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ON SOFT D^2-ALGEBRA AND SOFT D^2-IDEALS

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Abstract:

In this paper, we have studied some characterization of soft D^2-algebra, kernel, intersection, image, quotient D^2-algebra's and relations ship between D^2-algebra and D^2-ideals with suitable examples.

Index terms: d-algebra, soft set, kernel of d-algebra, image d-algebra & quotient d-algebra

1. Introduction:

Aktas and Cagman [2007] studied the basic concepts of soft set theory, and compared soft sets to fuzzy and rough sets, providing examples to clarify their differences. They also discussed the notion of soft groups. Molodtsov [1999] introduced soft set theory as an alternative approach to fuzzy set theory defined by Zadeh [1965]. After Molodtsov's study, many researchers have studied on set theoretical approaches and decision making applications of soft sets. For example Majiet.al [2011] defined some new operations of soft sets and gave a decision making method on soft sets. Y. Imai and K. Iseki [1996] introduced two classes of abstract algebra BCI-algebra and BCL –algebra. It is known as the notion of BCI –algebra is a generlisation of BCK-algebra. J. Neggers and S. Kim [1999] introduced the class of d-algebra which another generalization of BCK –algebra and investigated relations between d-algebras and BCK-algebra. In this paper, we have studied characterization of soft D^2-algebra, kernel, intersection, image, quotient d-algebra's and D^2-ideals with suitable examples.

2. Preliminaries:

2.1 Definition [D. A. Molodtsov]: A pair (δ,A) is called a soft set over U, where δ is a mapping given by $\delta : A \rightarrow P(U)$.

2.2 Definition [D. A. Molodtsov]: Let δ_A and δ_B be two soft sets over U such that $A \cap B \neq \Phi$. The restricted intersection of δ_A and λ_B is denoted by $\delta_A \Cup \lambda_B$ and is defined as $\delta_A \Cup \lambda_B = (H,C)$, where $C = A \cap B$ and for all $c \in C$, $H(c) = \delta(c) \cap \lambda(c)$.

2.3 Definition [D. A. Molodtsov]: Let δ_A and G_B be two soft sets over U such that $A \cap B \neq \Phi$. The restricted union of δ_A and λ_B is denoted by $\delta_A \cup_R \lambda_B$ and is defined as $\delta_A \cup_R \lambda_B = (H,C)$,where $C = A \cap B$ and for all $c \in C$, $H(c) = \delta(c) \cup \lambda(c)$.

2.4 Definition [D. A. Molodtsov]: Let δ_A and λ_B be soft sets over the common universe U and ψ be a function from A to B. Then we can define the soft set $\psi(\delta_A)$ over U, where $\psi(\delta_A) : B \rightarrow P(U)$ is a set valued function defined by $\psi(\delta_A)(b) = \bigcup\{\delta(a) \mid a \in A \text{ and } \psi(a) = b\}$,
If $\psi^{-1}(b) \neq \Phi$, = 0 otherwise for all $b \in B$. Here, $\psi(\delta_A)$ is called the soft image of F_A under ψ. Moreover we can define a soft set $\psi^{-1}(\lambda_B)$ over U, where $\psi^{-1}(\lambda_B) : A \rightarrow P(U)$ is a set-valued function defined by $\psi^{-1}(\lambda_B)(a) = \lambda(\psi(a))$ for all $a \in A$. Then, $\psi^{-1}(\lambda_B)$ is called the soft pre image (or inverse image) of λ_B under ψ.

2.5 Definition [P. K. Maji, R. Biswas]: Let δ_A and λ_B be soft sets over the common universe U and ψ be a function from A to B. Then we can define the soft set $\psi^{\star}(\delta_A)$ over U, where $\psi^{\star}(\delta_A) : B \rightarrow P(U)$ is a set-valued function defined by $\psi^{\star}(\delta_A)(b) = \bigcap\{\delta(a) \mid a \in A \text{ and } \psi(a) = b\}$, if $\psi^{-1}(b) \neq \Phi$,=0 otherwise for all $b \in B$. Here, $\psi^{\star}(\delta_A)$ is called the soft anti image of F_A under ψ.

2.6 Definition: A non-empty set X with constant 0 and a binary operation Δ is called D^2 –algebra if (i) $x^2 \Delta x^2 = 0$ (ii) $0 \Delta x^2 = 0$ (iii) $x^2 \Delta y^2 = 0$ and $y^2 \Delta x^2 = 0 \rightarrow x = y$ for all x,y in X.

2.7 Definition: Let X be a D^2-algebra and I be a subset of X. Then I is called D^2-idea of X if (i) $0 \in I$ (ii) $x^2 \Delta y^2 \in I$ and $y^2 \in I \rightarrow x \in I$. (iii) $x^2 \in I$ and $y^2 \in I \rightarrow x^2 \Delta y^2 \in I$.

2.8 Definition: Let (X, Δ) be a D^2-algebra and (I,Δ) be a D^2-ideal. Then $X/I = \{ a^2 \Delta I / a^2 \varepsilon X \}$ and $a^2 \Delta I = \{ a^2 \Delta x^2 / x \varepsilon I \}$, $0 \varepsilon a^2 \Delta I$. It follows that $(a^2 \Delta I) \Delta (b^2 \Delta I) = I \Delta (a^2 \Delta b^2)$.

3. Some Standard Propositions on D^2-Algebra:

3.1 Proposition:

The intersection of two D^2- algebra is a D^2 –algebra with respect to Δ and $\otimes$.

Proof:

Let (X, Δ) and $(Y, \otimes)$ be two D^2-algebra.
Here $0 \varepsilon X$ and $0^1 \ y \varepsilon Y$.
(i) Let $x^2 \varepsilon X \cap Y$. Thus $x^2 \varepsilon X$ and $x^2 \varepsilon Y$. so $x^2 \Delta x^2$ and $^{x2} \otimes x^2 = 0$.

(ii) Let x^2 ε X∩ Y and 0 ε X. Thus x^2 ε X and x^2 ε Y and $0 \Delta x^2 = 0$ and $0 \otimes x^2 = 0$.

(iii) Let x^2, y^2 ε X∩ Y implies x^2, y^2 ε X and x^2, y^2 ε Y, and $x^2 \Delta y^2 = y^2$. Since X is a D^2 –algebra. We have $x^2 = y^2$, $x^2 \otimes y^2 = 0$ and $y^2 \otimes x^2 = 0$ implies $x^2 = y^2$, which shows that x = y (since Yis a D^2-algebra).

Therefore X∩Y forms a D^2- algebra with respect to Δ and ⊗.

3.2 Proposition:

The union of two D^2- algebra (X, Δ) and (Y, Δ) form a D^2 –algebra with respect to Δ if one contained in other.

Proof:

Let X and Y be two D-algebra's. Here 0 ε X and 0^1 y ε Y.

(i) Let x^2 ε X U Y implies x^2 ε X or x^2 ε Y and so $x^2 \Delta x^2 = 0$ or $0 \Delta x^2 = 0$ implies $x^2 \Delta x^2 = 0$.

(ii) Let x^2 ε XUY implies x^2 ε X or y^2 ε Y implies $0 \Delta x^2 = 0$ or $x^2 \Delta 0 = 0$ implies $0 \Delta x^2 = 0$.

(iii) Let x^2, y^2 ε XUY implies x^2, y^2 ε X or x^2,y^2 ε Y. Since X is contained Y or Y is contained in X. (or) $x^2 \Delta y^2 = 0$, $y^2 \Delta x^2 = 0$ implies $x^2 = y^2$ where x^2 ε X and y^2 ε Y. Therefore union of two D^2-algebras is a D^2-algebra with respect to Δ.

3.3 Proposition:

Kernel of a D^2 –algebra homomorphism from X into Y is a D^2-algbera of X.

Proof:

Let f : (X, Δ) → (Y, ⊗) be a homeomorphisms and define $f(x^2 \Delta y^2) = f(x^2) \Delta f(y^2)$.

For all x^2, y^2 ε X, Kernel of I = {0 ε X / f(0) = 0^1}.

(i) $f(0) = f(0 \Delta 0) = f(0) \otimes f(0) = 0^1 \otimes 0^1 = 0^1$.

(ii) Let $x^2 \Delta y^2$ ε I, y^2 ε I, we claim x^2 ε I. $f(x^2) \otimes f(y^2) = 01$ implies $f(x^2) \otimes 0^1 = 0^1$. It follows that $0^1 \otimes f(x) = 0^1$ by (ii) of D^2-algebra; $f(x^2) = 0^1$ (iii) of D^2 –algebra, x^2 ε I.

(iii) Let x^2 ε I; y^2 ε x → $f(x^2) = 0^1$ implies I C x^2 → x^2 ε X ; y^2 ε Y → $x^2 \Delta y^2$ ε X. $f(x^2 \Delta x^2) = f(x^2) \otimes f(y^2) = 0^1 \otimes f(y^2) = 0^1$ by (ii) of D^2 –algebra implies $x^2 \Delta y^2$ ε I. Therefore I is a D^2-ideal of D^2-algebra X.

3.4 Proposition:

Let (X, Δ) be a D^2-algbera and (I, Δ) be a D^2-ideal of X. Then X /I form a D^2-algebra.

Proof:

Let $a^2 \Delta$ I ε X/I.

(i) $(a^2 \Delta I) \Delta (a^2 \Delta I) = I \Delta (a^2 \Delta a^2) = I \Delta 0^1$ (since $a^2 \Delta a^2 = 01$)

(ii) $0^1 \Delta a^2 = (0 \Delta a^2) \Delta (a^2 \Delta I) = I \Delta (0 \Delta a^2)$ (since $0 \Delta a^2 = 0$) $= I \Delta 0 = 0$.

(iii) $(a^2 \Delta I) \Delta (b^2 \Delta I) = 01$ and $(b^2 \Delta I) \Delta (a^2 \Delta I) = 0$

Now $(a^2 \Delta b^2) \Delta I = 0 \Delta I = 0$ and $(b^2 \Delta a^2) \Delta I = 0 \Delta I = 0$.

$(a^2 \Delta b^2) \Delta x^2 = 0$ for all x^2 ε I and $(a^2 \Delta b^2) \Delta 0 = 0$ (0 ε I). It gives that $0 \Delta (a^2 \Delta b^2) = 0$; $a^2 \Delta b^2$ ε X; 0 ε y.

$a^2 \Delta b^2 = 0$ and $b^2 \Delta a^2 = 0$ implies $a^2 = b^2$.

Now $a^2 \Delta I = b^2 \Delta I$. Therefore X/I is a D^2-algbera.

3.5 Proposition:

Every image of a homomorphic D^2-algebra is a D^2-Algebra.

Proof:

Let (X, Δ) be a D^2-algebra and f: X → Y be 1-1 homomorphism's f(0) = 0.

To show that $f(x^2)$ is a D^2-algebra with respect to ⊗.

(i) Let y^2 ε $f(x^2)$. Then there exists an element x^2 ε X, such that $f(x^2) = y^2$. But X is a D^2-algebra. $X^2 \Delta x^2 = 0$, $f(x^2 \Delta x^2) = f(0) = 0^1$, $y^2 \Delta y^2 = 0^1$ (i) is true for $f(x^2)$.

(ii) $0^1 \otimes y^2 = f(0) \otimes f(y^2) = f(0 \Delta x^2) = f(0)$ (X is D^2-algebra) $=0^1$ (ii) is true for $f(x^2)$.

(iii) Let $f(x^2)$, $f(y^2)$ ε f(X) , such that $f(x^2) = f(y^2) = 0^1 = f(y^2) = f(x^2)$.

$f(x^2 \Delta y^2) = f(0)$ implies $f(y^2 \Delta x^2) = f(0)$.
Since f is 1-1 , then $x^2 \Delta y^2 = 0$ and $y^2 \Delta x^2 = 0$.
X is a D^2 –algebra, $x^2= y^2$ implies $f(x^2) = f(y^2)$.

Conclusion:

Y.I mai and K.I seki [1996] introduced two classes of abstract algebra BCI-algebra and BCL –algebra. We have extended this concept into soft D^2 –algebra and soft D^2-ideals and relations ship between them.

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