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INTUITIONISTIC ANTIFUZZY SUB-BIGROUP

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Abstract:

In this paper we made an attempt to study the algebraic nature of intuitionistic antifuzzy sub-bigroup and intuitionistic bi-lower level subset of the antifuzzy sub-bigroup of the bigroup and discussed some of its properties with theorems.

Key Words: Fuzzy Sett, Antifuzzy Subgroup, Antifuzzy Subgroup of a Bi-Group, Antifuzzy Sub-Bigroup of a Group, Bi-Lower Level Subset, Homomorphism & Antihomomorphism

Introduction:

The concepts of fuzzy sets was introduced by Zaeh,Then it has become a vigorous area of research in engineering medical science,social science,graph theory etc,Rosenfeld gave the idea of fuzzy subgroups and Ranjith Biswas gave the idea of antifuzzy subgroups.

The notion of bigroups was first introduced by P.L.Maggu in 1994.W.B.Vasantha kandasamy and D.Meiyappan introduced concept of fuzzy sub-bigroup of a bi-group and antifuzzy sub-bigroup of a group.

The concepts of antifuzzy sub-bigroup of a bigroup and bi-lower subsets of an antifuzzy sub-bigroup and antifuzzy subgroup of a group introduced by R.Muthuraj, M.Rajinikannan, M.S.Muthuraman. we established the relationship between antifuzzy sub-bigroup of a group and antifuzzy subgroup of group.

In this paper we introduced the concept of intuitionistic antifuzzy sub-bigroup of a bigroup and bi-lower level subset of a antifuzzy sub-bigroup and prove some results with counter examples.Also we estsblished the intuitionistic bi-lower level subgroup.

1. Preliminaries:

This sections contains some definitions and results to be used in the sequel.

1.1 Definition: An Intuitionistic fuzzy subset (IFS) A in a set X is defined as an object of the form

$$A = \{< x, \mu_A(x), \gamma_A(x) > / x \in X\}$$

Where $\mu_A : X \to [0,1]$ and $\gamma_A : X \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively and for every $x \in X$ satisfying $0 \le \mu_A(x) + \gamma_A(x) \le 1$.

1.2 Definition: Let $G = (G, +, \bullet)$ be a bigroup. Then $A: G \to [0,1]$ be an intuitionistic fuzzy subset A then

$$A = \{< x, \mu_A(x), \gamma_A(x) > / x \in X\}$$

An intuitionistic fuzzy subset μ_A in a group G_1 is a membership function and γ_A in a group G_2 is a non-membership function and for every $\mu_A \in A_1 \& \gamma_A \in A_2$

Then $A_1 : G_1 \to [0,1]$ and $A_2 : G_2 \to [0,1]$ is said to be an Intuitionistic antifuzzy sub-bigroup of the bigroup of G. If there exists two fuzzy subsets $A_1 \in G_1$ and $A_2 \in G_2$.If the following conditions are,

i) $\mu_A(xy) \le \max \{\mu_A(x), \mu_A(y)\}$
ii) $\mu_A(x^{-1}) \le \mu_A(x)$
iii) $\gamma_A(xy) \ge \min \{\gamma_A(x), \gamma_A(y)\}$
iv) $\gamma_A(x^{-1}) \ge \gamma_A(x)$ for all $x, y \in G$

1.1.2 Example: Let G_1 = { 0 , a , b , a + b } be a group under the operation '+ ' with a + a = b + b = 0 and a + b = b + a

Let G_2 = { 1 , -1 , i , - i } be a group under the operation '•'

Define A : G →[0 ,1]

$$A(x) = \begin{cases} 0.2 \text{ for } x=0 \\ 0.3 \text{ for } x=a,1 \\ 0.4 \text{ for } x=b,a+b,-1 \\ 0.5 \text{ forx}=i,-i \end{cases}$$

It is easy to verify that A is an anti fuzzy sub-bigroup of the bigroup G, for we can find $A_1 : G_1 \to [0, 1]$ by

$$A_1(x) = \mu_A(x) = \begin{cases} 0.2 \text{ for } x = 0 \\ 0.3 \text{ for } x = a \\ 0.4 \text{ for } x = b, a+b \end{cases}$$

and $A_2 : G_2 \to [0 ,1]$ is given by

$$A_2(x) = \gamma_A(x) = \begin{cases} 0.3 \text{ f0r } x=1 \\ 0.5 \text{ for } x = i,-i \end{cases}$$

Clearly A is an intuitionistic anti fuzzy sub-bigroup of the bigroup G.

1.3 Definition: Let A be an intuitionistic antifuzzy subset of a set X.For $t \in [0,1]$ the lower level subset of A is the set

$$\overline{A_t} = \left\{ \frac{\mu_A(x) \le t \;\&\; \gamma_A(x) \ge t}{x} \in X \right\}$$

Where μ_A is a membership function and γ_A is a non-membership function.This is called an Intuitionistic antifuzzy lower level subset

1.4 Theorem: Every t – lower level subsets of an intuitionistic anti fuzzy sub-bigroup A of a bigroup G need not in general be a sub-bigroup of the bigroup G.

Proof:

We prove this by an example Take G= {-1,0,1} to be a bigroup under the operation ‘ + ‘ and ‘•’ where G_1 = {0} and G_2 = {-1,1} are groups respectively with respect to usual addition and usual multiplication.

Define A : G →[0 ,1]

$$A(x) = \begin{cases} 0.5 \text{ for } x = -1, 1 \\ 0.75 \text{ for } x = 0 \end{cases}$$

Clearly (A,+,•) is an intuitionistic anti fuzzy sub-bigroup of the bigroup (G, +,•) . Now consider The lower level subset $\bar{A}_t$ for t = 0.5 of the anti fuzzy sub-bigroup A

For t = 0.5, $\bar{A}_t$ = { x ϵ G : A(x) ≤ 0.5 }

$\bar{A}_t$ = { -1,1 }.

It is easy to verify that { -1,1 } is not a sub-bigroup of the bigroup (G, +, •), Hence the lower level subset $\bar{A}_t$ for t = 0.5 of an intuitionistic anti fuzzy sub-bigroup A is not a sub-bigroup of the bigroup (G,+,•).

1.5 Definition: Let G = ($G_1 \cup G_2$,+, •) be a bigroup and A = ($A_1 \cup A_2$) be an anti fuzzy sub-bigroup of the bigroup G. Then the Intuitionistic bi-lower level subset of the anti fuzzy sub-bigroup A of the bigroup G is defined as

$\bar{A}_t = \bar{A}_{1t} \cup \bar{A}_{2t}$ for

t ϵ [max {μ_A (e1), μ_A (e2) },1].

t ϵ [min {γ_A (e1), γ_A (e2) },1].

Where e_1 denotes the identity element of the group (G_1, +) and e_2 denotes the identity element of the group (G_2,•)

1.6 Theorem: Every intuitionistic bi-lower level subset of an intuitionistic anti fuzzy sub-bigroup A of a bigroup G is a sub-bigroup of the bigroup G.

Proof:

Let A = ($A_1 \cup A_2$) be an intuitionistic anti fuzzy sub-bigroup of a bigroup G = ($G_1 \cup G_2$,+, •)

Consider the intuitionistic bi-lower level subset $\bar{A}_t$ of an intuitionistic anti fuzzy sub-bigroup A for every

t ϵ [max {μ_A (e_1), μ_A (e_2) },1]

t ϵ [min {γ_A (e_1), γ_A (e_2) },1]

Where e_1 denotes the identity element of the group (G_1, +) and e_2 denotes the identity element of the group (G_2,•)

Then $\bar{A}_t = \bar{A}_{1t} \cup \bar{A}_{2t}$

Where $\bar{A}_{1t}$ and $\bar{A}_{2t}$ are subgroups of G_1 and G_2 respectively.

Hence by the definition of sub-bigroup $\bar{A}_t$ is a sub-bigroup of the bigroup (G,+, •).

1.7 Theorem: Let G be a bigroup and A_1, A_2 be fuzzy subsets of A such that A = ($A_1 \cup A_2$). The intuitionistic bi-lower level subset $\bar{A}_t$ of A is a sub-bigroup of G

t ϵ [max {μ_A (e_1), μ_A (e_2) },1], t ϵ [min {γ_A (e_1), γ_A (e_2) },1]

Where e_1 denotes the identity element of G_1 and e_2 denotes the identity element of G_2 respectively .Then A is an intuitionistic antifuzzy sub-bigroup of a bigroup G.

Proof:

Let G = (G_1 ∪ G2) be a bigroup.

Given that, The bi-lower level subset

$\bar{A}_t = \bar{A}_{1t} \cup \bar{A}_{2t}$ is a sub-bigroup of G.

Clearly, $\bar{A}_{1t}$ is a subgroup of G_1 ,

A_1 is an anti fuzzy subgroup of G_1.

Clearly, $\bar{A}_{2t}$ is a subgroup of G_2 ,

A_2 is an anti fuzzy subgroup of G_2.

Hence, A = ($A_1 \cup A_2$) and

Hence, A is an intuitionistics anti fuzzy sub-bigroup of G.

1.8 Theorem: Let A = ($A_1 \cup A_2$) be an intuitionistic anti fuzzy sub-bigroup of a bigroup G = ($G_1 \cup G_2$). Two intuitionistic bi-lower level subgroups $\bar{A}\alpha$, $\bar{A}\beta$,

α, β ∈ [max {μ_A (e_1), μ_A (e_2) },1],

$\alpha, \beta \in$ [min $\{\gamma_A\ (e_1), \gamma_A\ (e_2)\ \}$,1] where e_1 denotes the identity element of $G_1$and e_2 denotes the identity element of G_2 respectively with $\alpha < \beta$ are equal iff there is no x in G such that $\alpha <$ A(x) $\leq \beta$

Proof:

Let A = $(A_1 \cup A_2)$ be an intuitionistic anti fuzzy sub-bigroup of a bigroup G = $(G_1 \cup G_2)$

Consider, The two intuitionistic bi-lower level subgroups Ā α ,Ā β, $\alpha, \beta \in$ [max $\{\mu_A\ (e_1), \mu_A\ (e_2)\ \}$,1],

$\alpha, \beta \in$ [min $\{\gamma_A\ (e_1), \gamma_A\ (e_2)\}$,1]

Where, e_1 denotes the identity element of G_1 and e_2 denotes the identity element of G_2 respectively with $\alpha < \beta$

Let Ā α = Ā β. We have to prove that, there is no x in G such that $\alpha <$ A(x) $\leq \beta$

Suppose that, There is an x in G such that $\alpha <$ A(x) $< \beta$

Then, x ∈ Ā β and x ∉ Āα.

This implies, Āα ⊂ Ā β

Which contradicts the assumption that Ā α = Ā β .

Hence, There is no x in G such that $\alpha <$ A(x) $\leq \beta$.

Conversely, suppose that there is no x in G such that $\alpha <$ A(x) $\leq \beta$

Then, By definition Āα ⊂ Ā β ,

Let x ∈ Ā β and there is no x in G such that $\alpha <$ A(x) $\leq \beta$.

Hence, There is no x in G such that $\alpha <$ A(x) $\leq \beta$.

Conversely, suppose that there is no x in G such that $\alpha <$ A(x) $\leq \beta$.

Then,

By definition, Āα ⊂ Ā β ,

Let x ∈ Ā β and there is no x in G such that $\alpha <$ A(x) $\leq \beta$.

Hence, x ∈Āα.

(i.e) Ā β ∈ Āα.

Hence, Āα = Ā β

1.9 Definition: Let A = $(A_1 \cup A_2)$ be an intuitionistic anti fuzzy sub-bigroup of the bigroup G=$(G_1 \cup G_2)$. The sub-bigroups Āt for

t ϵ [max $\{\mu_A\ (e_1), \mu_A\ (e_2)\ \}$,1].

t ϵ [min $\{\gamma_A\ (e_1), \gamma_A\ (e_2)\ \}$,1].

Where, e_1 denotes the identity element of the group G_1 and e_2 denotes the identity element of the group G_2.This is called as Intuitionistic bi-lower level sub-bigroups of A.

1.10 Theorem: Any sub-bigroup H of a bigroup G can be realized as a Intuitionistic bi-lower level sub-bigroup of some intuitionistic anti fuzzy sub-bigroup of a bigroup G.

Proof:

Let G = $(G_1 \cup G_2$,+, •) be a bigroup.

Let H = $(H_1 \cup H_2$,+, •) be a sub-bigroup of G.

A_1 and A_2 be a fuzzy subsets of A defined by

$$\mu_A\ (x) = \begin{cases} 0 \text{ if } x \in H_1 \\ t \text{ if } x \notin H_1 \end{cases} \qquad \gamma_A\ (x) = \begin{cases} 0 \text{ if } x \in H_2 \\ t \text{ if } x \notin H_2 \end{cases}$$

where, t ϵ [max $\{\mu_A\ (e_1), \mu_A\ (e_2)\ \}$,1], t ϵ [min $\{\gamma_A\ (e_1), \gamma_A\ (e_2)\ \}$,1]. and $e_1$denotes the identity element of G_1 and $e_2$denotes the identity element of G_2 respectively

We shall prove that,

A = $(A_1 \cup A_2)$ is an intuitionistic anti fuzzy sub-bigroup of a bigroup G.

Suppose x , y ϵ H, Then

i) x , yϵ H_1 =>x + y ϵ $H_1$and x + (-y) ϵ H_1

μ_A (x) = 0 , μ_A (y) = 0 and μ_A (x +(- y)) = 0, then μ_A (x +(- y)) ≤max { μ_A (x) , μ_A (y) }.

ii)x , y ϵ H_2=>xy ϵ H_2 and xy^{-1} ϵ H_2

γ_A (x) = 0 , γ_A (y) = 0 and γ_A (xy^{-1}) = 0 , then γ_A (xy^{-1}) $\geq$ *min* { γ_A (x) , γ_A (y) }

iii) x ϵ $H_1$and y∉ H_1 =>x + y∉ $H_1$and x + (-y) ∉ H_1

μ_A (x) = 0, μ_A (y) = t and μ_A (x +(- y)) = t, then μ_A (x +(- y)) ≤max { μ_A (x) , μ_A (y) }

iv) x ϵ H_2 and y∉ H_2 =>xy ∉ $H_2$and xy^{-1} ∉ H_2

γ_A (x) = 0 , γ_A (y) = t and γ_A (xy^{-1}) = t , then γ_A (xy^{-1}) ≥ *min* { γ_A (x) , γ_A (y) }

Suppose x , y ∉ H , then

i)x , y ∉ H_1then x + y ϵ H_1 (or) x + y ∉ H_1

x , y ∉ H_1then x + (-y) ϵ H_1 (or) x +(- y) ∉ H_1

μ_A (x) = t , μ_A (y) = t and μ_A (x +(- y)) = 0 (or) t ,

Then, μ_A (x +(- y)) ≤max { μ_A (x) , μ_A (y) }.

ii)x , y ∉ H_2=>xy ϵ H_2(or) xy ∉ H_2

x , y ∉ H_2=>xy^{-1}ϵ H_2 (or) xy^{-1} ∉ H_2

$\gamma_A(x) = t$, $\gamma_A(y) = t$ and $\gamma_A(xy^{-1}) = 0$ (or) t, then $\gamma_A(xy^{-1}) \geq min\ \{\gamma_A(x), \gamma_A(y)\}$
Thus in all cases

$(A_1, +)$ is an anti fuzzy subgroup of $(G_1, +)$ and $(A_2, \bullet)$ is an anti fuzzy subgroup of $(G_2, \bullet)$

Hence A = $(A_1 \cup A_2)$ is an intuitionistic anti fuzzy sub-bigroup of a bigroup G.For this anti fuzzy sub-bigroup,

$$\bar{A}_t = \bar{A}_{1t} \cup \bar{A}_{2t} = H.$$

Conclusion:

In this paper we developed Intuitionistic antifuzzy subgroup of the group it has been some results of Intuitionistic antifuzzy sub-bigroup.This concepts can further generalised Intuitionistic antifuzzy level sub-bigroups new results in our future works.

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