A New Keynesian Model with Wealth in the Utility Function

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This paper extends the textbook New Keynesian model by introducing wealth, in the form of government bonds, in households' utility function. This extension modifies the properties of the New Keynesian IS curve: the real interest rate is now negatively related to output instead of being constant, equal to the time discount rate. As a result, when price rigidity and marginal utility of wealth are sufficient, the equilibrium admits a unique steady state, and this steady state is always a source, whether the economy is at the zero lower bound or away from it. These properties greatly simplify the analysis of the zero lower bound, and they eliminate several zero-lower-bound pathologies, such as the forward-guidance puzzle.

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1. Introduction

The New Keynesian model is the keystone of modern monetary economics. Given this position, we would expect the textbook version of the model to offer a good description of basic macroeconomic phenomena, to operate in an intuitive way, and to have some predictive power, for instance to forecast the effect of policy changes. While the model does very well in most situations, at the zero lower bound it is plagued by a handful of well-documented pathologies.

Indeed the dynamical system describing the New Keynesian equilibrium is surprisingly complex: it sometimes has two steady states, one at the zero lower bound and the other away from it (Benhabib, Schmitt-Grohe, and Uribe 2001); and a zero-lower-bound steady state is always a saddle, leading to indeterminacy (Woodford 2001). These properties make it difficult to study the zero lower bound: researchers are forced to construct scenarios where shocks and policies follow sophisticated patterns over time; they cannot simply study the effects of shocks or policies by comparative statics. Besides, the results are sensitive to seemingly innocuous changes in the temporal patterns followed by shocks and policies (Cochrane 2017; Wieland 2018). And the properties of the dynamical system occasionally yield puzzling properties at the zero lower bound, such as the forward-guidance puzzle: changes in monetary policy announced far enough in advance have implausibly large effects on output and inflation (Del Negro, Giannoni, and Patterson 2012; Carlstrom, Fuerst, and Paustian 2015).

This paper proposes a simple remedy to these pathologies. First, we notice that if the Phillips curve is flat enough, the zero-lower-bound steady state disappears. In a New Keynesian steady state the real rate equals the time discount rate. If the nominal rate is zero, then inflation necessarily is minus the time discount factor. But if the Phillips curve is flat enough—for instance if it is near horizontal around zero inflation—and the time discount rate is large enough, then there is no point on the Phillips curve that can generate an inflation of minus the time discount rate. The literatures measuring the slope of the Phillips curve and the time discount rate, surveyed by Mavroeidis, Plagborg-Moller, and Stock (2014) and Frederick, Loewenstein, and O'Donoghue (2002), suggest that the Phillips curve is flat enough for the steady state to disappear.

Accordingly we focus on the parametric configuration under which the textbook model is left with a unique, well-behaved, but non-zero-lower-bound steady state. We then assume that people derive utility not only from consumption but also from holding wealth, which in the New Keynesian model takes the form of government bonds. Since bonds are in zero net supply, the steady-state IS curve imposes that output is decreasing in the real interest rate—as in the old IS-LM model. In contrast, in the textbook model the IS curve does not involve output at all—it imposes that the real interest rate equals the time discount rate. Now, in the New Keynesian model like in any model of equilibrium, demand and supply curves have to cut each other in the right way for the equilibrium to behave well. By tilting the IS curve sufficiently, a zero-lower-bound steady state appears when aggregate demand is weak enough (when demand is strong we have a regular steady state). Furthermore because the IS curve has tilted demand and supply curves cut each other in the right way. This flips the properties of the dynamical system at the zero lower bound, making the steady state a source and thus curing the New Keynesian pathologies. This requires the IS curve to tilt enough, but as prices become more rigid, the required marginal utility becomes vanishingly small. We find that the requirement is likely to be satisfied in practice.

Although the assumption of wealth in the utility function is atypical in modern economics, many founding fathers of economics and sociology recognized that there were many other motives for accumulating wealth beyond providing for future consumption.[1] Steedman (1981) offers a fascinating survey of the views of Smith, Ricardo, J.S. Mill, Marshall, Veblen, Fisher, and Keynes. They believed that wealth was valuable for several non-consumption reasons: wealth brings social status; wealth brings political power and influence; and wealth is a gauge of success in life. For instance, Fisher wrote in *The Theory of Interest* that “A man may include in the benefits of his wealth . . . the social standing he thinks it gives him, or political power and influence, or the mere miserly sense of possession, or the satisfaction in the mere process of further accumulation” (p. 17, n. 5). Fisher’s perspective is particularly interesting, since the modern view of saving as a way to improve one’s consumption stream is often attributed to him. Fisher’s view that people derive “satisfaction in the mere process of further accumulation” is aligned the thesis developed by Weber in *The Protestant Ethic and the Spirit of Capitalism*: Weber argued that the Protestant ethics favored frugality and asceticism and thus dignified the accumulation of wealth. In this paper we do not precisely model the various ways in which wealth may bring utility; instead we make the reduced-form assumption that wealth enters the utility function.[2]

In the New Keynesian model wealth takes the form of government bonds, so in practice we assume that households derive utility from consumption and from real bond holdings. This assumption can also be justified on financial grounds: government bonds are safer and more liquid than other assets, which can be captured in reduced form by introducing bonds in the utility function, as in Poterba and Rotemberg (1987) and Krishnamurthy and Vissing-Jorgensen

[1]Modern researchers have started to explore the implications of this assumption: for economic growth (Kurz 1968; Zou 1994), for life-cycle consumption and saving (Carroll 2000; Francis 2009), for asset pricing (Bakshi and Chen 1996; Gong and Zou 2002), and for capital taxation (Piketty and Saez 2013; Saez and Stantcheva 2016).

[2]See Cole, Mailath, and Postlewaite (1992) for a possible microfoundation of wealth in the utility function. They develop a model in which wealthier individuals have higher social rankings, allowing them to marry wealthier partners. The partners form a wealthier household, so they consume more. They show that in reduced form—abstracting from the marriage market—people behave as if both consumption and wealth entered the utility function.

(2012).[3] In fact, Krishnamurthy and Vissing-Jorgensen show that a bond-in-the-utility-function model resolves a number of asset-pricing puzzles. In addition, Fisher (2015) and Campbell et al. (2017) show that in a quantitative New Keynesian model, the shock responsible for most short-run fluctuations can be interpreted as a shock to the marginal utility of bonds. And in a comparable model, Del Negro et al. (2017) show that changes in the marginal utility of bonds explain medium-run variations in interest rates.

In this paper we address the New Keynesian pathologies by extending the New Keynesian model in such a way that it always features a unique, well-behaved steady state, and this steady state is at the zero lower bound when aggregate demand is low enough. Before us, Eggertsson and Mehrotra (2014), Gabaix (2016), Diba and Loisel (2017), and Cochrane (2018) have also altered the New Keynesian model to obtain a well-behaved zero-lower-bound steady-state equilibrium (Eggertsson and Mehrotra call this steady state a "secular stagnation"). Eggertsson and Mehrotra modify the IS curve by introducing an overlapping-generation structure, and the Phillips curve by introducing downward nominal wage rigidity. Gabaix modifies the IS curve by introducing boundedly rational households, and the Phillips curve by introducing boundedly rational firms. Diba and Loisel introduce a banking sector to the model, and assume that the central bank sets both the interest rate on bank reserves and the stock of bank reserves. The presence of a banking cost modifies the Phillips curve; the spread between the interest rates on bonds and bank reserves modifies the IS curve. Finally, Cochrane (2018) maintains the IS and Phillips curves but appends the fiscal theory of the price level to the model. Our paper, then, shows that to obtain a well-behaved zero-lower-bound steady state it is sufficient to tilt the IS curve; it is not necessary to tilt the Phillips curve. Moreover, the paper shows that this can be done while maintaining all the features of the textbook model: we only introduce a new argument (bonds) into households' utility, and we do not need to modify one step of the model's typical derivations.

In this paper we address the New Keynesian pathologies by introducing wealth in the utility function, thus tilting the IS curve. Methodologically, our work is related to two groups of recent papers using similar approaches to tackle related problems. First, in non-New-Keynesian models, Ono and Yamada (2012), Michaillat and Saez (2014), and Michau (2015) show that introducing wealth in the utility function generates well-behaved zero-lower-bound steady states. These results are not directly portable to the New Keynesian model, however, because these results require strong forms of wage or price rigidity: in Ono and Yamada wages are constrained to follow an exogenous time path; in Michaillat and Saez inflation is fixed; and in Michau nominal wages are downward rigid. Second, McKay, Nakamura, and Steinsson (2016) and Bilbiie (2017) tackle

[3]The argument developed by Feenstra (1986) implies that models in which bonds provide transaction services, such as in Bansal and Coleman (1996), are observationally equivalent to bond-in-the-utility-function models.

the forward-guidance puzzle by introducing heterogeneous agents facing borrowing constraints into the New Keynesian models. While these models and ours are very different, they share the property that the aggregate IS curve is discounted (McKay, Nakamura, and Steinsson 2017; Diba and Loisel 2017). What does not appear in these papers is that if the IS curve is sufficiently discounted, the New Keynesian model inherits a well-behaved zero-lower-bound steady state, which cures all its pathologies.

2. The Model

We extend a textbook New Keynesian model by assuming that households derive utility not only from consuming goods and services but also from holding real wealth. Our starting point is the model in Gali (2008, chap. 3), ever-so-slightly modified to facilitate exposition. First, the model is set in continuous time instead of discrete time. Second, households are self-employed, as in Benhabib, Schmitt-Grohe, and Uribe (2001), instead of being hired by firms on a competitive labor market. Third, price rigidity comes from a quadratic price-adjustment cost, as in Rotemberg (1982), instead of Calvo (1983) pricing.[4]

2.1. Assumptions

The economy is composed of a measure 1 of self-employed households. Each household $j \in [0, 1]$ produces $y_j(t)$ units of a differentiated good or service, sold to other households at a price $p_j(t)$. All households use an identical technology, represented by the production function

$$y_j(t) = ah_j(t), \tag{1}$$

where the parameter $a > 0$ represents the level of technology, and the variable $h_j(t)$ is household j's hours of work. Household j incurs a disutility $h_j(t)$ from producing goods and services.[5] The goods and services produced by households are imperfect substitutes for one another, so each household exercises some monopoly power on the goods market. In pricing their goods, however,

[4]These modifications leave the equilibrium conditions, and hence the results, unaffected. In particular, after linearization, Calvo and Rotemberg pricing lead to the same Phillips curve (Roberts 1995, pp. 976–979).

[5]We assume a linear disutility from labor to obtain a linear Phillips curve in the phase diagrams describing the equilibrium. This assumption is not uncommon in the New Keynesian literature (for example, Nakamura and Steinsson 2010, p. 976), and can be justified by the indivisible-labor argument proposed by Rogerson (1988, pp. 13–14). On the other hand, the assumption implies an infinite Frisch elasticity of labor supply, at odds with existing microestimates: Chetty et al. (2013, table 2) reports a median microestimate of the Frisch elasticity for aggregate hours of 0.9. We have repeated the analysis with a finite Frisch elasticity. The Phillips curve becomes convex, but the results remain broadly the same.

households incur a quadratic price-adjustment cost: when household j changes its price at a rate $\pi_j(t) \equiv \dot{p}_j(t)/p_j(t)$, it incurs a disutility

$$\frac{\gamma}{2}\pi_j(t)^2.$$

The parameter $\gamma > 0$ governs the cost to change prices and thus determines the rigidity of prices. If $\gamma \to 0$, prices can be adjusted at no cost, so prices will be perfectly flexible. If $\gamma \to \infty$, adjusting prices is infinitely costly, so prices will be fixed.

Besides working, each household consumes goods and services produced by other households. Household j buys amounts $c_{jk}(t)$ of the different goods $k \in [0,1]$. Household j's consumption of the different goods is aggregated into a consumption index

$$c_j(t) \equiv \left[\int_0^1 c_{jk}(t)^{(\epsilon-1)/\epsilon} dk\right]^{\epsilon/(\epsilon-1)}, \tag{2}$$

where $\epsilon > 1$ is the elasticity of substitution between the different goods. Then the household derives utility $\ln(c_j(t))$ from the consumption index. With such a consumption index, the relevant price index is

$$p(t) \equiv \left[\int_0^1 p_j(t)^{1-\epsilon} di\right]^{1/(1-\epsilon)}; \tag{3}$$

when households optimally allocate their consumption expenditure across goods, p represents the price of one unit of the consumption index. The inflation rate is $\pi(t) \equiv \dot{p}(t)/p(t)$.

Finally, households can hold government bonds earning a nominal interest rate $i(t)$, determined by the central bank. Household j holds a nominal quantity of bonds $b_j(t)$. The household holds bonds partly to smooth consumption over time and partly because it derives utility

$$u\left(\frac{b_j(t)}{p(t)}\right)$$

from its real wealth $b_j(t)/p(t)$, where the function u is increasing and concave. The law of motion of nominal bond holdings is

$$\dot{b}_j(t) = i(t)b_j(t) + p_j(t)y_j(t) - \int_0^1 p_k(t)c_{jk}(t)dk. \tag{4}$$

Lastly, household j chooses paths $\{y_j(t)\}$, $\{p_j(t)\}$, $\{h_j(t)\}$, $\{\pi_j(t)\}$, $\{c_{jk}(t)\}$ for all $k \in [0,1]$, and $\{b_j(t)\}$ to maximize the discounted sum of instantaneous utilities

$$\int_0^\infty e^{-\delta t}\left[\ln(c_j(t)) + u\left(\frac{b_j(t)}{p(t)}\right) - h_j(t) - \frac{\gamma}{2}\pi_j(t)^2\right]dt, \tag{5}$$

where $\delta > 0$ is the time discount factor. There are several constraints: the law of motion (4), the law of motion $\dot{p}_j(t) = \pi_j(t)p_j(t)$, the production function (1), and households' demand for good j. The demand for good j takes the form $y_j(t) = y_j^d(p_j(t), t)$, where y_j^d is a decreasing function of p_j that we will compute. Finally, the household takes as given initial bond holdings $b_j(0)$ and price $p_j(0)$, and is subject to a borrowing constraint preventing Ponzi schemes.

2.2. Optimal Consumption and Pricing

We determine households' consumption and pricing by solving their maximization problem. To solve household j's problem, we write the current-value Hamiltonian:

$$\begin{aligned}\mathcal{H} &= \ln(c_j(t)) + u\left(\frac{b_j(t)}{p(t)}\right) - \frac{y_j^d(p_j(t),t)}{a} - \frac{\gamma}{2}\pi_j(t)^2 \\ &\quad + \eta_j(t)\pi_j(t)p_j(t) + \zeta_j(t)\left[i(t)w_j(t) + p_j(t)y_j^d(p_j(t),t) - \int_0^1 p_k(t)c_{jk}(t)dk\right]\end{aligned}$$

with control variables $c_{jk}(t)$ for all $k \in [0,1]$ and $\pi_j(t)$, state variables $b_j(t)$ and $p_j(t)$, and costate variables $\eta_j(t)$ and $\zeta_j(t)$. To simplify we have used the production and demand constraints to substitute $y_j(t)$ and $h_j(t)$ out of the Hamiltonian. To ease notation, we now drop the time index t.

The first optimality condition is $\partial\mathcal{H}/\partial c_{jk} = 0$. Using (2), we obtain

$$\frac{1}{c_j}\left(\frac{c_{jk}}{c_j}\right)^{-1/\epsilon} = \zeta_j p_k. \tag{6}$$

Appropriately integrating this expression over all $k \in [0,1]$, and using the definitions of the consumption and price indexes, we find that $\zeta_j = 1/(pc_j)$. Using this result and (6), we obtain the usual demand for good k:

$$y_k^d(p_k) \equiv \int_0^1 c_{jk}dj = c \cdot \left(\frac{p_k}{p}\right)^{-\epsilon}, \tag{7}$$

where $c \equiv \int_0^1 c_j dj$ measures aggregate consumption. We use this expression for $y_k^d(p_k)$ in household k's Hamiltonian. We also obtain $\int_0^1 p_k c_{jk}dk = pc_j$: the price of one unit of consumption index is indeed p.

The second optimality condition is $\partial\mathcal{H}/\partial b_j = \delta\zeta_j - \dot{\zeta}_j$, which implies

$$-\frac{\dot{\zeta}_j}{\zeta_j} = i + \frac{u'\left(b_j/p\right)}{p\zeta_j} - \delta.$$

Using $1/\zeta_j = pc_j$, we obtain the consumption Euler equation:

$$\frac{\dot{c}_j}{c_j} = r + u'\left(b_j/p\right) c_j - \delta, \tag{8}$$

where $r \equiv i - \pi$ is the real interest rate. This Euler equation describes the time path of household j's consumption; in general equilibrium it will yield the IS curve. The Euler equation is modified because real wealth enters the utility function: the consumption path is governed not only by the gap between real interest rate and time discount rate, but also by the marginal rate of substitution between real wealth and consumption, $u'(b_j/p)c_j$.

The third optimality condition is $\partial\mathcal{H}/\partial\pi_j = 0$, which implies $\eta_j p_j = \gamma\pi_j$. We differentiate this equation with respect to time:

$$\frac{\dot{\eta}_j}{\eta_j} = \frac{\dot{\pi}_j}{\pi_j} - \pi_j. \tag{9}$$

The fourth optimality condition is $\partial\mathcal{H}/\partial p_j = \delta\eta_j - \dot{\eta}_j$, which implies

$$\pi_j - \frac{(\epsilon-1)y_j\zeta_j}{\eta_j p_j}\left[p_j - \frac{\epsilon}{\epsilon-1}\cdot\frac{1}{a\zeta_j}\right] = \delta - \frac{\dot{\eta}_j}{\eta_j}.$$

Using $\zeta_j = 1/(pc_j)$, $\eta_j p_j = \gamma\pi_j$, and (9), we obtain the pricing equation:

$$\dot{\pi}_j = \delta\pi_j + \frac{(\epsilon-1)y_j}{\gamma c_j}\left[\frac{p_j}{p} - \frac{\epsilon}{\epsilon-1}\cdot\frac{c_j}{a}\right]. \tag{10}$$

This equation describes the time path of the price set by household j; in general equilibrium it will yield the Phillips curve. The pricing equation has two interesting special cases. Without price-adjustment cost ($\gamma = 0$), it implies

$$\frac{p_j}{p} = \frac{\epsilon}{\epsilon-1}\cdot\frac{c_j}{a}. \tag{11}$$

In that case, as usual, the household sets its real price p_j/p at a markup $\epsilon/(\epsilon-1) > 1$ over its real marginal cost of producing goods and services—defined as the ratio between the marginal rate of substitution of consumption for leisure, c_j, and the marginal product of labor,

a. With infinite price-adjustment cost ($\gamma \to \infty$), the equation implies $\dot{\pi}_j = \delta\pi_j$, or equivalently $\pi_j(t) = \pi_j(0)e^{\delta t}$. The value of $\pi_j(0)$ is given by another optimality condition: the transversality condition $\lim_{t\to+\infty} e^{-\delta t}\eta_j(t)p_j(t) = 0$. As $\eta_j(t)p_j(t) = \pi_j(t)\gamma = \pi_j(0)e^{\delta t}\gamma$, the transversality condition implies $\pi_j(0) = 0$. Thus, with infinite price-adjustment cost, $\pi_j(t) = 0$ for all t: the price $p_j(t)$ is completely fixed. Finally, when $\gamma \in (0, +\infty)$, the price $p_j(t)$ is somewhat rigid: the inflation path is governed by the gap between the actual real price and the profit-maximizing real price, given by (11).

The four conditions that we have just described, together with the transversality conditions $\lim_{t\to+\infty} e^{-\delta t}\eta_j(t)p_j(t) = 0$ and $\lim_{t\to+\infty} e^{-\delta t}\zeta_j(t)b_j(t) = 0$, are necessary and sufficient for a maximum to the household's problem.

2.3. General Equilibrium

We now describe the general equilibrium. We focus on a symmetric equilibrium, in which all the households have the same initial wealth and set the same initial price. In such an equilibrium, all households set the same price, work the same amount, have the same consumption of goods and services, and hold the same wealth. All the variables are the same for all households and firms, so we drop the subscripts j and k.

First, government bonds are in zero net supply: $b = 0$. Second, production and consumption of each goods and services are equal: $y = c$. Third, the central bank follows a simple interest-rate rule, subject to the zero lower bound on nominal interest rates:

$$i = \max\left\{0, i^* + \phi\pi\right\}. \tag{12}$$

The parameter i^* is the nominal interest rate with zero inflation, and the parameter $\phi \geq 0$ governs the response of monetary policy to inflation. When $i^* + \phi\pi$ is negative, the zero lower bound is binding, and $i = 0$. Fourth, consumption and saving follow (8), which yields the IS curve:

$$\frac{\dot{y}}{y} = i - \pi + u'(0)y - \delta. \tag{13}$$

The presence of wealth in the utility function introduces the term $u'(0)y$ in the IS curve, which will modify in many ways the properties of the New Keynesian model. Last, pricing follows (10), which yields the usual Phillips curve:

$$\dot{\pi} = \delta\pi - \frac{\epsilon}{\gamma a} \cdot y + \frac{(\epsilon - 1)}{\gamma}. \tag{14}$$

The New Keynesian general equilibrium is therefore be described by three variables: y, π, and i. These three variables are given by three equations: a monetary-policy rule and two differential equations, the IS and Phillips curves. The nominal interest rate can be substituted out of the IS curve using the monetary-policy rule to obtain a new differential equation: the aggregate demand (AD) curve. The AD curve has two distinct expressions, depending on the inflation rate. When $\pi < -i^*/\phi$, the economy is at the zero lower bound ($i = 0$), and the AD curve is

$$\frac{\dot{y}}{y} = -\pi + u'(0)y - \delta. \tag{15}$$

When $\pi > -i^*/\phi$, the economy is away from the zero lower bound ($i = i^* + \phi\pi > 0$), and the AD curve is

$$\frac{\dot{y}}{y} = i^* - \delta + (\phi - 1)\pi + u'(0)y. \tag{16}$$

3. Revisiting New Keynesian Properties

We use our model to revisit several New Keynesian properties: number of steady states, requirements for equilibrium determinacy (Taylor principle), effect of monetary policy on interest rates (Fisherian effects), effect of technology shocks at the zero lower bound and away from it (paradox of toil), effect of aggregate demand shocks (paradox of thrift), and effects of monetary policy announcements (forward guidance puzzle).

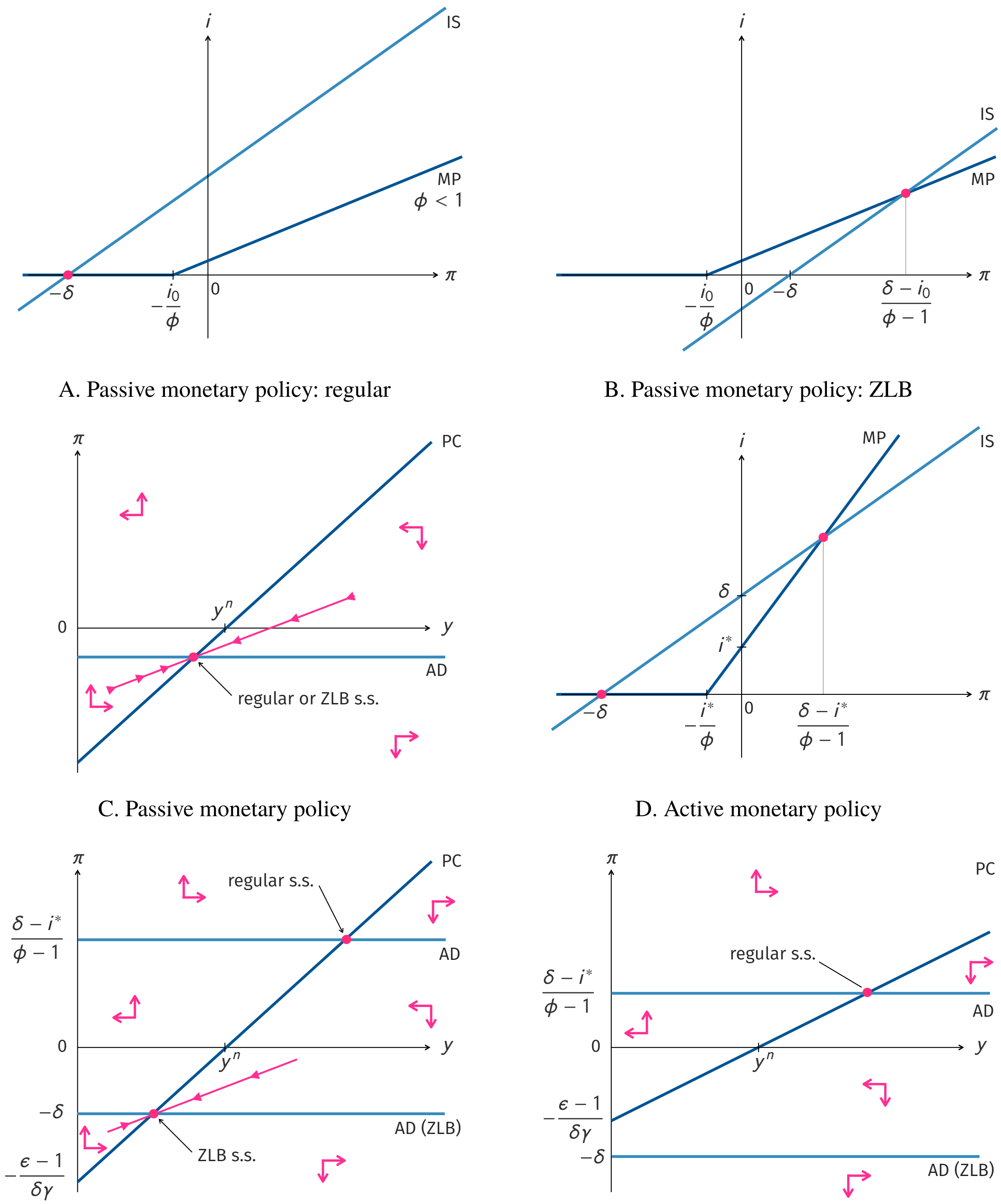
3.1. Steady States and Determinacy

We quickly revisit the steady-state properties of the standard New Keynesian model. We denote the steady-state values of the nominal interest, output, and inflation by $\dot{i}$, $\dot{y}$, and $\dot{\pi}$. We solve for the steady state in two steps. First, without wealth in the utility function, the steady-state IS curve is

$$\bar{i} = \delta + \overline{\pi}. \tag{17}$$

This IS curve imposes that in steady state the real interest rate, $\overline{r} = \bar{i} - \overline{\pi}$, equals the time discount factor, δ. Combining the IS curve with the monetary-policy rule, given by (12), yields inflation and interest rate. Then, we obtain output by plugging inflation into the steady-state Phillips curve:

$$\overline{\pi} = \frac{\epsilon}{\delta\gamma a}\left(\overline{y} - y^n\right) \tag{18}$$



A. Passive monetary policy: regular

B. Passive monetary policy: ZLB

C. Passive monetary policy

D. Active monetary policy

E. Active monetary policy: low price rigidity

F. Active monetary policy: high price rigidity

Figure 1. Equilibrium Dynamics Without Wealth in the Utility Function

Notes: The MP line represents the monetary-policy rule (12). The IS line represents the steady-state IS curve (17). The AD lines represents the steady-state AD curve, given by $\overline{\pi} = \delta$ at the zero lower bound and $\overline{\pi} = (\delta - i^*)/(\phi - 1)$ away from the zero lower bound. The PC line represents the steady-state Phillips curve (18).

where

$$y^n \equiv \frac{\epsilon - 1}{\epsilon} a \tag{19}$$

is the natural level of output: the level prevailing when prices are flexible ($\gamma = 0$). The output y^n increases with competitiveness of the economy (ϵ) and technology (a).

When monetary policy is passive ($\phi < 1$), we obtain a unique inflation and interest rate. These can either be at the zero lower bound: $\bar{i} = 0$ and $\overline{\pi} = -\delta$. Or they can be away from it:

$$\bar{i} = \frac{\phi\delta - i^*}{\phi - 1} \qquad \text{and} \qquad \overline{\pi} = \frac{\delta - i^*}{\phi - 1}. \tag{20}$$

These two cases are illustrates in panels A and B in figure 1). As shown in figure 1), panel C, a steady state exists only if steady-state inflation is high enough to cross the steady-state Phillips curve for positive output. However, in any case, the steady state with passive monetary policy is never interesting because it is always locally a saddle. Hence the equilibrium is indeterminate: there are infinitely many equilibrium paths converging to the steady state. This result can be formally established by linearizing the dynamical system generated by the AD and Phillips curves:

$$\begin{bmatrix} \dot{y} \\ \dot{\pi} \end{bmatrix} = \mathbf{A} \begin{bmatrix} y - \overline{y} \\ \pi - \overline{\pi} \end{bmatrix}. \tag{21}$$

The matrix $\mathbf{A}$ is given by

$$\mathbf{A} \equiv \begin{bmatrix} 0 & \theta \\ -\epsilon/(\gamma a) & \delta \end{bmatrix},$$

where $\theta \equiv -\overline{y}$ at the zero lower bound and $\theta \equiv (\phi - 1)\overline{y}$ away from the zero lower bound. In both cases, the trace of $\mathbf{A}$ is positive: $T = \delta > 0$. In both cases, $\theta < 0$, so the determinant is also negative: $D = \theta\epsilon/(\gamma a) < 0$. This implies that the eigenvalues of $\mathbf{A}$ are necessarily real and of opposite sign: the steady state is necessarily a saddle of the dynamical system. This is the well-know Taylor principle: when monetary policy is passive (in particular, at the zero lower bound) the equilibrium is indeterminate (Woodford 2001).

With passive monetary policy, the pathologies of the zero lower bound also spread to normal times. To tackle the issue, we follow the literature and throughout the paper make the following assumption:

ASSUMPTION 1. *Monetary policy is active:* $\phi > 1$.

With active monetary policy, the central bank systematically raises the nominal interest rate more than one-for-one with inflation. Under this type of policy, steady states away from the zero lower bound will be well behaved. It also seems that in the United States after 1980, the Federal Reserve has been following such an active policy (Clarida, Gali, and Gertler 2000). Before proceeding, we limit the range of monetary-policy rules to ensure that at least one steady state exists:

ASSUMPTION 2. *The intercept of the monetary-policy rule is low enough to satisfy the following two conditions:*

$$i^* < \phi\delta \tag{22}$$

$$i^* < \delta + \frac{(\epsilon - 1)(\phi - 1)}{\delta\gamma}. \tag{23}$$

These conditions are necessary for a steady state to exist. Indeed, if the first is not satisfied then the IS and MP lines do not cross in panel D of figure 1; and if the second is not, then both AD lines are below the PC line in panels E and F of figure 1. From now on we assume that these conditions are always satisfied.

With active monetary policy the MP line in figure 1 becomes steeper and, as discovered by Benhabib, Schmitt-Grohe, and Uribe (2001), there are always two pairs of inflation and interest rates satisfying the monetary-policy rule and steady-state IS curve. These two solutions are illustrated in panel D of figure 1. The fact that two inflation rates satisfy both the monetary-policy rule and steady-state IS curve imply that the model admits two steady states—as long as the output level given by the steady-state Phillips curve and these inflation rates is positive. It turns out that this is not always the case.

When prices are relatively flexible (low γ), the stead-state Phillips curve is steep—in fact it becomes vertical when prices become completely flexible. Under assumption 2 the top branch of the AD curve crosses the Phillips curve once. This is a regular steady state. The bottom branch crosses the Phillips curve if prices are flexible enough: $\gamma < (\epsilon - 1)/\delta^2$. The resulting steady state is a zero lower bound. The situation with flexible prices and two steady states is depicted in panel E of figure 1. This is the situation on which Benhabib, Schmitt-Grohe, and Uribe (2001) focus. The regular steady state is source so it is locally determinate: as long as the equilibrium is constrained to remain in the vicinity of the steady state, the equilibrium is unique; the equilibrium path is to jump to the steady state and remain there. Determinacy is easy to establish following the same method as above: since $\phi > 1$, $\theta > 0$, and the determinant of the matrix $\mathbf{A}$ is $D = \theta\epsilon/(\gamma a) > 0$. Since both determinant and trace are positive, the steady state is a source, ensuring determinacy.

The zero-lower-bound steady state, on the other hand, is a saddle, exactly like in the case of passive monetary policy. This implies that the equilibrium is globally indeterminate. Furthermore, Benhabib, Schmitt-Grohe, and Uribe (2001) show that there are infinitely many trajectories starting in the neighborhood of the regular steady state and converging to this zero-lower-bound steady state. Because so many things are possible in these conditions, it is complicated to use the model for its intended purposes: develop intuition or make predictions.

As long as the model retains two steady states, it will be plagued by indeterminacy. The first step then is to remove one of the steady states by assuming that prices are rigid enough:

ASSUMPTION 3. *The price-adjustment cost is high enough:*

$$\gamma > \frac{\epsilon - 1}{\delta^2}. \tag{24}$$

The graphical interpretation of the assumption is that the Phillips curve is flat enough that it never crosses the zero-lower-bound AD curve, $\overline{\pi} = -\delta$. This scenario with high price rigidity is illustrated in panel F of figure 1. In that case the model has a unique steady state, and the steady state is source, so the equilibrium is unique: the equilibrium path is to jump to the steady state and remain there. In section 4 we will review a large amount of evidence on price rigidity and the time discount rate. We will find that except for extreme estimates of price rigidity and discount rate, (24) is satisfied in the data. Thus it seems reasonable to eliminate the zero-lower-bound steady state this way. Furthermore, the remaining steady state is the steady state on which most of the literature focuses.

Of course the issue with this parametric configuration is that the steady state can never be a zero lower bound. These properties make it difficult to study the zero lower bound: researchers cannot use typical tools such as comparative statics (for unanticipated permanent shocks) or phase-diagram analysis (for anticipated or temporary shocks). Instead, researchers are forced to construct scenarios where shocks follow sophisticated patterns over time, bringing the economy to the zero lower bound for only a few periods, and where policy changes are tightly synchronized to the shocks.[6]

Moreover, because there is no well-behaved zero-lower-bound steady state, the properties of the dynamical system generated by the AD and Phillips curves are such that the results obtained during zero-lower-bound episodes are very sensitive to tiny changes in the temporal path of shocks and policies. For instance, Cochrane (2017) shows that at the zero lower bound different policies that generate the same equilibrium path for nominal interest rates have tremendously

[6]For examples of such scenarios, see Eggertsson and Woodford (2003, pp. 167–172), Eggertsson and Woodford (2004, p. 78), Eggertsson and Woodford (2006, pp. 94–98), Christiano, Eichenbaum, and Rebelo (2011, p. 90), Eggertsson (2010, pp. 7–8), Woodford (2011, pp. 16–20), Eggertsson (2011, pp. 69–71).

different effects on output and inflation. Wieland (2018) shows that the persistence of fiscal policy at the zero lower bound has dramatic effects on the government-spending multiplier: increasing the duration of spending first raises the multiplier, possibly very sharply; then, when duration reaches a certain threshold, multipliers become small and even negative.

We have introduced wealth in the utility function to remedy these issues. However, an infinitely small amount of marginal utility of wealth will not do. The AD line needs to cross the PC line from below in panel F of figure 1, which requires sufficient marginal utility to tilt the AD line enough. In fact, with low marginal utility of wealth, the model retains the properties of the textbook model. We therefore make the following assumption:

ASSUMPTION 4. *The marginal utility of wealth is high enough:*

$$u'(0) > \frac{\epsilon}{\delta\gamma a}. \tag{25}$$

Without utility of wealth the steady-state IS curve imposes that the real interest rate equals the time discount factor. This leads to a degenerate steady-state AD curve: the AD curve it does not involve output at all—-it only pins down two inflation rates (one at the zero lower bound and one away from it), as showed by figure 1. With wealth in the utility function, the steady-state IS curve becomes

$$u'(0)\overline{y} = \delta - \overline{r}. \tag{26}$$

In steady state output is a decreasing function of the real interest rate—as in the old IS-LM model.

With this new steady-state IS curve, the steady-state AD curve is a dogleg, as illustrated by figure 2. When $\overline{\pi} > -i^*/\phi$, the economy is away from the zero lower bound and the AD curve is downward-sloping:

$$\overline{\pi} = \frac{\delta - i^*}{\phi - 1} - \frac{u'(0)y}{\phi - 1}. \tag{27}$$

Then, for $\overline{\pi} < -i^*/\phi$, the economy is at the zero lower bound and the AD curve is backward-bending:

$$\overline{\pi} = u'(0)\overline{y} - \delta. \tag{28}$$

In our New Keynesian model with wealth in the utility function, the equilibrium has the following properties:

PROPOSITION 1. *The dynamical system generated by the Phillips curve* (14) *and the AD curve* (15)–(16) *admits a unique steady state. Steady-state output and inflation are given by*

$$\overline{y} = \frac{\alpha}{\alpha + \beta} \cdot y^d + \frac{\beta}{\alpha + \beta} \cdot y^n \tag{29}$$

$$\overline{\pi} = \frac{1}{\alpha + \beta} \cdot \left(y^d - y^n\right) \tag{30}$$

where y^n is given by (19)*; $\alpha \equiv \delta\gamma a/\epsilon$; $y^d \equiv (\delta - i^*)/u'(0)$ away from the zero lower bound, and $y^d \equiv \delta/u'(0)$ at the zero lower bound; $\beta \equiv (\phi - 1)/u'(0)$ away from the zero lower bound, and $\beta \equiv -1/u'(0)$ at the zero lower bound. The steady state is a zero lower bound if and only if*

$$u'(0) > \frac{\phi\delta - i^*}{\phi y^n - \alpha i^*}. \tag{31}$$

At the zero lower bound or away from it, the steady state is a source. Hence, the equilibrium is always unique: the equilibrium path is to jump to the steady state and remain there.

The proof of the proposition is simple, and can be sketched using figure 2. First, assumption 2 ensures that the top intercept of the AD line with the y-axis is above the intercept of the PC line. Second, assumption 3 ensures that the bottom intercept of the AD line with the y-axis is below the intercept of the PC line. Since the AD line is a dogleg, it necessarily cuts the PC line exactly once: the steady state is unique. The expressions for steady-state output and inflation are obtained from (18) and (28)–(27) and simple algebra. The kink in the AD line occurs at inflation $\overline{\pi} = -i^*/\phi$, and (31) is obtained by checking that at $\overline{\pi} = -i^*/\phi$ the PC line implies higher output than the AD line.

Next the source property can be seen by studying using the different equations (14) and (15)–(16) to plot the directions of the trajectories. More formally, around the steady state the nonlinear dynamical system generated by the Phillips curve (14) and the AD curve (15)–(16) is linearized as (21), where the matrix $\mathbf{A}$ becomes

$$\mathbf{A} \equiv \begin{bmatrix} u'(0)\overline{y} & \theta \\ -\epsilon/(\gamma a) & \delta \end{bmatrix}.$$

The trace of the matrix remains positive: $T = u'(0)\overline{y} + \delta > 0$. But now the determinant is also positive because assumption 4 holds: $D = \delta u'(0)\overline{y} + \epsilon\theta/(\gamma a) > \delta\overline{y} \cdot [u'(0) - \epsilon/(\delta\gamma a)] > 0$. We infer that the two eigenvalues of the matrix $\mathbf{A}$ have a positive real part, such that the dynamical system is a source. If the eigenvalues are real and positive, the system is a nodal source; if they are complex conjugates with a positive real part, the system is a spiral source. Both cases are possible, but either way the equilibrium is determinate.[7]

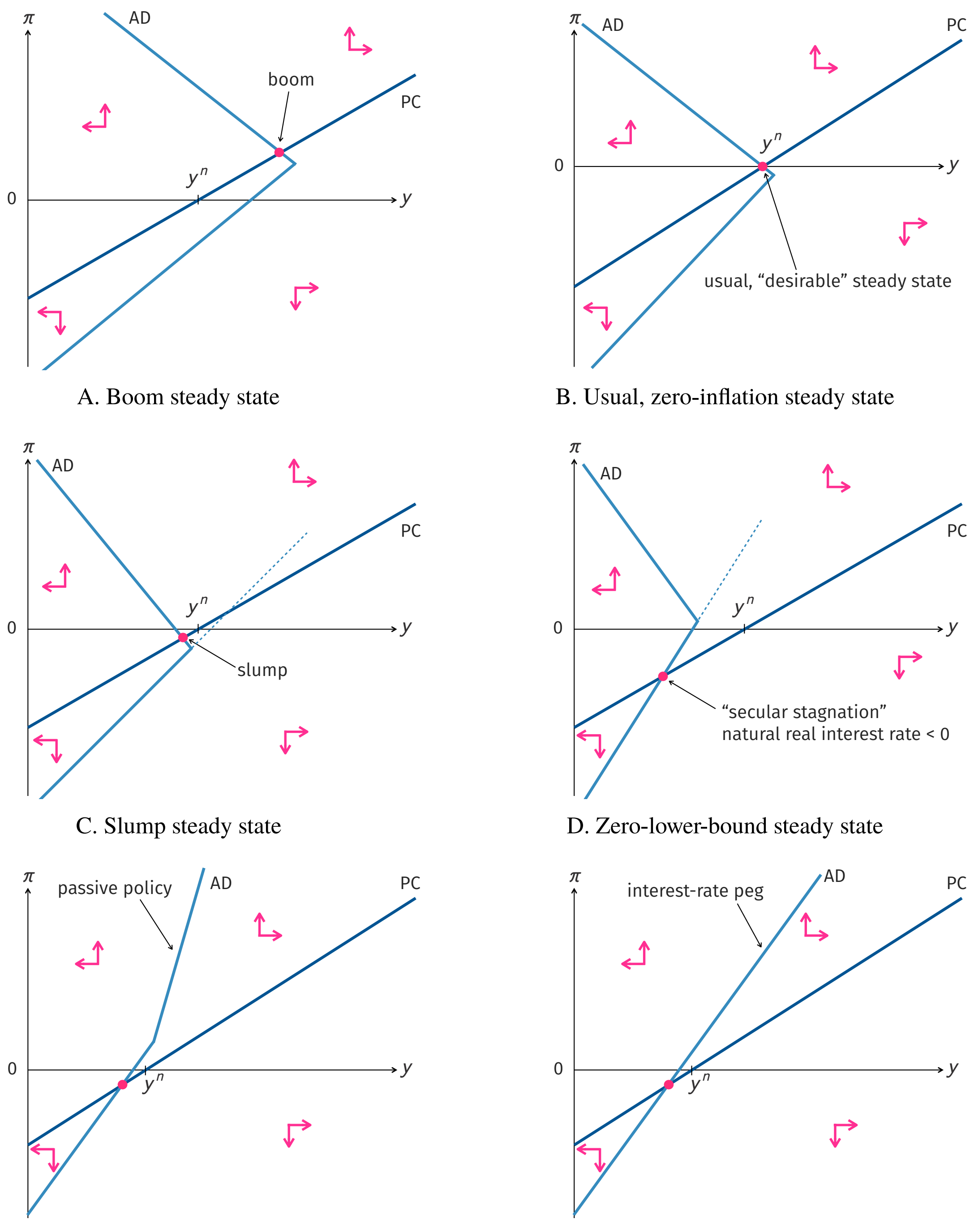
The proposition occasions several remarks. First, steady-state output is a weighted average of y^n and y^d. The quantity y^d is stead-state output when prices are fixed ($\gamma \to \infty$ and thus $\alpha \to \infty$). When prices are fixed, producers of goods and services produce supply whatever is required to meet demand. Hence y^d is determined by the demand side of the economy: monetary policy (i^*) and marginal utility of wealth ($u'(0)$). On the other hand the quantity y^n is stead-state output when prices are flexible ($\gamma = 0$ and thus $\alpha = 0$). When prices are flexible, producers of goods and services determine prices and output to maximize profits. Hence y^n is determined by the supply side of the economy: competitiveness of the economy (ϵ) and technology (a). Expression (29) shows how supply-side and demand-side factors jointly determine output when prices are somewhat rigid.

Second, steady-state inflation is determined by the gap between demand-side output y^d and natural output y^n. As $\alpha + \beta > 0$ under assumption 4, inflation is positive if demand-side output is greater than natural output and negative if demand-side output is lower than natural output. At the limit where price adjustment is infinitely costly, steady-state inflation is zero ($\gamma \to \infty$ and thus $\alpha \to \infty$).

Third, steady-state equilibria describe all possible stages of the business cycle, as illustrated in figure 2. Panel A describes a boom: the economy is away from the zero lower bound, output is above its natural level, and inflation is positive. In that situation, monetary policy can bring the economy back to a steady state with zero inflation, which is often seen as desirable. (With production subsidies undoing the monopolistic distortions, this steady state is indeed efficient.) This is achieved by raising i^*, which shifts the top branch of the AD curve inward, as showed by (27). The resulting zero-inflation steady state is showed in panel B.

Next, panel C of figure 2 describes a regular slump: while the economy is away from the zero lower bound, output is below its natural level, and inflation is negative. In that situation, monetary policy could bring the economy back to the zero-inflation steady state by lowering i^* and thus shifting the top branch of the AD curve outward. This type of slump is not very problematic because monetary policy can undo it and fully stabilize the economy. This is possible as long as aggregate demand is sufficient: $\delta/u'(0) > y^n$.

[7]To obtain determinacy we also need to assume that the solution $[y(t), \pi(t)]$ to the dynamical system remains bounded. Then, since neither variable is predetermined at time 0, the equilibrium is determinate when the dynamical system is a source. In that case, the equilibrium jumps to its steady state from any initial position. If the system were a sink or a saddle instead, the equilibrium would not be determinate: output and inflation could jump to many possible positions at time 0, while respecting all equilibrium conditions at time 0 and after that. The assumption that output and inflation remain bounded is typical in the New Keynesian literature, but there is not always a good justification for it (Cochrane 2011). In our model explosive solutions sometimes violate the transversality conditions of the households' maximization problem, in which case they can be ruled out.



A. Boom steady state

B. Usual, zero-inflation steady state

C. Slump steady state

D. Zero-lower-bound steady state

E. Passive monetary policy

F. Interest-rate peg

Figure 2. Equilibrium Dynamics with Wealth in the Utility Function

Notes: The AD line represents the steady-state AD curve, given by (28) at the zero lower bound and (27) away from the zero lower bound. The PC line represents the steady-state Phillips curve, given by (18).

Things are different when aggregate demand is too low: $\delta/u'(0) < y^n$. Then, as illustrated in panel D of figure 2, no monetary policy can bring the economy from a slump back to the desirable steady state. The economy is stuck in a zero-lower-bound slump. This steady state corresponds the secular stagnation of Eggertsson and Mehrotra (2014): the natural interest rate $r^n = \delta - u'(0)y^n$ is negative. Critically, unlike in the textbook New Keynesian model, equilibria at the zero lower bound are determinate: if the central bank sets the nominal interest rate to zero, the equilibrium simply jumps to its steady-state position, exactly as it would away from the zero lower bound. This result implies that absent shocks or policy interventions, the zero lower bound would last forever: no market forces would pull the economy away from it. Among New Keynesian models, ours shares the property of having a well-behaved zero-lower-bound steady state with those developed by Eggertsson and Mehrotra (2014), Gabaix (2016), Diba and Loisel (2017), and Cochrane (2018).

The proposition shows that once wealth enters the utility function the equilibrium of the New Keynesian model can be determinate at the zero lower bound. This result implies that the Taylor principle is invalid once the marginal utility of wealth is sufficient (once (25) holds). To obtain determinacy at the zero lower bound the stead-state AD curve (describing the $\dot{y} = 0$ locus) and the stead-state Phillips curve (describing the $\dot{\pi} = 0$ locus) must cross each other in the right order: the AD curve must cross the Phillips curve from below, as in figure 2, panel D. Adding wealth in the utility function makes the AD curve upward sloping at the zero lower bound, instead of being flat (compare panel D in figure 2 and panel E in figure 1). Once the marginal utility of wealth is sufficiently large, the AD curve becomes steeper than the Phillips curve, and the dynamical system becomes a source around the zero-lower-bound steady state. The argument clarifies why it is not necessary to modify both AD and Phillips curves to obtain a well-behaved zero-lower-bound steady state. The only thing that matters is the slope of the AD curve relative to the Phillips curve matters; so it is sufficient to tilt the AD curve until it cross the Phillips curve from below.

Of course making the Phillips curve flatter helps, but without some tilt of the AD curve it is impossible to construct a well-behaved zero-lower-bound steady state (since the steady-state Phillips curve cannot be downward sloping). Indeed the marginal utility of wealth $u'(0)$ required to satisfy assumption 4 decreases with price rigidity. When prices are completely rigid (which occurs if the price-adjustment cost γ is infinite), an infinitesimal marginal utility of wealth is sufficient to obtain a well-behaved zero-lower-bound steady state. Formally, the condition (25) becomes $u'(0) > 0$. Conversely, when prices are perfectly flexible (which occurs if $\gamma = 0$), no marginal utility of wealth is sufficient to obtain a well-behaved zero-lower-bound steady state: condition (25) becomes $u'(0) > \infty$. So our approach continues to work at the fixed-price limit, but not at the flexible-price limit. In that it differs from the approach taken by Cochrane (2018), which works at the flexible-price limit but not the fixed-price limit.

Additionally figure 2 displays two types of monetary policy not considered in the paper. Panel E illustrates a passive monetary policy ($\phi < 1$) away from the zero lower bound. Panel F illustrates an interest-rate peg ($\phi = 0$). The figure shows that the dynamical properties of the equilibrium are not modified by these policies: the steady state remains unique, and it remains a source. Hence the equilibrium remains unique even if monetary policy is passive or a peg. The uniqueness of the steady state is not a robust result, however: with a convex steady-state Phillips curve, a second steady state would appear when monetary policy is passive away from the zero lower bound.

3.2. Monetary Policy Shocks and Fisherian Effects

We now consider the effect of an unanticipated, permanent shock to monetary policy: a decrease in i^*. We know that such a shock has no effect at the zero lower bound (see figure 3, panel B); so we assume that the economy in not at the zero lower bound. The economy is initially in a steady-state position, and since the shock is unanticipated and permanent, the economy jumps to a new stead-state position. Hence, output and inflation are given by (29) and (30). Interest rates are given by monetary policy: the nominal interest rate by $i = i^* + \phi\pi$ and the real interest rate by $r = i^* + (\phi - 1)\pi$.

PROPOSITION 2. *An unanticipated, permanent decrease in i^* leads to higher output, higher inflation, lower nominal interest rate, and lower real interest rate.*

The comparative statics described in the proposition are illustrated in figure 3, panel A. The decrease in i^* shifts the top branch of the AD curve outward: interest rates are lower, which makes consumption more desirable relative to holding wealth and stimulates aggregate demand. The economy moves along the Phillips curve, and both output and inflation increase: as households produce more, they do not need to reduce prices as much as before, or they need to increase prices at a faster rate than before, which either way leads to higher inflation. As i^* falls but π rises, the response of the interest rates are ambiguous. But we can show that under assumption 4, the effect of i^* always dominates, such that both nominal and real interest rates fall.

The effect of monetary policy described in the proposition is aligned with the empirical evidence (for example, Christiano, Eichenbaum, and Evans 1999, 2005): when monetary policy sets a lower nominal interest rate, output and inflation rise while the real interest rate falls.

The effects of monetary policy with wealth in the utility function are quite different than those in the textbook model. Indeed, without wealth in the utility, the equilibrium real interest rate equals the time discount rate δ. So, to begin with, monetary policy has no effect on the real interest rate. However, even without wealth in the utility, a decrease in i^* still shifts up the top branch of AD curve (which is horizontal in that case). Hence, a decrease in i^* leads to higher inflation and output. Since the real interest rate does not change but inflation rises, the nominal interest rate necessarily rises. This Fisherian effect does not happen in our model: a decrease in i^* lowers the real interest rate enough that the nominal interest also falls.

Because of the Fisherian effect, the textbook model implies that when monetary policy sets a higher nominal interest rate, output and inflation rise. This prediction is counterfactual, and goes against the common narrative about monetary policy. Hence with the textbook model it is not possible to use permanent shocks to model monetary policy: we must resort to temporary shocks. Even temporary shocks must be designed in a specific way to obtain realistic predictions: because persistent shocks yield the same predictions as permanent shocks, policy shocks must be modeled as very transient. As in the analysis of the zero lower bound, the analysis of monetary policy is complicated and possibly obscured because researchers have to impose complex temporal patterns to policy shocks. For example, the usual story behind the zero lower bound does not fit with the textbook model: the central bank raises output and inflation by raising the nominal interest rate—not lowering it—so in fact the central bank is never constrained by the zero lower bound when stimulates the economy. In the textbook model it is not the case that the central bank reduces the nominal interest rate to stimulate the economy, until the zero lower bound becomes binding. But this logic does operate in our model.

3.3. Aggregate Demand Shocks and the Paradox of Thrift

We study the effect of an unanticipated, permanent shock to aggregate demand: an increase in the marginal utility of wealth $u'(0)$. Once again, the economy is initially in a steady-state position and it jumps to a new stead-state position after the shock. Such an aggregate demand shock has the same effects at the zero lower bound and away from it:

PROPOSITION 3. *An unanticipated, permanent increase in the marginal utility of wealth leads to lower output and lower inflation.*

The comparative statics described in the proposition are illustrated in figure 3, panels C and D. The increase in $u'(0)$ rotates both branches of the AD curve inward: the marginal utility of wealth is higher, which makes consumption less desirable relative to holding wealth and depresses aggregate demand. At the zero lower bound or not, the economy moves along the Phillips curve, and both output and inflation decrease.

A decrease in the marginal utility of wealth describes a typical negative aggregate-demand shock. The shock operates exactly as in the Keynesian paradox of thrift: when the marginal utility of wealth goes up, households want to save more and accumulate more wealth. But aggregate financial wealth is fixed since government bonds are in fixed supply. Hence, in general equilibrium, the only way to hold more wealth relative to consumption is to reduce consumption, following exactly the logic of the paradox of thrift. If such negative aggregate-demand shock is strong enough, it may bring the economy to the zero lower bound (for instance, starting from panel C in figure 3 and moving to panel D).

3.4. Technology Shocks and the Paradox of Toil

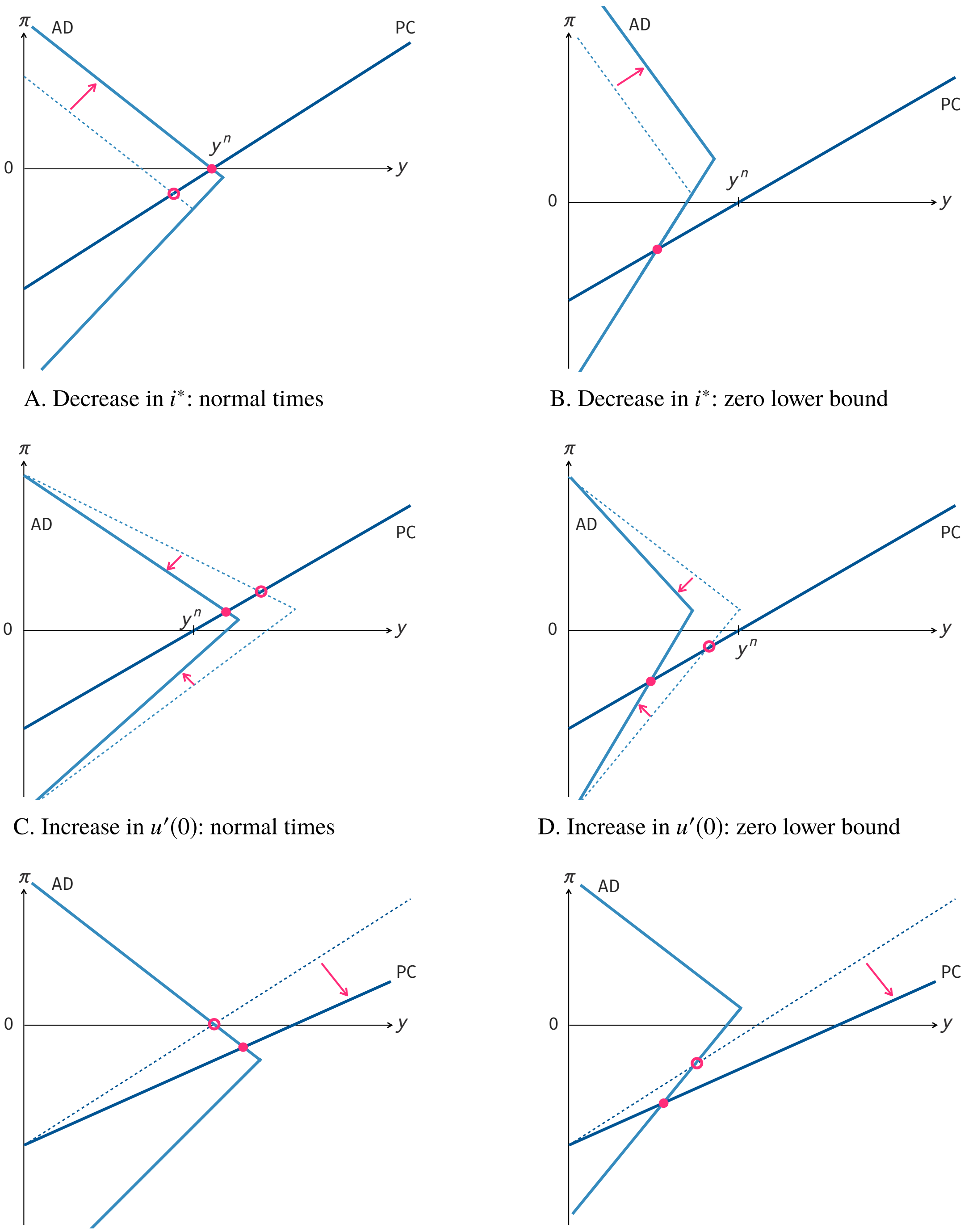
We study the effect of an unanticipated, permanent shock to technology: an increase in a. The economy is initially in a steady-state position and it jumps to a new stead-state position after the shock. Such a technology shock has different effects at the zero lower bound and away from it:

PROPOSITION 4. *An unanticipated, permanent increase in technology leads to lower hours worked and lower inflation. It leads to higher output away from the zero lower bound, but lower output at the zero lower bound.*

The comparative statics described in the proposition are illustrated in figure 3, panels E and F. The increase in technology rotates the Phillips curve downward: with higher technology level, real marginal costs are lower, which tends to reduce inflation. The natural level of output is accordingly higher. The economy therefore moves along the AD curve. Away from the zero lower bound, the AD curve is downward sloping so the technology shock leads to higher output and lower inflation (panel E). Since output increases less than proportionally to technology, hours worked actually fall. At the zero lower bound, the AD curve is upward sloping (backward bending) so the technology shock leads to lower output, lower hours worked, and lower inflation (panel F).

The reduction in output after an increase in technology at the zero lower bound is the paradox of toil, first described by Eggertsson (2010). An increase in technology has an adverse effect on output at the zero lower bound because it reduces inflation and thus raises the real interest rate (as the nominal interest rate is stuck at zero). A higher real interest rate in turn depresses aggregate demand. This does not occur in normal times because then a reduction in inflation leads to lower nominal and real interest rates (recall that $r = i^* + (\phi - 1)\pi$ with $\phi - 1 > 0$).

It may seem strange that higher technology always reduces hours worked—especially because in flexible-price models (such as real-business-cycle models) higher technology leads to higher hours worked. But this property is in fact quite realistic: Basu, Fernald, and Kimball (2006) show



A. Decrease in i^*: normal times

B. Decrease in i^*: zero lower bound

C. Increase in $u'(0)$: normal times

D. Increase in $u'(0)$: zero lower bound

E. Increase in a: normal times

F. Increase in a: zero lower bound

Figure 3. Effects of Monetary-Policy, Technology, and Marginal-Utility-of-Wealth Shocks

Notes: The AD line represents the steady-state AD curve, given by (28) at the zero lower bound and (27) away from the zero lower bound. The PC line represents the steady-state Phillips curve (18).

using US data that when technology improves hours worked fall sharply and output changes little. It is only after several years that output rises significantly.

3.5. Forward Guidance

Away from the zero lower bound monetary policy perfectly controls output and inflation by setting the nominal interest rate, but of course at the zero lower bound monetary policy loses that power. A policy that can be used instead is forward guidance: monetary policy commits today to a policy that will stimulate the economy in the future. In the textbook New Keynesian model, however, the impact of forward guidance on output is unrealistically strong—this is the forward-guidance puzzle discovered by Del Negro, Giannoni, and Patterson (2012) and Carlstrom, Fuerst, and Paustian (2015). This puzzle seems to arise because of the dynamical properties of the equilibrium at the zero lower bound. With wealth in the utility function, the zero-lower-bound steady state is well behaved, and the puzzle disappears.

To begin with, we consider the following scenario: The economy is at the zero lower bound. At time 0, people learn that at time T in the future, a permanent, positive aggregate-demand shock will hit the economy. This could be a reduction in the marginal utility of wealth or an increase in the time discount rate. At the same time, the central bank announces that it will not counterbalance this shock, such that the economy will be permanently tighter from time T onward. For instance, the central bank could commit to keeping the nominal interest rate at zero despite higher output and inflation. The following proposition describes what happens under this scenario of permanent forward guidance.

PROPOSITION 5. *Let $y^T > y(0)$ and $\pi^T > \pi(0)$ be steady-state output and inflation after time T. Then under permanent forward guidance, output and inflation have the following dynamics:*

- *Output and inflation jump up at $t = 0$. The size of the jump is decreasing with T. The jump becomes vanishingly small as $T \to \infty$, while output and inflation jump all the way to y^T and π^T when $T \to 0$.*

- *For $t \in (0, T)$, output and inflation increase over time, but they remain below y^T and π^T.*

- *As $t \to T$, output and inflation converge to y^T and π^T. When $t = T$, output and inflation are exactly at y^T and π^T. The AD curve shifts out at the same time, such that the equilibrium is exactly in steady state.*

- *For $t > T$, output and inflation remain at y^T and π^T.*

There is no forward-guidance puzzle here. An announcement about a future economic boom does stimulate output and inflation, but output and inflation always remain below the levels they would reach during the boom. Moreover the instantaneous effect on inflation and output becomes vanishingly small as the boom is scheduled further in the future. The dynamics of output and inflation are illustrated in figure 4, panel A.

We obtain the same type of results if the future boom engineered by the central bank is temporary instead of permanent. Imagine again that the economy is at the zero lower bound. At time 0, people learn that between times T and $T + \Delta$ in the future, a positive aggregate-demand shock will hit the economy. At time 0 the central bank also announces that it will not counterbalance this shock, such that the economy will be permanently tighter between times T and $T + \Delta$. For instance, the central bank could commit to keeping the nominal interest rate at zero during the boom. The following proposition describes what happens under this scenario of temporary forward guidance.

PROPOSITION 6. *Let $\overline{y}$ and $\overline{\pi}$ be initial steady-state output and inflation. Let y^T and π^T be steady-state output and inflation under the parameters prevailing between times T and $T + \Delta$. Then output and inflation have the following dynamics:*

- *Output and inflation jump up from $\overline{y}$ and $\overline{\pi}$ at $t = 0$. The size of the jump is decreasing with T and increasing with Δ. The jump becomes vanishingly small when $T \to \infty$ and $\Delta \to 0$. Output and inflation jump all the way to $\overline{y}$ and $\overline{\pi}$ when $\Delta \to \infty$.*
- *For $t \in (0, T]$, output increases over time but remains below y^T. For $t \in (0, T)$, inflation first increases and then decreases over time; inflation always remains between $\overline{\pi}$ and π^T.*
- *At $t = T$, the IS curve shifts out. For $t \in (T, T + \Delta)$, output and inflation decrease over time, but they remain above $\overline{y}$ and $\overline{\pi}$.*
- *As $t \to T + \Delta$, output and inflation converge to $\overline{y}$ and $\overline{\pi}$. When $t = T + \Delta$, output and inflation are exactly at $\overline{y}$ and $\overline{\pi}$. The IS curve shifts back in at the same time, such that the equilibrium is exactly in steady state.*
- *For $t > T + \Delta$, output and inflation remain at $\overline{y}$ and $\overline{\pi}$.*

There is even less of a forward-guidance puzzle with a temporary boom engineered by the central bank. An announcement about a future, temporary boom does stimulate output and inflation, but output and inflation always remain below the levels they would have reached if the future boom had been permanent. Moreover, the instantaneous effect on inflation and output becomes vanishingly small as the boom is scheduled further in the future. Finally, the maximum

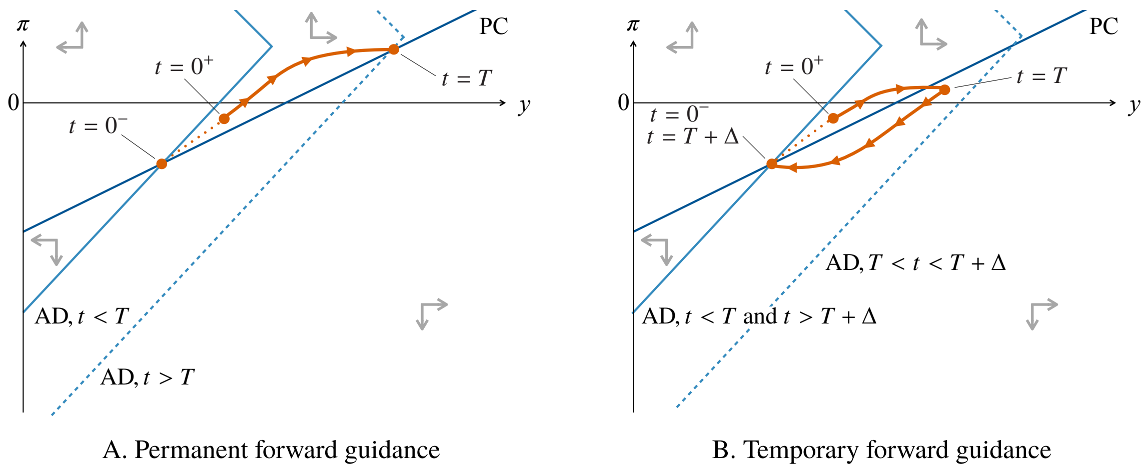


Figure 4. The Effect of Forward Guidance on Output and Inflation

effect on output (achieved at $t = T$) becomes vanishingly small as the duration of the future boom is shortened. The dynamics of output and inflation with an anticipated, temporary boom are illustrated in figure 4, panel B.

Of course our model is not the first to resolve the forward-guidance puzzle. The papers that we have discussed with a well-behaved zero-lower-bound steady state all solve the puzzle (see Eggertsson and Mehrotra 2014; Gabaix 2016; Diba and Loisel 2017; Cochrane 2018). This is because, once the zero-lower-bound steady state is a source, the effect of forward guidance is bound to vanish when the policy is scheduled for a later date in the future. The logic behind this claim is illustrated in figure 4. Other papers have attenuated the puzzle by modifying the Phillips curve (for example, Carlstrom, Fuerst, and Paustian 2015), modify both IS and Phillips curves (for example, Angeletos and Lian 2016; Del Negro, Giannoni, and Patterson 2012), modifying only the IS curve (for example, Bilbiie 2017; McKay, Nakamura, and Steinsson 2016), or taking different approaches (for example, Farhi and Werning 2017; Gertler 2017).

4. Empirical Support for the Main Assumptions

The analysis relies on key assumptions: assumption 3 imposes that prices are rigid enough relative to the time discount rate, and assumption 4 that the marginal utility of wealth is large enough relative to price rigidity. Here we survey micro and macro evidence on the time discount rate, price rigidity, and marginal utility of wealth, and conclude that these assumptions are quite reasonable.

4.1. Time Discount Rate and Marginal Utility of Wealth

A vast literature has attempted to estimate time discount rates. Frederick, Loewenstein, and O'Donoghue (2002, sec. 6) provide a comprehensive survey of the estimates obtained using a wide variety of methods: real-world behavior and elicitation using field or laboratory experiments. Table 1 in Frederick, Loewenstein, and O'Donoghue (2002) reports all the studies surveyed and the range of estimates obtained in each one. Different studies may obtain a range of estimates under different specifications for the estimation, different assumptions about observed behavior, or different experimental setups. There is a lot of variation in the estimates obtained, but what matters for us is that the vast majority of the estimates points to high time discounting, much higher than prevailing market interest rates. To obtain an average estimate for the discount rate, we first take the average estimate in each of the 43 studies covered. We then take the median of these average estimates: this midpoint estimate is $\delta = 35\%$.

The studies discussed in the survey usually assume that people use a single rate to exponentially discount future utility. This is also the assumption that made in all the New Keynesian literature. This discounted-utility model is subject to many anomalies, such as hyperbolic discounting. Recent papers allow for discounting much more general than exponential discounting. They also attempt to separate between time discounting and risk aversion, which were sometimes mingled in previous studies. Andersen et al. (2014) survey recent studies, based on laboratory experiments with real (not hypothetical) incentives, that take all these considerations into account. In table 3, they report the range of estimates obtained in 16 studies. Taking the average estimate in each study, and then the median of these averages, we obtain a midpoint estimate of $\delta = 43\%$.

Overall, a time discount rate of $\delta = 40\%$ seems like a reasonable midpoint estimate from the literature. Combining this estimate we evidence on real interest rates, we can get evidence on the marginal utility of wealth. Indeed, the steady-state IS curve (26) implies that when the economy is in a natural state with zero inflation, which it should be on average if the central bank is effective, then

$$\delta - r^n = u'(0)y^n, \tag{32}$$

where r^n is the natural rate of interest and y^n the natural level of output. Del Negro et al. (2017, fig. 1) and Holston, Laubach, and Williams (2017, fig. 1) estimate the average US natural interest rate around $r^n = 2\%$. Hence, the average marginal utility of wealth is such that $u'(0)y^n = 38\%$. The interpretation is that the average marginal rate of substitution between real wealth and consumption is about 38%.

4.2. Price Rigidity

There is also a vast literature estimating price rigidity using micro and macro data.

Price rigidity is estimated using macro data by estimating the slope of the New Keynesian Phillips Curve. The Phillips curve (14) can be rewritten

$$\dot{\pi} = \delta\pi - \kappa \cdot \frac{y - y^n}{y^n},$$

where the natural level of output y^n is given by (19) and

$$\kappa \equiv \frac{\epsilon - 1}{\gamma}. \tag{33}$$

The literature estimates a discrete-time version of this Phillips curve:

$$\pi(t) = (1 - \delta)\pi(t + 1) + \kappa \cdot \frac{y(t) - y^n}{y^n}, \tag{34}$$

Estimates of the coefficient κ in front of the output gap—the relative deviation of output from the natural level of output—indicate the rigidity of prices, and are directly relevant to assess our assumptions.

Mavroeidis, Plagborg-Moller, and Stock (2014) reviews the literature estimating the New Keynesian Phillips curve, and propose their own estimates. For basic specification of the New Keynesian Phillips curve, similar to (34), estimates vary between $\kappa = 0.5\%$ and $\kappa = 5\%$ (an outlier is $\kappa = 8\%$), with a median estimate across 16 studies of $\kappa = 3\%$ (fig. 3). Their own estimate is slightly lower: $\kappa = 1.8\%$ (table 3). We use $\kappa = 3\%$ as a midpoint estimate.

We can also measure $\kappa = (\epsilon - 1)/\gamma$ differently: by obtaining microestimates of ϵ and γ. Using firm-level data, De Loecker and Eeckhout (2017) measure the goods-market markup in the United States. They find that the average markup $\epsilon/(\epsilon - 1)$ hovers between 1.2 and 1.3 in the 1950–1980 period before continuously rising to 1.7 in the 1980–2014 period (fig. 1). Since 1990 the average markup is around 1.5, implying an average elasticity $\epsilon = 3$, which we use as our midrange estimate. In addition, following Michaillat (2014, p. 206), we calibrate the price-adjustment cost to $\gamma = 61$. This estimate is obtained from microevidence collected by Zbaracki et al. (2004): using time-and-motion methods, they study the pricing process of a large industrial firm and measure the physical, managerial, and customer costs of changing prices. Combining these estimates for ϵ and γ yields $\kappa = (3 - 1)/61 = 3.3\%$. This number is very close to the median estimate in the Phillips-curve literature.

4.3. Validating the Assumptions

Assumption 3 imposes that prices are rigid enough relative to the time discount rate: $\delta^2 > \kappa$, where $\kappa = (\epsilon - 1)/\gamma$ is the coefficient on output gap in the Phillips curve. We have found a midrange estimate of $\delta = 0.4$, which implies $\delta^2 = 0.16$. This is much higher than the midrange estimate of $\kappa = 0.03$. So for midrange estimates, assumption 3 is easily verified. Thus, for midrange estimates of δ and κ, the New Keynesian model's ill-behaved, zero-lower-bound steady state (depicted in figure 1, panel E) does not exist.

Of course there is significant uncertainty about the estimates of κ and δ. But assumption 3 seem to hold in most situations. It holds for values of κ at the high end of available estimates, around $\kappa = 0.05$, which is below 0.16. The rich robustness analysis conducted by Mavroeidis, Plagborg-Moller, and Stock (2014) shows that it is not impossible to obtain estimates of κ above 0.16, but it is quite unlikely. We turn to alternative estimates of δ: In Frederick, Loewenstein, and O'Donoghue (2002), the bottom third of estimates is below $\delta = 0.19$. In Andersen et al. (2014), it is the bottom 10% of estimates that is below $\delta = 0.19$. This lower estimate of δ implies $\delta^2 = 0.036$, which remains above the median estimate of κ.

Assumption 4 imposes that the marginal utility of wealth is high enough relative to price rigidity: $u'(0) > \kappa/(\delta y^n)$, which we rewrite using (32) as $(\delta - r^n)\delta > \kappa$. The estimates of δ and κ that we have just discussed, together with $r^n = 2\%$, imply that assumption 4 also holds for most estimates. With $\delta = 0.4$, we have $(\delta - r^n)\delta = 0.15$, above most possible estimates of κ. With the lower estimate $\delta = 0.19$, we have $(\delta - r^n)\delta = 0.032$, above the median estimates of κ.

To conclude, both assumption 3 and assumption 4 hold for most estimate of the time discount rate and price rigidity. The assumptions would be violated only with κ at the high end of available estimates and simultaneously δ at the low end of available estimates.

5. Conclusion

This paper extends the textbook New Keynesian model by introducing wealth, in the form of government bonds, in households' utility function. For a realistic amount of price rigidity, the equilibrium admits a unique steady state, and this steady state is a source, implying that the equilibrium is unique. Furthermore, when the marginal utility of wealth is high enough, the equilibrium is at the zero lower bound.

These properties improve the description of the zero lower bound and facilitate its analysis: The zero lower bound is not necessarily short-lived; it can last forever. The effects of shocks and policies at the zero lower bound can be studied with comparative statics (for permanent shocks) or with a phase diagram (for temporary or anticipated) shocks. Explosive behavior at the zero lower bound disappears: inflation and output do not collapse if the zero lower bound last forever; there is no forward-guidance puzzle; the government-spending multiplier is never particularly large.

Furthermore, with wealth in the utility function, monetary policy has more realistic effects on interest rates. In the textbook model, an expansionary shift in monetary policy leads to higher output and inflation but also higher nominal interest rates. This Fisherian effect arises because the real interest rate does not respond to monetary policy. It runs contrary to the intuition that central banks stimulate the economy by lowering nominal interest rates. It is also inconsistent with the usual story behind the zero lower bound: that the central bank lowers nominal interest rates to stimulate the economy but at some point becomes constrained by the zero lower bound. With wealth in the utility function the equilibrium real interest rate is negatively related to equilibrium output. Hence an expansionary shift in monetary policy leads to higher output and inflation but also lower nominal and real interest rates, in line with common intuition about the short-run effects of monetary policy.

Finally, introducing wealth in the utility function allows to reconcile single-digit market interest rates with the double-digit time discount rates measured in laboratory, field, and natural experiments. Although the return on wealth is much lower than their time discount rate, people are willing to hold wealth because they derive direct utility from holding wealth.

Unlike other approaches generating a well-behaved zero-lower-bound steady state in the New Keynesian model, ours does not require bounded rationality, overlapping generations, or heterogeneous agents: we remain within the typical rational, infinite-horizon, representative-agent paradigm. Of course real people have bounded rationality, a finite lifetime, and heterogeneous characteristics. They also care about wealth in and of itself. We think that introducing wealth in the utility function could be a useful methodological advance because it remedies the pathologies of the New Keynesian model at the zero lower bound while altering the textbook model only in the most limited way.

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