rotational shear of poloidal fields, and poloidal magnetic fields, which are generated by the twisting of toroidal fields, initially propagated toward the equator with a phase velocity in fairly good agreement with that inferred from the solar butterfly diagram.

Unfortunately, our simulated poloidal fields would not continue to propagate due to our inadequate numerical resolution of the shear and transport processes in the inner part of the shell that twist toroidal fields into new poloidal fields. Without this effect the toroidal field propagation would also soon terminate. As a result, we were unable to simulate a complete magnetic cycle. The problem is that we need to explicitly resolve the large amplitude, giant-cell convection in the outer part of the shell in order to simulate the small amplitude, convective overshooting into the stable region below.

Better numerical resolution is needed in the inner half of the shell. Not only would this improve the dynamo simulations, it would allow a more realistic representation of convective penetration into a very subadiabatic region. Increasing the number of Chebyshev polynomials we use from 33 to 65 may give us enough resolution; but the corresponding increase in computer time by about a factor of eight for the same amount of simulated time would be prohibitively expensive. The work described in Paper I, Paper II, and in this paper already represents several hundred hours of Cray computer time. A more practical approach would be to use a radial mapping that enhances the resolution in the inner part of the shell while reducing the resolution in the outer part.

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Although our dynamic dynamo simulations have demonstrated only an initial tendency, they suggest that the solar dynamo could be operating at the base of the convection zone if the bulk of the convection zone is too turbulent to maintain a large-scale dynamo. (Note that our simulated differential rotation and helicity profiles are the opposite of that assumed in most kinematic dynamo models of the Sun.) On the other hand, although we feel that improving our numerical resolution would enable us to simulate magnetic cycles in this region, we can not be certain of this until improved simulations are made.

The hypothesis that the solar dynamo is seated at the base of the convection zone also has some problems. One is that, when the longitudinal component of the longitudinally averaged magnetic field on the Sun (inferred from the polarity configuration of sunspot groups and bipolar magnetic regions) is in the direction of rotation, the radial component (obtained from Mt. Wilson magnetograms) is directed downward; and vice versa (Stix 1976). Although this constraint is satisfied in Figure 1, it no longer is at the end of our simulation. On the other hand, it is difficult to estimate how the turbulent convection zone may distort the large-scale magnetic structure generated at the base of the convection zone. However, this leads to another problem. It is uncertain how Hale's polarity law (Hale 1924) for sunspot groups observed on the solar surface could be maintained by flux tubes rising through the convection zone. Since the polarity law is so well maintained, one assumes the magnetic flux tubes are still connected to the main toroidal field below the surface. However, it is difficult to imagine how a perturbation loop in the toroidal field could extend up through the entire convection zone, maintaining a relatively small horizontal separation between the two polarities, without undergoing magnetic reconnection.

Another possibility worth considering is that the solar dynamo may be operating in the outer 5% of the solar radius which has been excluded in our model because of the high numerical resolution requirements there. Angular velocity may be increasing with depth in the top layer as suggested by the rotational frequency spitting of solar oscillations (Duvall *et al.* 1984), by the difference between sunspot and Doppler rotation velocities (Snodgrass 1983; Howard *et al.* 1983), by thin shell Boussinesq simulations (Gilman and Foukal 1979), and by linear anelastic simulations (Glatzmaier and Gilman

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1982). Helicity in this region should be negative (positive) in the northern (southern) hemisphere; however, it is uncertain how effective helicity is when convective motions and magnetic flux tubes are spatially separated. In addition, magnetic buoyancy considerations suggest that a magnetic cycle of 22 years would be difficult to achieve if the dynamo were operating just below the solar surface (Parker 1975). If the solar dynamo were operating in this thin top layer, the dynamics of the small-scale flux tubes certainly would be an important feature.

Although progress is being made, better simulations and observations are required before we can feel confident about our understanding of the basic properties of the solar dynamo which may be much more complicated and possibly very different than what current dynamo theory would lead one to believe.

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