

This is a draft of a chapter/article that has been accepted for publication by Oxford University Press in the Oxford Handbook of Algorithmic Music, edited by Alex McLean and Roger Dean and published in 2018.

## Chapter 4

### Origins of Algorithmic Thinking in Music

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#### Abstract

Musicians' relationships with algorithms have deep precedents in the confluence of music and mathematics across millenia and across cultures. Technological and musico-mathematical precedents in the ancient world predate the Arabic etymology of the term 'algorithm'. From Guido d'Arezzo's hand, to rule systems in music theory and 18th century ars combinatoria, there is a rich background to 20th century rule-led music making. Robotic music too has its mechanical precedents, and the link from the great automata builder Vaucanson to early programmable weaving looms shows an interesting proto-computational thread. The likes of Ada Lovelace's writing, Joseph Schillinger's composition system and John Pierce's 1950 stochastic music science fiction article provide productive insight into the origins of later algorithmic music. Indeed, the world's musics as a whole reveal a panoply of interesting practices, such as campanology, Nzakara court harp music, time structures in Indian classical music, and many more examples of the rich combinations of music and mathematics often predating musical computer science.

#### Keywords

Algorithmic composition, musical algorithms, historical roots, music and mathematics, musical automata

#### Introduction

'...the phono-lecturer came to the main theme of the evening - to our music as a mathematical composition (mathematics is the cause, music the effect). The phono-lecturer began the description of the recently invented musicometer.

"...By merely rotating this handle anyone is enabled to produce about three sonatas per hour. What difficulties our predecessors had in making music! They were able to compose only by bringing themselves to attacks of inspiration, an extinct form of epilepsy." '

Extract from Yevgeny Zamyatin's dystopian novel *We* (1921) (Zamyatin 1924, p. 17)

The roots of algorithmic composition in music are elaborate, and twine through many intriguing early experiments, some well known, and some more surprising. In this chapter, we shall consider the union of music and mathematics, before exploring early algorithmic procedures for music generation, and surveying mechanical music precedents. A wider sense of music within world cultures outside the Western canon is also encountered. The whole text is intended to open the reader up to the deep-rooted foundations of more recent computer-led automatic composition, and show the rich connection to general musical endeavour in human history.

## **Music and Mathematics**

The confluence of music and mathematics isn't just a case of smart people demonstrating a joint aptitude to the two subjects through self-disciplined practice, but a far-reaching historical interaction (Benson 2007, Loy 2007, Harkleroad 2006, Fauvel et al. 2003, O'Keeffe 1972). From Pythagorean joint investigation of numerical and musical whole number ratios (Crocker 1964) to the medieval quadrivium teaching of music alongside arithmetic, geometry, and astronomy, to the intensive combinatorics of more recent music theory mathematics problems (Friperfinger 1999) and computer music as a discipline (Moore 1990), the physics, technology and theory of music making is replete with mathematical links. Coxeter (1968) even advances the theory that the times and places producing great mathematicians also tend to be those producing great musicians, though such circumstances are more likely linked to a general background of appropriate socioeconomic circumstances for such studies.

Interaction is two directional. J.S.Bach, perhaps the most influential and respected of all composers, was highly mathematically aware, oft noted through the evidence of his contrapuntal wizardry and musical puzzles, a mind in Hofstadter's well known book on a league with arch-logician Kurt Gödel (Hofstadter 2000). Scientists have dabbled with musical construction; both Kepler and Newton devised scales, Kepler from ratios of planetary orbit minimum and maximum speeds (Field 2003), and Newton with a seven note diatonic in analogy to the seven colour spectrum (Bibby 2003).

The direction of influence on development in music and mathematics also goes both ways. Perhaps most often, music theory has reacted to innovations in mathematics (Nolan 2002); Catherine Nolan gives an example of great pertinence to this present volume. Mersenne, soon after the introduction of combinatorial methods in Western mathematics, calculated for his *Harmonie Universelle* (1636) the number of possible serialist melodies of from 1 to 22 notes (in his case, permutations of the order of diatonic notes over a three octave gamut); his table of course lists the permutation note counts as the factorials from 1! to 22!.

There are also occasions of novel mathematical results in musical theory, from the aforementioned Pythagorean investigations which discovered the harmonic mean, to campanology's anticipation of group theory within the design of change ringing permutation chains (Roaf and White 2003), or a claim for an

11<sup>th</sup> century Venn diagram (Edwards 2011). The more recent theory of maximally even scales has been noted to provide a solution to a problem in Ising model spin configurations in physics (Douthett and Krantz 2008), and the related Euclidean rhythms have many linked applications in scientific disciplines (Demaine et al. 2009).

The 20<sup>th</sup> century pre-occupation with formalist and computational method, closely detailed elsewhere in this handbook, was often carried out with an awareness of the historical precedents. Aside from Schoenberg's sense of historical inevitability for the twelve tone method, Xenakis is the modernist composer perhaps most closely associated with cross-fertilisation between the fields of mathematics and music (Xenakis 1992, Harley 2004). Yet as a Greek expat, he was highly aware of the ancient heritage of Greek mathematical music, and often returns to early Greek natural philosophers in his writing.

For the interested reader, Gareth Loy's two volumes of 'musimathics' (Loy 2007) provide a strong introduction to areas of mathematics with musical application, with a particular emphasis on acoustics, signal processing, and elements of tuning and algorithmic composition. David Benson's university course in music and mathematics is covered by a freely available online book (<https://homepages.abdn.ac.uk/mth192/pages/html/maths-music.html>) and associated Cambridge University Press publication (Benson 2007), though is perhaps less immediately accessible to the non-specialist mathematician.

### **Pre-computer algorithmic composition precedents**

The standard touchpoints for algorithmic composition include the vowel to pitch algorithm of Guido d'Arezzo (1026) and the fad for 'ars combinatoria', the musical dice games of the later 18<sup>th</sup> century (Roads 1996, Loy 2007, Nierhaus 2009, Collins 2010). Hedges (1978) notes twenty examples of the latter being published following Kirnberger's brilliantly titled *The Always Ready Polonaise and Minuet Composer* (1757); Nolan (2000) makes clear that ars combinatoria is a more general trend within musical treatises of the time, dealing with the combinatorial possibilities of musical material within established template musical structures, as a stimulant to the work of composers.

Further procedural stimulations for composition in the 18<sup>th</sup> century include the use of divination, as in Vogt's 1719 casting of hobnails to furnish melodic contours (Loy 2007, volume 1, p. 294), and Hayes' splattering of paint on a musical score to determine notes, as suggested in his 1751 pamphlet 'The Art of Composing Music by a Method Entirely New, Suited to the Meanest Capacity' (Hiller and Isaacson 1979, p. 52). These were acts more than two hundred years ahead of John Cage's conceptions of chance operations, Cage also being beaten to the idea by his idol Marcel Duchamp's *Erratum Musical* (1913) where notes are drawn from a hat! Athanasius Kircher had been a century ahead of his time in the 1650 tract *Musurgia Universalis* when he described a music generating machine, the arca musarythmica, a box full of options for different components necessary to a composition, halfway between Guido's method and the musical dice games.

Some authors have seen the mechanical potential, without building the machine. One of the most celebrated anticipations of the possibilities of

computational music, more poignant for the fact that Babbage's computer was never built, is Ada Lovelace's footnote prediction on the composition of 'elaborate and scientific pieces of music of any degree of complexity' (Collins 2010). John R. Pierce, famous as director of research at Bell Labs, sponsor of Max Mathew's early computer music and the coiner of the term transistor, had his own prescience (pun intended). Writing in November 1950 in *Astounding Science Fiction* (Pierce 1968), Pierce outlined the potential of Shannon's new information theory to generate music according to Markov chains. Ahead of Hiller and Isaacson's celebrated 1956 computer-generated string quartet experiments, Pierce was carrying out experiments with human calculators on Markov music generation (Hiller and Isaacson 1979, p. 33). The potential for more advanced computer music yet was also being discussed; as Turing said of Shannon, reputedly during a second world war lunch conversation, impressed by his verve in plotting a course for artificial intelligence, 'Shannon wants to feed not just data to a Brain [computer], but cultural things! He wants to play music to it!' (Hodges 2012, p. 251).

In as much as computer era algorithmic composition is one manifestation of radical experimental music technique, we might go on to consider many further precedents through novel compositional ideas. Lejaren Hiller points to mappings from data to music, such as renaissance 'eye music' experiments as early graphical scores, and Charles Ives' 1907 baseball game sonification (Hiller and Isaacson 1979, pp. 47-48): 'notes set on paper like men on a football field' (ibid, p.48). Karlheinz Essl dwells on serialism in particular in his chapter survey of algorithmic composition (Essl 2007), though he finds literary precedent in Goethe. We might also point to Earle Brown's experiments with statistical generation of graphical scores in 1952, ahead of Xenakis' own paper and pen stochastic music experiments (Xenakis only gained access to a computer in 1962) (Collins 2010). Reginald Smith Brindle (1956) highlights the new formal techniques inspiring the integral serialists, and the necessity for some element of perceptual accessibility in human terms: 'A 'computational' composer is in the position of a designer who uses a kaleidoscope to discover new and striking patterns. Such use is legitimate, but the patterns only captivate us if there is a certain element of familiarity inter-woven in their strangeness' (Smith Brindle 1956, p. 356).

One author whose conception of mathematical constructions for music composition are key here is Joseph Schillinger, who was influential on a generation of composers in the 1930s, including Gershwin, but who fell into relative obscurity in later decades (Glinsky 2000, pp. 131-135, Brodsky 2003). Two enormous posthumously edited volumes are not enough to fully describe his composition system (Schillinger 1946); his *The Mathematical Basis Of The Arts* includes strong speculation on the possibilities for automated music composition (Schillinger 1948). In a review of the latter text, whilst disparaging much of its writing style, John Myhill (1950) is impressed by the scientific approach to aesthetics, and sees no loss of human choice in the automations, or 'artomations'. Human intervention has merely moved back to the setting up of the system: 'It is in the *presetting of the controls* of the machine that the "freedom" or "individuality" of the artist expresses itself' (Myhill 1950, p. 113), a position much echoed in later understanding of the act of algorithmic composition. Schillinger himself writes evocatively of the potential for musical

machines, the never built Musamatons. Categorising forms of machine creation for the arts he notes:

‘3. Instruments for automatic composition of music:

a. limited to specified components, such as rhythm, melody, harmony, harmonization of melodies, counterpoint, etc.

b. combining the above functions, and capable of composing an entire piece with variable tone qualities (choral, instrumental chamber music, symphonic and other orchestral music)

4. Instruments for automatic variation of music of the following types:

a. quantitative reproductions and variations of existing music

b. modernizing old music

c. antiquating modern music

5. Instruments of groups 3 and 4, combined with sound production for the purpose of performance during the process of composition or variation.

6. Semi-automatic instruments for composing music. These instruments will be used as a hobby for everyone interested in musical composition, whether amateur or professional, and will not require any special training. The prospective name for instruments of this type will be “Musamaton.” ’ (Schillinger 1948, p. 673)

## **Musical Automation**

Automation is an essential characteristic of much algorithmic music; a process runs independent of human gestural energy, with or without higher-level human intervention. David Cope takes the Aeolian harp as a paradigmatic example (Cope 1991), but we might also dig into the more substantial engineering precedents. Self-playing mechanical musical instruments, also known as musical automata, have a long and distinguished history of over a millennium (Fowler 1967, Ord-Hume 1973, Kapur 2005). Founded on clockwork mechanisms, and making heavy use of drum roll sequencing, early automata range from miniature music boxes, through humanoid figures, to larger scale orchestrion automatic orchestras.

It is striking that in a text on mechanical precedents to the mid 20<sup>th</sup> century computer, Teun Koetsier (2001) is unable to avoid multiple references to musical automata. The link of the famous 18<sup>th</sup> Century automata builder Jacques Vaucanson to the programmable loom is notable. On the back of the international fame of his automatic flute player, digesting and defecating duck, and pipe and tabor robot, Vaucanson was invited in 1740 to look into automation for looms in the French weaving industry. Though his designs were

not implemented immediately, making their way to an institutional attic, they were later rediscovered. They became one of the stimulations (alongside innovations from other French engineers working in the earlier eighteenth century) to Jacquard in the creation of his eponymous loom (1801) with its punchcard control mechanism. The punchcard system would go on to inspire Babbage, and thence to 20<sup>th</sup> century computing.

The earliest confirmed musical automata mentioned in the Koetsier article shows the strong Islamic engineering link. Even before Al-Jazari's drinking party robots (circa 1200), and nine hundred years before Vaucanson, the ninth century Musa brothers of Baghdad deployed hydraulics in the construction of a flute playing robot!

Nonetheless, even if indebted to Islamic science, the flourishing of mechanical music in Europe followed the 13<sup>th</sup> century development of the mechanical clock. By the time of Shakespeare at the bridge between the 16<sup>th</sup> and 17<sup>th</sup> centuries, automata were well established. Adam Max Cohen (2011) notes that amongst the nobility, 'Some of the most popular automata were Christ figures on the cross, clockwork Madonnas, trumpeters, men on horses, and even men sailing ships ... clockwork beasts had eyes that shifted back and forth with the tick-tock of their verge-and-foliot escapements, and some moved or performed on the hour ... With early clockmakers crafting increasingly elaborate and increasingly lifelike automata it was only a matter of time before a few attempted to build life-sized human and animal automata'.

Riley (2009) makes the interesting point that to the audiences of the early nineteenth century, mechanical music was not a novelty, but a well known avenue, often unregarded or unsurprising, and certainly not controversial in the terms of later debates around technology threatening human jobs. Whilst certain musical engineering innovations could have a short-term run of success, the age of *ars combinatoria* was ending. Much of the more ornate work may have been fuelled by fads amongst the nobility, and was undermined by the rise of the bourgeoisie and the industrial revolution. Illustrating this theory of public automata ennui, he writes of a celebrated 1814 concert containing Beethoven symphonic premieres:

'And on this spectacular and much-heralded program, following the joyful Seventh and before the concluding Battle Symphony, a ten-minute performance by Mälzel's Automatic Trumpet Player, accompanied by full orchestra, in marches by Ignace Pleyel and Jan Dussek. After the first piece, someone, presumably Mälzel, would have gone out on stage, opened the Trumpet Player's back to replace the Pleyel cylinder with the Dussek cylinder, inserted a large crank into the figure's thigh (Arrington 57), and wound it up for its second performance. This mechanical interlude was duly recorded in the press of the day and in the memoirs of several participants; but in none of the eyewitnesses' accounts is there any suggestion that the android's appearance on the program was strange or unexpected or interesting or uninteresting.' (Riley 2009, p. 372).

The most fantastical plot recounted by Riley (2009) is the interaction of Johann Nepomuk Mälzel and Diederich Winkel. In 1815, Mälzel was on tour with the Panharmonicon, his second automatic orchestra. Visiting Amsterdam, he also

visited the workshop of Winkel, where he discovered (i.e., stole) the idea for an accurate clockwork metronome. Whilst Mälzel grabbed the patents and the international market for the metronome, and even gave his name to the metronome's tempo markings, Winkel's revenge was to out-do the Panharmonicon. Winkel's mechanical Componium, premiered in 1821, combining the idea of an orchestrion with the musical dice game, to create a musical automata capable of playing variations on a theme. It did this via an ingenious use of two synchronised barrel rolls, alternating two measure phrases; whilst one played back, the other was silent (having no pegs in that section), and slid horizontally on a random walk to select its next material. A revised estimate of the combinatorial capability of this fantastic device puts the number of possible variations at a modest 256 million or so, with a prediction of 41 years of continuous play before any full sequence would be repeated (Bumgardner 2013). This combination of algorithmic composition and musical robotics is a startling early precedent, but has been little publicised; the machine itself was a modest initial success, but with the death of its inventor, fell into obscurity (the gutted machine survives in the Brussels Museum of Musical Instruments).

The metronome itself, perhaps more than automata, brought home the mechanisation of musical process, since it was the subject of mass production, and mass adoption. The changes in performance practice brought about by the arrival of the metronome as a central musical teaching aid are documented by Alexander Bonus (2014) and reveal the tensions between an absolute sense of musical time fitting the industrial-urban-scientific age, and a nostalgia for the looser human time of musical tradition. The protagonists of the two sides of this debate prefigure the more 20<sup>th</sup> century anxieties around musical technology, whether silent film musicians losing jobs to the talkies, or MIDI threatening the musician's union. The heel and toe clog dancing of female machine operators in Lancashire dating back to the 1820s, and subsequent machine inspired choreography might provide a further link to the mechanization of the arts (Radcliffe and Angliss 2012); the clog link evokes sabotage by sabot, and the Luddites of a similar time period (1810s) as the metronome's arrival. Ultimately, the metronome won: metronome time guiding human musicianship is throughout the click tracks and computer time of modern recording process, and indispensable to conservatoire practice.

Aside from street organs and orchestrions, musical automata had a further flowering with the 1890 to 1920 fad for player pianos, an out-growth of bourgeoisie aspiration to home pianism sneaking in before the booms in radio and higher quality record players. The vast majority of early automata may be 'reprogrammable' in the sense of changing sequencer data by substituting a new cylinder, altering peg positions on an existing cylinder, or punching new holes in a piano roll, but they are not programmable in any more profound sense. Indeed, it is surprising how much music for mechanical musical instruments imitates human speed music making, at least before the hyper-virtuosity of Nancarrow (Gann 1995). Nancarrow began his player piano studies around the middle of the 20<sup>th</sup> century, coincidentally mirroring the rise of the electronic computer, and perfectly anticipating the inhumanly fast sequencing capabilities so unleashed.

## Ethnomusiconomy

In contrast to musicology as the humanities activity of 'speaking' (logos) about music, Gareth Loy suggests musiconomy as more suitable to investigation of the 'laws' (nomos) of music (Loy 2007). Extending this from ethnomusicology, we might consider the sub-discipline of *ethnomusiconomy* as pondering generative laws within the world's musics. Chemillier (2002) has treated the situation of 'ethnomathematics' as applied within ethnomusicology, and provides a wonderful example of Nzakara court harp music, founded on string combinatory patterns.

Mathematical thinking is hardly limited to Western music. The thousand year old Tibetan Buddhist tradition of *Rol Mo* is detailed by Ellingson (1979), who observes compositions based around two cymbals. The music places an emphasis on time spacing of events, and the timbral effects possible by altering the location of strikes on the cymbals and the after strike interaction of the two cymbals in proximity. In *Days of the Waxing Moon*, a sequence of repetitions for events uses an arithmetic series run from 1 to 15 by steps of 2. *Invitation to Mahākāla* uses an accelerating 'countdown' beginning by stepping through 180, 170, ..., 20, 15, then from 10 down to 1. The same piece also includes an action sequence based on the symbolism of a hexagram inscribed in a circle, where cymbal strikes draw out the geometric figure in space. This sort of gesture piece gazumps recent experiments in action composition in contemporary Western music!

Indian classical music is full of complicated temporal, pitch and timbral structures; an introduction to rhythmic tāla alone might take a whole book (Clayton 2008). Perhaps the most developed research project in algorithmic modelling of North Indian (Hindustani) music is that of Kippen and Bel (1992); the associated Bol Processor software has been made available as an open source project, and its modelling tools are generally amenable to many musics (<http://bolprocessor.sourceforge.net>). North Indian music is not alone in its complexity and inspiration; Virtual Gamelan Graz is a project seeking heightened understanding of compositional rules and the sound world within Indonesian gamelan (Grupe 2008). Godfried Toussaint (2013) has researched many world rhythms, especially with respect to those expressible as a subset within a cycle of isochronous pulses (see also Demaine et al. 2009); he provides an analysis of the clave timeline pattern, and its historical evolution.

It may be tempting, though it is fallacious, to believe that because a music is amenable to mathematical modelling, that such modelling constitutes a final proof of that music's construction. This is especially dangerous to ethnomusicologists when devoid of the further cultural factors. Steven Feld has cautioned on the uncritical and often unscientific adoption of linguistic models in his field; just because a given music can be represented via a particular model, does not mean that the model is thereby proved the only analytical approach, nor that the model is the original representation in culture (Feld 1974)! Nonetheless, mathematically minded music theorists and ethnomusicologists have found much of interest in the world's musics beyond Western ideas. Participants within alternative musical culture are themselves adopting and adapting formal methods to treat their own musics (see for instance Sen and Haihong 1992, Kippen and Bel 1994), though the vast majority of algorithmic



composition research, much like music psychology research, is limited to the Western canon.

## Conclusions

Algorithms for music have rich precedents, across cultures and across eras. The expansion of scientific knowledge has often interacted with music's cultural evolution, especially where developments in technology have been necessitated by artistic concerns, such as the novelty of musical automata, or where scientific investigation of a physical phenomenon is intimately tied to aural resultant, such as the Greek study of stringed instruments. We have seen that music technology is sometimes unshocking, falling into a trend of the day such as the clockwork universe, and sometimes more threatening to older tradition (whilst attractive to others), such as the metronome's new time regulation.

The wealth of precursors may act as a check on the hyperbole of some algorithmic composers. Should we be surprised at the unsurprising nature of generative music, when only one presentation/realisation can occur at a time, and it is so hard in human terms to hear out the immense combinatorial space of possibility, or to program for substantial perceptual variation? Instead, one might only see the relationship to that existing generative system par excellence, human improvisation. The fate of Winkel's Componium should be born in mind.

One unfortunate aspect of the survey carried out here is the reduced role of female composer-engineers. Although there are electronic computer generated music pioneers such as Laurie Spiegel and the lesser known Harriet Padberg (Ariza 2011), Lancashire clog dancers and Ada Lovelace are not enough to avoid the sense of male preserve in earlier historical time. At least the present era is replete with more equal opportunity. Confronting the challenges of algorithmic composition will without doubt be led by female composers in the coming decades.

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