A State-Queueing Model of Thermostatically Controlled Appliances

Ning Lu, *Member, IEEE,* and David P. Chassin, *Member, IEEE*

***Abstract*—This paper develops a state-queueing model to analyze the price response of aggregated loads consisting of thermostatically controlled appliances (TCAs). Assuming a perfectly diversified load before the price response, we show that TCA setpoint changes in response to the market price will result in a redistribution of TCAs in on/off states and therefore change the probabilities for a unit to reside in each state. A randomly distributed load can be partially synchronized and the aggregated diversity lost. The loss of the load diversity can then create unexpected dynamics in the aggregated load profile. Raising issues such as restoring load diversity and damping the peak loads are also addressed in this paper.**

***Index Terms*—Demand-side management program, load control, load modeling, price responsive load, power distribution, state-queueing model, thermostatically controlled appliances.**

I. Introduction

DEMAND-SIDE management (DSM) programs [1] and load response, in general, have long been considered effective strategies to alleviate excessive price volatility and its adverse impact on electricity markets when there is a shortage of generation and transmission capacity. In a bid-based market, increased demand elasticity has a moderating effect on suppliers, reducing their ability to exert market power and set the price above the competitive price. Because loads can be viewed as negative generation, price-responsive loads can also bid in the ancillary service market to enhance system reliability. However, the influences of the load-responsive programs on the overall load profile are not well understood. Issues such as the loss of load diversity caused by synchronous on/off equipment behaviors, load peak shaving percentages, and peak-load shifting time need to be examined to ensure that load response is an effective way to reduce the system stress and enhance the system reliability, without introducing detrimental effects to the system.

There are a number of DSM programs that claim to be able to respond to market price and significantly reduce the system load on call. Depending on the nature of electricity usage, responses to the spot prices can be classified into four different categories: curtailment [2], substitution [3], storage [4], and shift. Load shift is done by pre-consumption or post-consumption of the electricity to reduce the power consumption during the anticipated peak-price period. An important feature of this type of load management program is that it targets the cyclic loads such as thermostatically controlled appliances (TCAs). The power consumption is determined by the appliances' ability to "coast" through the peak-price period, rather than the substitution of other energy sources for electricity. Therefore, the on-peak consumption can be shifted rather than simply reduced, and the electricity will have to be consumed either earlier or later.

Probabilistic calculations of aggregate storage loads [10] and duty cycle approach [11] have been developed for aggregate cycling load models. These models are based on the end-use data analysis. Historical data collected at the substation level are used to account for nonengineering factors such as weather patterns and customer behaviors. The drawback of these models is that they are empirically driven and noninteractive. The thermodynamic and cyclic character of the loads and their controls is not considered either.

This paper focuses on the behavior of TCAs when they are capable of price response, as is the case with building heating ventilation and air conditioning (HVAC) systems. Based on the thermal model developed in [5], we study the load diversity and state shift behavior of HVAC systems after a change in energy price, in response to which their thermostat setpoints are changed. Physically-based methodologies for synthesizing the hourly residential HVAC load, such as was developed by Chan *et al.* [13] can be used to evaluate the impacts of load management programs. An estimate of the diversified load from a limited number of load shapes of individual households was used to form a load-diversification model. Our approach differs by assuming a uniformly randomized load as an initial condition, from which we derive the probability distribution function (pdf) after the setpoint changes. This reveals the fundamental reason for the reduced load diversity in large scale direct load control systems, which has been observed and discussed by Weller in [12]. Based on the pdf, the resulting load behaviors are successfully captured. A computationally efficient state-queueing model is developed to model the price response of TCAs. Because TCAs contribute to a large fraction of the total residential load, this research can be applied to create simulation tools to study the effectiveness of residential load-response programs, which are important in the establishment of sustainable and successful DSM programs.

II. Characteristics of TCAs

TCAs include residential HVAC systems, electric water heaters, and refrigerators. Because the characteristics of these TCAs are similar and therefore can be modeled in a similar fashion, this paper focuses on HVAC models for illustrative

Manuscript received January 13, 2004. This work was supported by the Pacific Northwest National Laboratory, operated for the U.S. Department of Energy by Battelle Memorial Institute, under Contract DE-AC06-76RL01830.

The authors are with the Energy Science and Technology Division, Pacific Northwest National Laboratory, Richland, WA 99352 USA (e-mail: ning.lu@pnl.gov; David.Chassin@pnl.gov).

Digital Object Identifier 10.1109/TPWRS.2004.831700

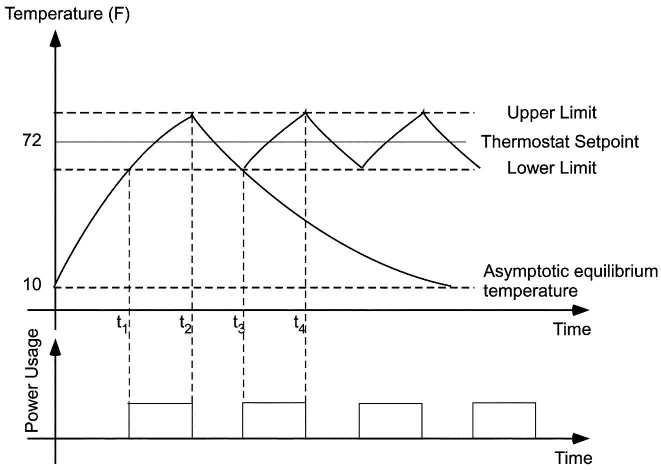

Fig. 1. Differential models of controlled thermal behavior.

purposes. Fig. 1 shows the thermal behavior of an HVAC unit over time. The rising curves indicate the operation of the HVAC heater, when the home heating system is "on", and the falling curves represent the cooling down period, when the home heating system is "off". During the summer air-conditioning months, the situation is reverse and the "on" period occurs during the cooling cycle. As the HVAC unit cycles, the temperature in the house rises and falls accordingly. The cycling points of the HVAC unit are shown as times t_1, t_2, t_3, and t_4 [5].

The upper and lower limits are set by the thermostat setpoint, and changing the setpoint allows one to regulate the power consumption of the HVACs. Because the asymptotic equilibrium temperatures are generally far beyond these limits for appropriately sized equipment, the exponential rising curve and falling curve are almost linear between the upper limit and the lower limit, as shown in Fig. 1. To simplify the analysis, a linear approximation of HVAC thermal characteristics, as shown in Fig. 2, is used in our model.

III. Price Responses of HVACs

An HVAC unit can respond to market price p by rising or lowering its setpoint T, which will then determine its power output P. Therefore, we have control functions (1) and (2), as follows:

$$T = f(p) \tag{1}$$

$$P = f(T). \tag{2}$$

Fig. 3 shows several possible T versus p curves, based on which HVAC units can respond to market price changes. For example, based on curve 1, the setpoint is 72 °F for an HVAC unit when market price is between 50 and 100 \$/MWh. When the price rises higher than 100 $\$/\mathrm{MWh}$, the thermostat setpoint of the unit will be lowered to 69 °F.

This paper focuses on solving (2) to calculate the aggregated TCA load P after the setpoint T changes. The TCAs are assumed to have same thermal characteristics and use the same control functions.

Two cases are studied in the following sections: the TCA response to a price increase and the TCA response to a price decrease.

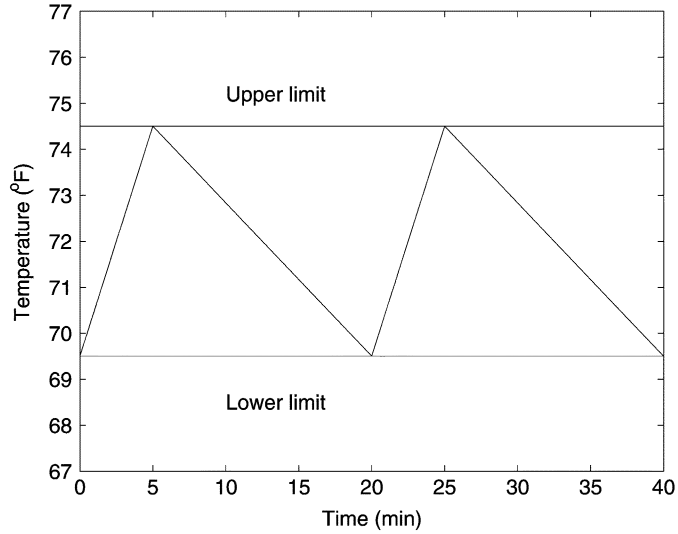

Fig. 2. The simplified thermal characteristic of an HVAC.

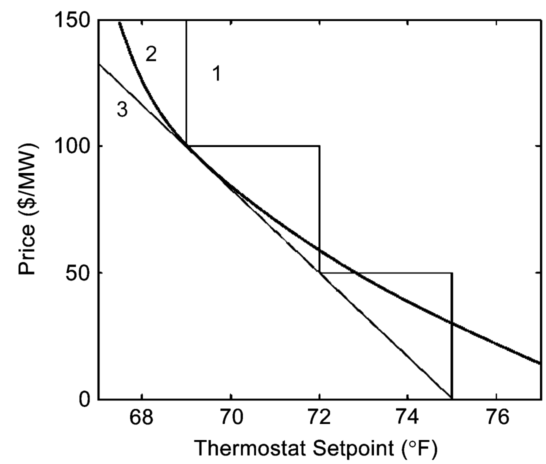

Fig. 3. Setpoint changes in respond to price changes.

A. Initial Conditions

We start the derivation with a simple example. We observe a system containing N HVAC units with the initial thermal states shown in Fig. 4, where a unit will be "on" for 5 min and "off" for 15 min for a given ambient temperature and a given setpoint. T_+ and T_- are the upper and lower temperature limit for a given setpoint T. A state is then defined by both the temperature and the on/off status of a unit. We subdivide the time cycle into 20 states of equal duration such that there are five distinct "on" states and 15 distinct "off" states in a temperature range of $[T_-, T_+]$. Initially, we assume a uniformly diversified load and the units are distributed uniformly among all 20 states. If the whole time period is divided into 20 time steps, then at time Step 1, we will have nearly equal numbers of HVAC units in each of the 20 states. At the end of each time step, units will move one state ahead. For example, at time Step 2, units in State 1 will move to State 2, units in State 2 to State 3, and units in State 20 to State 1, as shown in Table I.

Assume the power of each unit P is equal. The total load at each time step is simply the total number of units in all the "on" states (States 1–5).

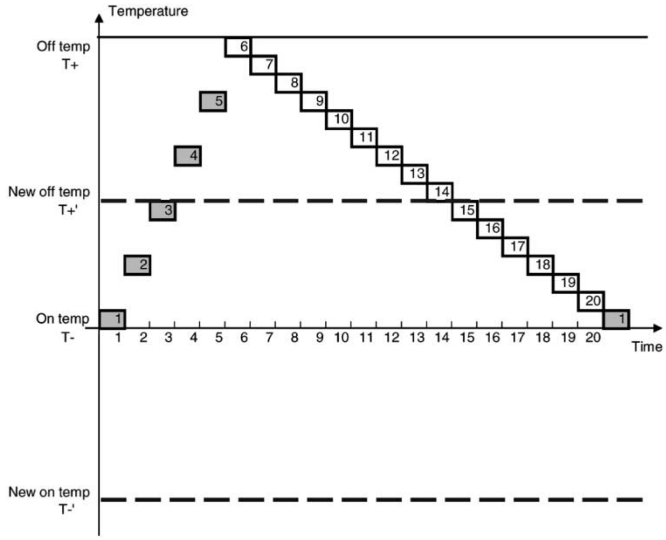

Fig. 4. The states of an aggregated TCA load.

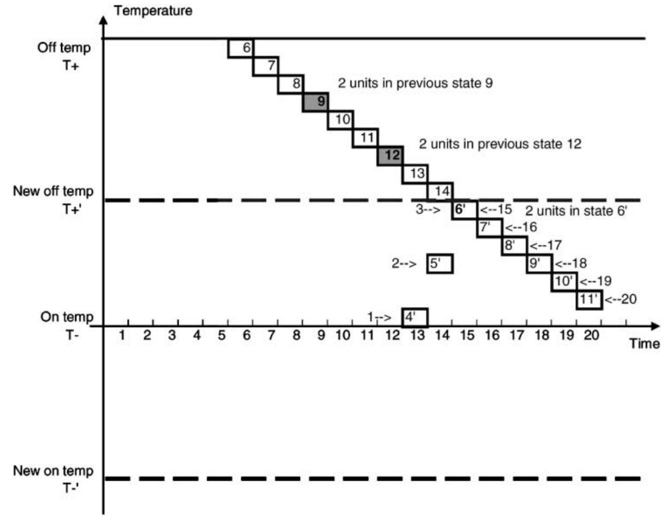

Fig. 5. The state degeneracy.

TABLE I
STATE DISTRIBUTION OF THE HVAC UNITS

	State	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	1
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	1	2
	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	1	2	3
	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	1	2	3	4
	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	1	2	3	4	5
	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	1	2	3	4	5	6
	7	8	9	10	11	12	13	14	15	16	17	18	19	20	1	2	3	4	5	6	7
	8	9	10	11	12	13	14	15	16	17	18	19	20	1	2	3	4	5	6	7	8
	9	10	11	12	13	14	15	16	17	18	19	20	1	2	3	4	5	6	7	8	9
	10	11	12	13	14	15	16	17	18	19	20	1	2	3	4	5	6	7	8	9	10
	11	12	13	14	15	16	17	18	19	20	1	2	3	4	5	6	7	8	9	10	11
	12	13	14	15	16	17	18	19	20	1	2	3	4	5	6	7	8	9	10	11	12
	13	14	15	16	17	18	19	20	1	2	3	4	5	6	7	8	9	10	11	12	13
	14	15	16	17	18	19	20	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	15	16	17	18	19	20	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	16	17	18	19	20	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	17	18	19	20	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
	18	19	20	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
time	**19**	20	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
	20	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

When the units are uniformly distributed among the states, the expected load P_L can be calculated as:

$$E(P_L) = E(N_{\rm on})P = \frac{n_{\rm on}}{n}NP = \frac{5}{20}NP$$

where n is the total number of states and $n_{\rm on}$ is the number of "on" states. N is the number of units, $N_{\rm on}$ is the number of "on" units, and P is the power demand of each HVAC unit.

Table I shows the state evolution along the timeline. Columns represent time and rows represent the unit temperature corresponding to the states. The shaded states are the "on" states. One can calculate the aggregated output by summing up the power consumption of all the units in the "on" states.

For example, if there is a 1-kW unit in each of the 20 states, the aggregated output at any time is 5 kW because at any one time, there are five "on" states in the system.

B. State Distribution in Response to Price Increase

Let us assume that the TCAs respond to a price increase by lowering their setpoints (see Fig. 5), which will result in a lower on/off temperature limit.

A close examination of the moment when the change happens reveals a degeneracy of states: States 4 and 5 combine with States 12 and 9. Because the units in States 4 and 5 immediately turn off as a result of the shift-down of the "off" temperature threshold, they are now at the same temperature and are indistinguishable from States 12 and 9.

TABLE II
STATE DISTRIBUTION OF THE TCAs AFTER A PRICE INCREASE

State	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	# of "on"
1	4	5	6	off	off	off	off	off	off	off	off	off	off	off	6	7	8	9	10	11	2
2	5	6	7	off	off	off	off	off	off	off	off	off	off	6	7	8	9	10	11	12	1
3	6	7	8	off	off	off	off	off	off	off	off	off	6	7	8	9	10	11	12	13	0
4	7	8	9	6	off	off	off	off	off	off	off	6	7	8	9	10	11	12	13	14	0
5	8	9	10	7	off	off	off	off	off	off	6	7	8	9	10	11	12	13	14	15	0
6	9	10	11	8	off	off	off	off	off	6	7	8	9	10	11	12	13	14	15	16	0
7	10	11	12	9	6	off	off	off	6	7	8	9	10	11	12	13	14	15	16	17	0
8	11	12	13	10	7	off	off	6	7	8	9	10	11	12	13	14	15	16	17	18	0
9	12	13	14	11	8	off	6	7	8	9	10	11	12	13	14	15	16	17	18	19	0
10	13	14	15	12	9	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	0
11	14	15	16	13	10	7	8	9	10	11	12	13	14	15	16	17	18	19	20	1	1
12	15	16	17	14	11	8	9	10	11	12	13	14	15	16	17	18	19	20	1	2	2
13	16	17	18	15	12	9	10	11	12	13	14	15	16	17	18	19	20	1	2	3	3
14	17	18	19	16	13	10	11	12	13	14	15	16	17	18	19	20	1	2	3	4	4
15	18	19	20	17	14	11	12	13	14	15	16	17	18	19	20	1	2	3	4	5	5
16	19	20	1	18	15	12	13	14	15	16	17	18	19	20	1	2	3	4	5	6	6
17	20	1	2	19	16	13	14	15	16	17	18	19	20	1	2	3	4	5	6	7	7
18	1	2	3	20	17	14	15	16	17	18	19	20	1	2	3	4	5	6	7	8	8
19	2	3	4	1	18	15	16	17	18	19	20	1	2	3	4	5	6	7	8	9	9
20	3	4	5	2	19	16	17	18	19	20	1	2	3	4	5	6	7	8	9	10	9
21	4	5	6	3	20	17	18	19	20	1	2	3	4	5	6	7	8	9	10	11	8
22	5	6	7	4	1	18	19	20	1	2	3	4	5	6	7	8	9	10	11	12	8
23	6	7	8	5	2	19	20	1	2	3	4	5	6	7	8	9	10	11	12	13	7
24	7	8	9	6	3	20	1	2	3	4	5	6	7	8	9	10	11	12	13	14	6
25	8	9	10	7	4	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	6
26	9	10	11	8	5	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	5
27	10	11	12	9	6	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	3
28	11	12	13	10	7	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	2
29	12	13	14	11	8	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	1
30	13	14	15	12	9	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	0

Furthermore, under the new setpoint, if we redefine the states in the same pattern used for the original control band, then an "out-of-regime" state transition occurs when the setpoint changes. For example, the previous States 1 and 2 now become States $4'$ and $5'$. However, as shown in Fig. 5, previous States 6–14 have to decay into the new state regime because the temperatures of the units are higher than the temperature range set by the new setpoint, therefore placing them outside the new regime of states. The temperatures of the units in these "out-of-regime" states will eventually decay into the new temperature setting range shown in Table II. Because the transition of these out-of-regime states into allowed states is deferred, the overall load profile will experience a transient reduction, the length of which is determined by the temperature decay rate. After all the states degeneracy and phase alignments are complete, the system will reach a steady state, which is also shown in Table II. If there is one unit in each state prior to the

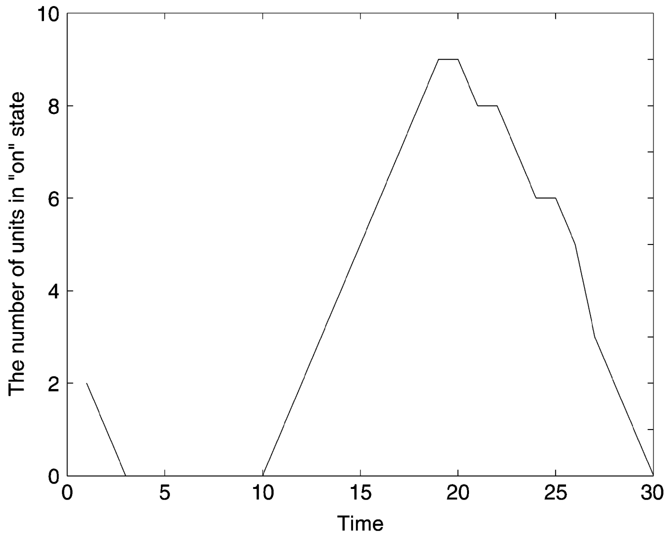

Fig. 6. The number of units in "on" state after a price increase.

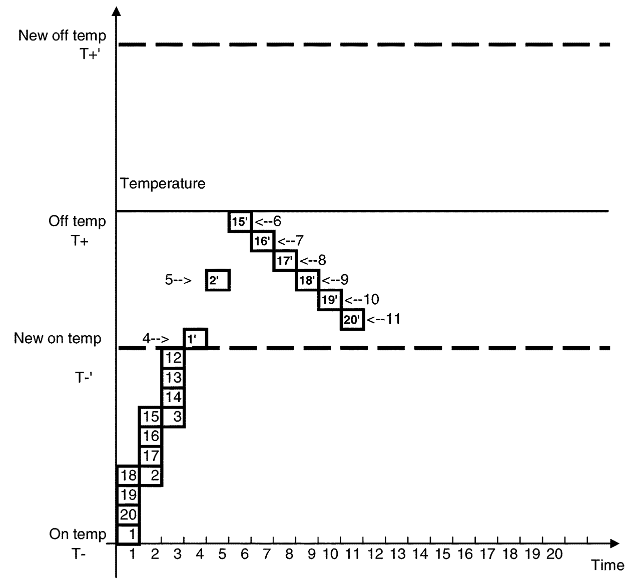

Fig. 7. The state degeneracy after a price reduction.

setpoint change, then the number of units that are "on" at each time step is shown in Fig. 6.

We make three observations based on these results.

1) There is a significant reduction of the load immediately after the lowering of the setpoints because the HVAC units at a temperature higher than T'_+ will turn off under the new setting. The remaining "on" units will turn off shortly, when they reach their new T_+ (Table II, times 1 and 2)
2) Because the cooling period is much longer than the heating period, the load output remains zero for a considerable length of time depending principally on how far the setpoint drops (Table II, times 3–8)
3) When the "off" units reach the lower temperature limit, they turn on, and the load begins to increase. Because of state degeneracy, there are more units in some states than in others', the number of units that remain in "on" states is no longer a constant average value along the timeline. Assuming there is one unit in each state prior to the setpoint change, then after the setpoint change, the total number of "on" units will have a maximum at eight and a minimum at one, as shown in Table II and Fig. 6 after time eight. Recall in Section III-A, the total number of "on" units is five all the time. Because $P_L = N_{\text{on}}P$ in the aggregated load profile, we will expect a dynamic similar to that shown in Fig. 6.

C. State Distribution in Response to Price Decrease

We can perform a similar analysis for the TCA response to a price decrease. We assume that the thermostat setpoint will be raised when the energy price drops. Raising the setpoint results in higher "off" temperature and "on" temperature limits of the TCAs. The degeneracy and the shift of the states are shown in Fig. 7 and Table III.

There are three observations based on the results.

1) Initially, there is a sharp increase of the load because all the TCAs at a temperature lower than T'_+ will turn on under the new setting, while the units that were on remain on. The peak load can reach much higher than the peak load that initiated the price increase.
2) The transient period is shorter than the previous case because the heating period is significantly shorter than the cooling period for HVAC units.
3) Fig. 8 shows that the synchronization behaviors are stronger than the previous case, which creates a much higher and longer peak and valley period in the overall load profile.

This reveals an interesting phenomenon. If we decrease the price prior to the expected peak price period, we may reduce the load during the peak price period *without the need for a price increase*. This "pre-loading" can be effective; however, the risk is that subsequently many HVAC units will turn on and off at the same time. This can put a significant stress on the distribution network. In the next section, we will discuss this problem further.

TABLE III
STATE DISTRIBUTION OF THE TCAs AFTER A PRICE REDUCTION

	State	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	# of "on"
Transient State	1	on	on	on	1	2	15	16	17	18	19	20	1	on	on	on	on	on	on	on	on	14
	2	on	on	1	2	3	16	17	18	19	20	1	2	1	1	1	on	on	on	on	on	15
	3	on	1	2	3	4	17	18	19	20	1	2	3	2	2	2	1	1	1	on	on	16
	4	1	2	3	4	5	18	19	20	1	2	3	4	3	3	3	2	2	2	1	1	17
Steady State	5	2	3	4	5	6	19	20	1	2	3	4	5	4	4	4	3	3	3	2	2	17
	6	3	4	5	6	7	20	1	2	3	4	5	6	5	5	5	4	4	4	3	3	16
	7	4	5	6	7	8	1	2	3	4	5	6	7	6	6	6	5	5	5	4	4	12
	8	5	6	7	8	9	2	3	4	5	6	7	8	7	7	7	6	6	6	5	5	7
	9	6	7	8	9	10	3	4	5	6	7	8	9	8	8	8	7	7	7	6	6	3
	10	7	8	9	10	11	4	5	6	7	8	9	10	9	9	9	8	8	8	7	7	2
	11	8	9	10	11	12	5	6	7	8	9	10	11	10	10	10	9	9	9	8	8	1
	12	9	10	11	12	13	6	7	8	9	10	11	12	11	11	11	10	10	10	9	9	0
	13	10	11	12	13	14	7	8	9	10	11	12	13	12	12	12	11	11	11	10	10	0
	14	11	12	13	14	15	8	9	10	11	12	13	14	13	13	13	12	12	12	11	11	0
	15	12	13	14	15	16	9	10	11	12	13	14	15	14	14	14	13	13	13	12	12	0
	16	13	14	15	16	17	10	11	12	13	14	15	16	15	15	15	14	14	14	13	13	0
	17	14	15	16	17	18	11	12	13	14	15	16	17	16	16	16	15	15	15	14	14	0
	18	15	16	17	18	19	12	13	14	15	16	17	18	17	17	17	16	16	16	15	15	0
	19	16	17	18	19	20	13	14	15	16	17	18	19	18	18	18	17	17	17	16	16	0
	20	17	18	19	20	1	14	15	16	17	18	19	20	19	19	19	18	18	18	17	17	1
	21	18	19	20	1	2	15	16	17	18	19	20	1	20	20	20	19	19	19	18	18	3
	22	19	20	1	2	3	16	17	18	19	20	1	2	1	1	1	20	20	20	19	19	8
	23	20	1	2	3	4	17	18	19	20	1	2	3	2	2	2	1	1	1	20	20	13
	24	1	2	3	4	5	18	19	20	1	2	3	4	3	3	3	2	2	2	1	1	17
	25	2	3	4	5	6	19	20	1	2	3	4	5	4	4	4	3	3	3	2	2	17
	26	3	4	5	6	7	20	1	2	3	4	5	6	5	5	5	4	4	4	3	3	16
	27	4	5	6	7	8	1	2	3	4	5	6	7	6	6	6	5	5	5	4	4	12
	28	5	6	7	8	9	2	3	4	5	6	7	8	7	7	7	6	6	6	5	5	7
	29	6	7	8	9	10	3	4	5	6	7	8	9	8	8	8	7	7	7	6	6	3
	30	7	8	9	10	11	4	5	6	7	8	9	10	9	9	9	8	8	8	7	7	2

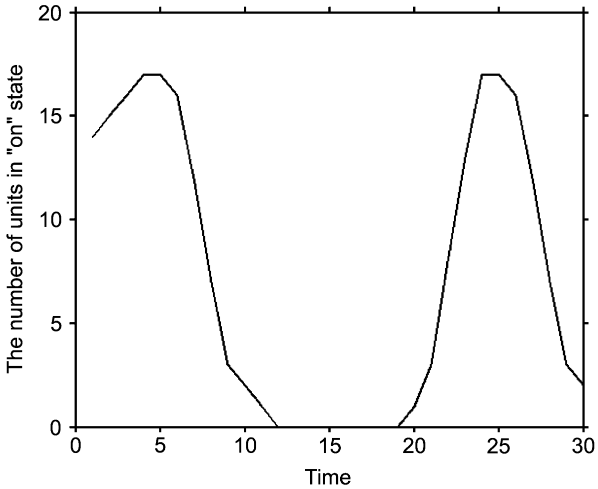

Fig. 8. The number of units in "on" state after a price reduction.

IV. The Price-Response Impacts on Load Diversities

On a distribution feeder, the demand is not simply an aggregation of randomly distributed individual load demand. The correlation between loads must be accounted for. Diversity factors as a function of number of customers are used in distribution circuit loading calculations [6]. Diversity factor k_d is defined as [7]–[9]

$$k_d = \frac{\sum_{i=1}^{n} D_i}{D_g}$$

where D_i is the maximum demand of load i and D_g is the coincident maximum demand of a group of n loads. k_d will have a value between $[1, +\infty]$. There are two extreme cases. If k_d is 1, all the individual loads are at "on" states. If k_d is infinity, all the individual loads are at "off" states. The lower value k_d has, the more individual loads are at "on" states.

Assume that a feeder's load consists of N HVAC units. Each unit has a cycling time of τ with an "on" period of τ_{on} and an "off" period of τ_{off}. Also assume that there are n states along the period of τ and each state evolves to the next state after the same deterministic time interval. Initially, if the units are uniformly distributed among each state, then the probability for a unit to reside in any of the n states is simply $1/n$. The diversity factor is calculated by

$$k_d = \frac{NP}{N\frac{n_{\text{on}}}{n}P} = \frac{NP}{N\frac{\tau_{\text{on}}}{\tau}P} = \frac{\tau}{\tau_{\text{on}}}.$$

For the HVAC example given in Section III, there are 20 states consisting of 5 "on" states and 15 "off" states; therefore, the probability for a unit to be in any of the 20 states is $1/20$ and the diversity factor is 4.

To illustrate the impact of the price response on load diversity, we examine the case where TCA setpoints are lowered in response to a price increase. After lowering the TCA setpoints, two things will happen (see Fig. 5). First, the states will immediately divide into in-regime states and out-of-regime states. Second, some states will be depopulated and some will have population increases. The setpoint changes will force the units whose temperatures are above the new off-temperature limit to turn-off; therefore, they will "move" to the "off" states from the out-of-regime state. If the ratio of the setpoint change is

$$k = \frac{T^{+} - T^{+'}}{T^{+} - T^{-}}$$

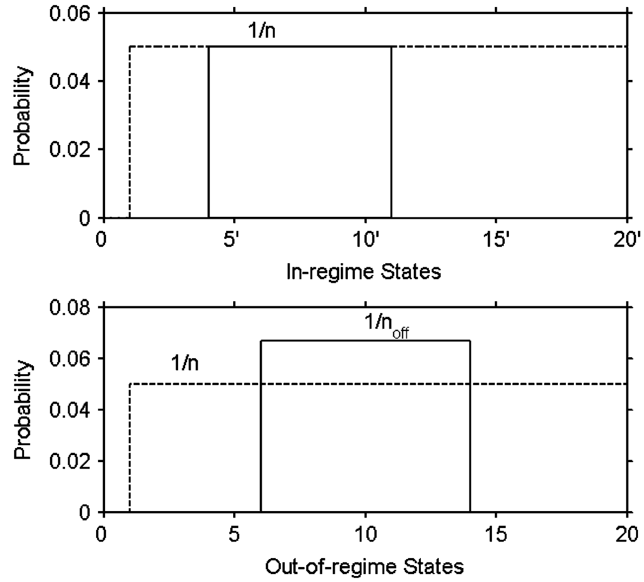

Fig. 9. The pdf of the in-regime states and the out-of-regime states of the example in Section III.

then the probability of finding a unit in out-of-regime states (States 6–14) is

$$\Pr\{x > T^{+}\} = \frac{kn_{\text{on}}\frac{N}{n} + kn_{\text{off}}\frac{N}{n}}{kn_{\text{off}}N} = \frac{1}{n_{\text{off}}}$$

where n_{off} is the number of "off" states (Fig. 9). Note that $1/\text{n}_{\text{off}}$ is greater than $1/\text{n}$, showing an increased state occupancy for out-of-regime states. The remaining states with a temperature within the new settings are in-regime (States $4'$–$11'$). Because the number of units in an in-regime state is unchanged, the probability for a unit to reside in an in-regime state is also unchanged and remains at $1/n$, as shown in Fig. 9. However, those states below the old on-temperature limits are not occupied yet. So, in a sense, they are depopulated.

The decay process will merge the out-of-regime states with the in-regime states and will repopulate some of the depopulated states among the in-regime states. The overall effect after the price-response signal is that the pdf is no longer a uniform distribution but a superposition of the two pdfs.

The probability for a unit to be in state x after lowering TCA setpoints at the end of the transient is

$$\begin{aligned}
& k\tau_{\mathit{off}} \leq \tau_{on} - k\tau_{on}, \\
& \quad \Pr\{x|t \leq k\tau\} = 0, \\
& \quad \Pr\{x|k\tau < t \leq \tau_{on}\} = \frac{1}{n}, \\
& \quad \Pr\{x|\tau_{on} < t \leq \tau_{on} + k\tau_{\mathit{off}}\} = \frac{1}{n} + \frac{1}{n_{\mathit{off}}} \\
& \quad \Pr\{x|\tau_{on} + k\tau_{\mathit{off}} < t \leq \tau\} = \frac{1}{n} \\
& k\tau_{\mathit{off}} > \tau_{on} - k\tau_{on} \\
& \quad \Pr\{x|t \leq \tau_{on}\} = 0 \\
& \quad \Pr\{x|\tau_{on} < t \leq k\tau\} = \frac{1}{n_{\mathit{off}}} \\
& \quad \Pr\{x|k\tau < t \leq \tau_{on} + k\tau_{\mathit{off}}\} = \frac{1}{n} + \frac{1}{n_{\mathit{off}}} \\
& \quad \Pr\{x|\tau_{on} + k\tau_{\mathit{off}} < t \leq \tau\} = \frac{1}{n}
\end{aligned}$$

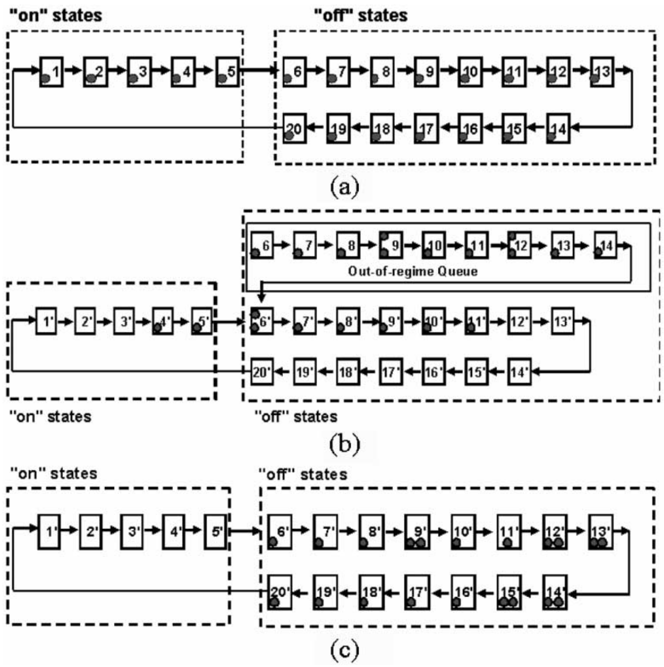

Fig. 10. A queue representation of the state transitions: price-increase response case.

where

$$n_{\text{off}} = \frac{\tau_{\text{off}}}{\tau} n.$$

The probability of a unit staying in a specific state can then be calculated, and one can estimate the number of units in each state. By summing up the number of units in all the "on" states, the total power consumption can be estimated accordingly. However, because the units are no longer uniformly distributed in each state, the total number of "on" units is no longer a constant average value along the time line. The aggregated output will change with respect to time.

The whole process can be more clearly represented by using a queue with a deterministic service time, as shown in Fig. 10 for the example given in Section III. As shown in Fig. 10(a), assume that initially, there is one unit (represented by a dot) in each of the 20 states (represented by a box). The setpoint changing process can be represented by two queues [Fig. 10(b)]: an out-of-regime queue and an in-regime queue. When the setpoint change happens, previous States 1 and 2 become in-regime States 4 and 5, States 3 and 4 degenerate to out-of-regime States 9 and 12, and State 5 degenerates to in-regime State 6.

After all the units in the out-of-regime queue evolve to the in-regime queue, the transient process will be over [Fig. 10(c)]. The transient time is then $k\tau_{\text{off}}$. Based on the queue structures shown in Fig. 10 and corresponding to different setpoint change ratio k in response to price increases, one can create an in-regime queue and an out-of-regime queue, as shown in Table IV, and simulate the whole system evolution along the timeline.

The total power demand will then be determined by the number of units in "on" states at any time. Because the "on" state will always be in States 1–5, a sum of machines in these states will yield the total demand.

The inverse load diversity factor $1/k_d$ is calculated by dividing the aggregated load with maximum demand *NP* (Fig. 11).

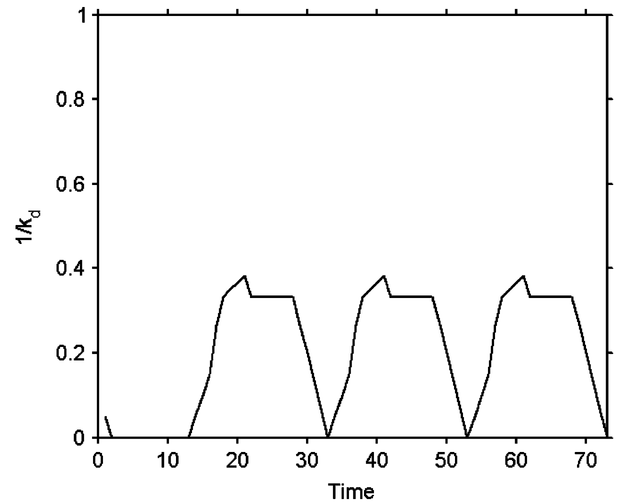

Fig. 11. The diversity factors after a price increase.

TABLE IV
STATE REDISTRIBUTION IN RESPONSE TO A PRICE INCREASE

k	In-regime Queue	Out-of-regime Queue
0.2	2'-3'-5'-6'-7'-8'-9'-10'-11'-12'-13'-14'-15'-16'-17'-18'	6-7-8-6'
0.4	3'-4'-5'-6'-7'-8'-9'-10'-11'-12'-13'-14'-15'	6-7-8-9-10-11-6'
0.6	4'-5'-6'-7'-8'-9'-10'-11'-12'	6-7-8-9-10-11-12-13-14-6'
0.8	5'-6'-7'-8'-9'	6-7-8-9-10-11-12-13-14-15-16-17-6'
1	0	6-7-8-9-10-11-12-13-14-15-16-17-18-19-20-6'
1.2	0	6-7-8-9-10-11-12-13-14-15-16-17-18-19-20-o-o-6'

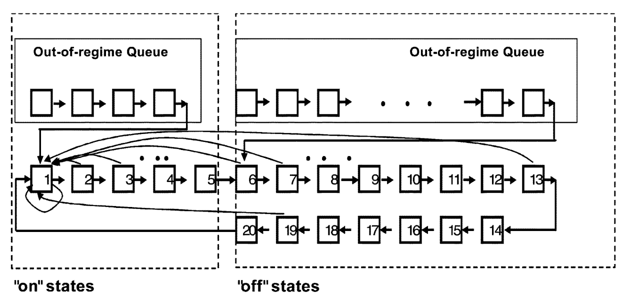

Fig. 12. The structure of the queue representation of TCAs.

Note that the diversity factor is no longer a constant as in the perfectly random distribution case. There are periods when the diversity factors are infinity, and we observe a synchronized "off" behavior. For some periods, the diversity factors are lower than the average at 4, where we observe stronger synchronized "on" behaviors. The results demonstrate that the price response has a significant impact on the load diversity. The remaining question then is whether and how the diversities are recovered after the response to a price change.

V. THE DAMPING PROCESS

Over time, random events such as the hot water usage or door openings will naturally randomize any synchronous behavior of water tanks, HVAC units, or refrigerators and consequently damp the oscillation. Randomness brought by different types

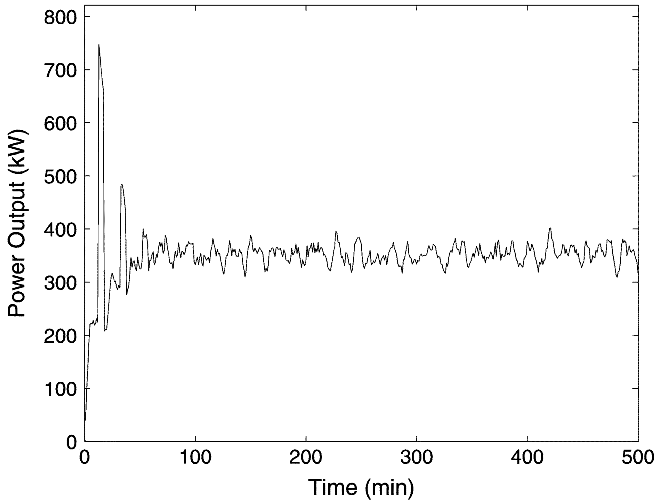

Fig. 13. The damping of load response.

and sizes of TCAs as well as randomness caused by environmental differences may also contribute to the damping process. Our present model simulates the behaviors of TCAs having the same type and size with similar environmental conditions. Therefore, the damping in our model is mainly caused by random events. These random events cause state jumps of the unit in the queue. For example, taking a shower draws water from the hot water tank and so the tank temperature drops out-of-regime. For simplification purpose, assume that each state except State 20 has a probability of p to return to State 1 and a probability of $1-p$ to evolve to the next stage. The queue can be modified as shown in Fig. 12. Then, for a deterministic transition time, the transition matrix P is

$$P = \begin{bmatrix} p & 1-p & & \dots & \\ p & & 1-p & & \\ \dots & & & \dots & \\ p & & & & 1-p \\ 1 & & & \dots & \end{bmatrix}.$$

Therefore, the state occupancy N after i transitions can be calculated as

$$N_i = N_{i-1}P.$$

When there are m "on" states, the number of units in "on" states will be

$$N_{\text{on}} = \sum_{j=1}^{m} N_j.$$

Fig. 13 shows the damping effect of the given example HVAC units on a feeder. The load dynamic is damped in about three cycles with a p of $1/20$. The load diversity is then fully restored. In distribution systems, equipment are chosen based on an assumption that a load at the end of a feeder has certain load diversity. When all the loads turn on/off at the same time, the equipment maybe overloaded. Therefore, when designing a load-response program, the natural damping factor of the system needs to be evaluated. If the system itself can not damp the dynamics fast enough, mechanics can be engineered to diversify the load artificially.

VI. Conclusion and Future Work

This paper developed a state-queueing model to simulate the price response of a load consisting of thermostatically controlled appliances in a competitive electricity market. An aggregated load consists of thousands of TCAs, while the number of states a TCA can reside in may be no more than 100. We expect that applying a queue representation brings a computational advantage over simulating the behavior of each individual unit.

By analyzing the load shifts caused by the setpoint changes in response to price, the impacts on load diversity are studied. The results indicate that by responding to price changes, a diversified TCA type of load becomes synchronized, and their behaviors present a dynamic response. Therefore, to design a successful load-response program for aggregated TCAs, one needs to examine the load shifting characters to ensure that the shifted load peaks will occur after the peak-price time. The synchronized load peak can be much higher than that of the diversified load. The stress on the distribution system should also be considered. The methodology developed in this research is expected to be used to create DSM simulation tools that are able to take the load-shifting behavior into consideration.

Future work is needed to develop a feeder equivalent model consisting of different types of TCAs with a consideration of different setpoint changes in response to a price change. Because there is randomness from sources other than customer consumptions, our future models for the damping process will take into account the randomness caused by different types and sizes of HVAC units and different housing environments.

Acknowledgment

The authors would like to thank their colleague S. E. Widergren for his comments and suggestions.

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Ning Lu (M'98) received the B.S.E.E. degree from Harbin Institute of Technology, Harbin, China, in 1993, and the M.S. and Ph.D. degrees in electric power engineering from Rensselaer Polytechnic Institute, Troy, NY, in 1999 and 2002, respectively.

Currently, she is a Research Engineer with the Energy Science and Technology Division, Pacific Northwest National Laboratory, Richland, WA. She was with Shenyang Electric Power Survey and Design Institute from 1993 to 1998. Her research interests include modeling and analyzing deregulated electricity markets.

David P. Chassin (M'03) received the B.S. degree in building science from Rensselaer Polytechnic Institiute, Troy, NY.

He is a Staff Scientist with the Energy Science and Technology Division, Pacific Northwest National Laboratory, Richland, WA, where he has worked since 1992. He was Vice President of Development for Image Systems Technology from 1987 to 1992, where he pioneered a hybrid raster/vector computer aided design (CAD) technology called CAD Overlay™. He has experience in the development of building energy simulation and diagnostic systems, leading the development of Softdesk Energy and DOE's Whole Building Diagnostician. He has served on the International Alliance for Interoperability's Technical Advisory Group and Chaired the Codes and Standards Group. His recent research focuses on emerging theories of complexity as they relate to high-performance simulation and modeling.