Matching Models Under Scrutiny: Understanding the Shimer Puzzle

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Abstract

Two papers have recently questioned the quantitative consistency of the search and matching model. Shimer (2005) has argued that a text-book matching model is unable to explain the cyclical variation of unemployment and vacancies in the U.S. economy. Costain and Reiter (2007) have found the existence of a trade-off in the model's performance: any attempt to change the calibrated values in order to amend such business cycle inability would jeopardize the model's predictions about the impact of unemployment benefits on unemployment. In surveying the literature originated in these findings, I distinguish three different avenues that have been followed to correct the model: change in wage formation, change in the calibration, changes in the model specification. The last approach seems to reach the best results both from a business cycle and from a microeconomic viewpoint.

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1 Introduction

In a influential paper, Shimer (2005) evaluates the business cycle performance of the search and matching model, nowadays the standard workhorse adopted by macro and labour economists to study aggregate labour markets[1]. The most important claims of his paper can be summarized as follows:

1. A textbook search and matching model is not able to explain the observed fluctuations of unemployment and vacancies in the U.S. economy in response to productivity shocks of plausible magnitude. The job-finding rate results 12 times more volatile in the data than in the model, whereas the standard deviation of the vacancies in the data is 10 times larger than in the model.

2. Such discrepancy stems from the wage formation assumptions. In a standard matching model, a Nash bargaining solution is introduced in order to share the total surplus between the firm and the worker. When a positive productivity shock hits the economy, wages instantaneously increase, dampening vacancy creation. Thus, vacancies (and, as a consequence, the job finding rate) are much more variable in the data than in the model.

3. The model also exhibits no propagation as it implies a contemporaneous correlation between the vacancy-unemployment ratio and productivity of 1, while in the data it is about 0.4.

The aim of this paper is to present the literature sprung from Shimer's findings. As far as the first two claims are concerned, I distinguish three different avenues that have been taken up by scholars.

In the first group, I list papers that agree with Shimer's first and second claims: the model fails to replicate the U.S. business cycle facts because wages are too responsive to changes in productivity. Hence, the most straightforward way to reconcile model and data is to modify the wage formation rules, for instance introducing some form of wage stickiness or imposing imperfect information in the firm-worker negotiation.

Although the sticky wage hypothesis has gained the initial attention of many economists, two caveats cast some doubts that it is the right answer

to the lack of amplification exhibited by the model. First, wage stickiness (or even a completely rigid wage) is not sufficient to fit the model with the data; a low calibrated profit share is also needed, so that the percentage increase in profits is large for a given percentage increase in productivity, and vacancy creation is boosted.[2] Second, as Pissarides (2007) and Haefke, Sonntag, and van Rens (2007) have recently pointed out, introducing sticky wages in matching models is difficult to justify on empirical ground. What is key in amplifying fluctuations is the cyclical behaviour of the wages of the newly hired workers. To augment the unemployment and vacancy volatility these wages must be nearly acyclical, while data show a near-proportional relation between them and labour productivity.

The second avenue, pursued by Hagedorn and Manovskii (2007a; b). They do not agree on Shimer's first two conclusions. According to them, the model is unable to match the data because of an erroneous parametrization of two key variables: the instantaneous utility of being unemployed and workers' bargaining power. With a higher calibrated value for the utility of unemployment and a lower bargaining power for workers, a standard matching model succeeds in replicating the observed business cycle fluctuations.

Even this second approach has been called in question on different grounds. Mortensen and Nagypál (2007) claim that Hagedorn and Manovskii's calibration postulates a too small and unrealistic difference between the instantaneous utility in employment and unemployment.

More importantly, Costain and Reiter (2007) verify the quantitative consistency of the model not only in response to productivity and separation rate shocks, but also to changes in the level of unemployment benefits (UBs). The conclusion is that any attempt to calibrate a standard matching model in order to match the business cycle unemployment and vacancies data produces unrealistic results about the effects of an increase in UBs on the unemployment rate, and *vice versa*. As a possible answer to the amplification puzzle, Costain and Reiter propose the introduction of embodied technological progress.

This naturally leads to a third way Shimer's findings have been confronted. What this vein of the literature (more or less) implicitly argues is that a standard matching framework contains some simplified assumptions

that inevitably jeopardize the quantitative consistency of the model. Enriching the basic set-up - considering for instance turnover costs, on-the-job search, or market power - would bridge the gap between data and theory.

This last approach seems the most successful in ameliorating the quantitative performance of the model, both at business cycle and at a policy analysis level. In particular, assuming cohort-specific productivity shocks as in Costain and Reiter (2007) and Reiter (2008) or introducing turnover costs as in Silva and Toledo (2007) improves the business cycle performance of the model and fits the microeconomic estimates, while keeping it analytically tractable.

A final remark concerns the lack of propagation exhibited by the model. Compared to the amplification puzzle explained above, this problem has gained much less attention among the scholars. Fujita (2003), Fujita and Ramey (2007b), and Hagedorn and Manovskii (2007b) have focused on this issue, caused by the excessive responsiveness of vacancies to the shocks. They show that the introduction of planning lags or increasing marginal costs in vacancy creation ameliorates the dynamics the model.

Related Surveys

The present paper is not the first survey that aims to present Shimer's claims and the subsequent literature they spurred.

In a detailed review, Hornstein, Krusell, and Violante (2005) clearly spell out the importance of a low profit share to better the performance of the model and explain the crucial differences between Shimer and Hagedorn and Manovskii's calibration. Since the publication of their article, however, many other papers have tried to answer to Shimer's points, focusing in particular on the aforementioned third approach. The present survey tries to account for this more recent strand of research.

Costain and Reiter (2007), Mortensen and Nagypál (2007), and Pissarides (2007), while taking part to the contest by proposing alternative ways to fit the model with the data, also provide helpful summaries of the state-of-the-art literature. Of course, the main objective of these papers is not to be a survey and some works are not presented there.

Finally, Yashiv (2007)'s survey covers a much wider subject, namely the recent advances in macroeconomic models with search frictions in the labour market. So, the so-called Shimer critique occupies only a fraction of the topics covered in his work[3].

Structure of the Paper

The paper is organized as follows. Section 2 describes the empirical and theoretical framework. Section 3 presents the quantitative inconsistency of the model. Sections 4, 5, and 6 respectively survey the three different approaches followed to react Shimer's claims. Section 7 deals with the propagation problem. Section 8 concludes.

2 Empirical and Theoretical Framework

2.1 U.S. Labour Market Facts

Table 1, taken from Shimer (2005), summarizes the statistics on the economic variables of interest. In bold are the numbers on which scholars have concentrated more. I first present Shimer's data collection method and then, in section 2.2, I briefly discuss the differences with the previous literature. The variables are the following: u unemployment, v vacancies, f the job-finding rate, s the separation rate, and p labour productivity. Data are quarterly and refers to the period from 1951 to 2003.[4]

What emerges from a first inspection of the data is the relatively high volatility of the level of unemployment and vacancies. The standard deviation of unemployment σ_u is equal to 0.19, meaning that this variable can be as much as 38 percent above or below trend. Since unemployment is counter-cyclical and vacancies are pro-cyclical, labour market tightness, defined as $\theta \equiv v/u$, is extremely pro-cyclical. Moreover, both unemployment and vacancies are very persistent variables, with an autocorrelation around 0.94.

Shimer assumes a constant returns to scale matching technology, $m(v, u)$, increasing and concave in both arguments, representing the measure of new jobs created as a function of the level of unemployment u and vacancies v. Because of the CRS assumption, the job-finding rate $f \equiv m(v, u)/u =$

		u	v	v/u	f	s	p
Standard deviation		**0.190**	**0.202**	**0.382**	**0.118**	**0.075**	**0.020**
Quarterly autocorrelation		0.936	**0.940**	0.941	0.908	0.733	0.878
Correlation matrix							
	u	1	**-0.894**	-0.971	-0.949	0.709	**-0.408**
	v	-	1	0.975	0.897	-0.684	**0.364**
	v /u	-	-	1	0.948	-0.715	**0.396**
	f	-	-	-	1	-0.574	0.396
	s	-	-	-	-	1	-0.524
	p	-	-	-	-	-	1

Table 1. Shimer's summary statistics. Quarterly U.S Data 1951-2003. All variables are reported in logs as deviations from an HP trend with smoothing parameter 10^5. Source: Shimer (2005).

$m(\theta, 1)$ is an increasing and concave function of θ only. Shimer gets an average monthly hazard rate around 0.45 and a standard deviation of 0.118^5. As Shimer points out, the high positive correlation between f and θ is a strong argument in favour of a constant returns to scale matching technology.

Once f_t and θ_t have been computed, Shimer looks at the matching function. With a Cobb-Douglas functional form, $m(v, u) = \mu u^\alpha v^{1-\alpha}$, two parameters need to be estimated, α and μ. Using his data on f_t and θ_t, he gets a value of α between 0.70 and 0.75, beyond the plausible range of 0.3 to 0.5 reported by Petrongolo and Pissarides (2001).

The separation rate is computed using another labour market flows equation and results equal to 0.034. Such variable results less volatile than the job finding rate and it presents a negative correlation with labour market tightness.

Labour productivity is computed as the ratio between real output and the number of workers in the non-farm business sector. Data show that labour productivity is positive correlated with tightness and, more crucially, its standard deviation is ten times lower than that of vacancies and almost twenty times lower than standard deviation of labour market tightness[6].

Not Shimer (2005), but some other papers I will survey later on, concentrate also on the volatility of the real wage with respect to productivity. In

this respect, a clear distinction has to be made. Scholars that look at time series statistics on the aggregate wage in the economy estimate an elasticity of the real wage with respect to productivity, denoted by η_{wp}, in a range between 0.3 and 0.7 and a ratio of the standard deviations σ_w/σ_p in a interval between 0.4 and 0.9 (Gertler and Trigari, 2006; Rotemberg, 2006; Hagedorn and Manovskii, 2007a; 2007b)[7].

Two more recents studies (Haefke, Sonntag, and van Rens, 2007; Pissarides, 2007), however, contend that for a correct comparison between the data and the search and matching model is important to distinguish between the volatility of the wages of newly hired workers and the volatility of the wages in ongoing matches. Examining various panel regressions of individual workers (Pissarides), or using micro-data from the CPS (Haefke *et al.*), both papers reach the same conclusion: the wages of newly hired workers are as much as volatile as labour productivity (implying an unit elasticity η_{wp}), whereas those in existing jobs are about half as cyclical (the same elasticity is around 0.5 for Pissarides, 0.27 for Haefke *et. al.*).

2.2   Job Creation vs. Job Destruction Volatility

The strong procyclicality of the job-finding rate and the weak countercyclicality of the job separation rate with respect to output contradict the conclusions reached by Blanchard and Diamond (1989) and (1990), Davis and Haltinwanger (1992) and Davis, Haltiwanger, and Schuh (1996) that consider job destruction as the main source of unemployment fluctuations. The question can be put in these terms: Shimer, both in his (2005) paper and in a subsequent one in which he uses gross flow data (Shimer, 2007) argues that recessions are essentially periods where it is extremely difficult to find a job, whereas most of the previous literature identifies recessions as periods mainly characterized by high job loss rates. What can explain such opposite results?

According to Shimer (2007), the reason lies on the different facts that he and David and Haltiwanger measure. Shimer considers the dynamic behaviour of monthly unemployment levels to get f_t and s_t, whereas Davis and Haltiwanger measure job creation and job destruction. The former is defined

as the net employment gains at establishments that experience positive net gains in a certain period; the latter as the net job losses at establishments experiencing negative net employment gains in a certain period. In the job separation estimates there are computed *both* the firings of existing employees *and* the decisions of not hiring new workers that replace quitters. Firings represent an increase in the separation rate but the decision of not hiring represents a decrease in the job-finding rate, that is therefore underestimated in Davis and Haltiwanger's analysis[8].

Even with respect to the previous gross worker flow data literature, the divergence in the results may depend on the different statistics measured. Much of this literature (such as Blanchard and Diamond, 1990) refer to the *levels*, that is $f_t u_t$ and $s_t e_t$, whereas Shimer's findings are about the *rates*, f_t and s_t; u_t and f_t moves in opposite direction over the cycle, so that $f_t u_t$ remain fairly stable even with a strongly procyclical job finding rate. Moreover, according to Shimer the gross flow literature also fails to account for time aggregation problems. For instance, models in which workers are not assumed to lose and find a job within the same period (a week, or a month) may deliver a biased measurement of the job finding rate.

Shimer's conclusions have not put an end to the dispute. A more recent strand of research (for instance, Davis, 2005; Yashiv, 2008; Fujita and Ramey, 2007; Elsby, Michaels, and Solon, 2007) has questioned his assertions, arguing that the volatility of the separation rate plays a decisive contribution to unemployment fluctuations.

Since the present survey primarily concentrates on the inconsistency between matching models and data and not on the measurement of the flows in and out of unemployment, space limitations do not allow me to discuss these papers. It is undeniable, however, that understanding the contribution of the separation rate to the unemployment variability has significant theoretical implications. Indeed, a separation rate strongly countercyclical would make much less justifiable to consider it exogenous as in a standard matching model. A definite word on the ins and the outs of unemployment - if the ins win, as the "old" literature concluded, or the "outs", as Shimer maintains, or we have a tie, as Fujita and Ramey (2007a), Elsby, Michaels, and Solon (2007) argue - is needed more than ever.

2.3 Policy Evaluation Estimates

The search and matching model can be quantitatively assessed also with respect to labour market micro data. We can consider for instance the estimates about the impact of the UBs both on unemployment duration and on the level of unemployment.

Because in a standard matching model labour supply is fixed and the UBs affect unemployment only via their negative impact on vacancy creation, the studies conducted by Layard, Nickell, and Jackman (1991) and Layard and Nickell (1999), that evaluate the general equilibrium effects of some labour market policies in various countries, appear the most appropriate references. They consider OECD data that go back to 1960 and get a semi-elasticity of unemployment with respect to the UI benefits replacement ratio around 1.3. Costain and Reiter (2007) also run some cross-country regressions on the basis of Layard and Nickell's dataset and obtain a semi-elasticity close to 2.0. In surveying other recent estimates, Baker *et al.* (2003) find values not substantially larger than those obtained by Layard and Nickell.

2.4 Theoretical Framework

The model built up by Shimer is a standard matching framework with the addition of a stochastic economy-wide shock in two parameters, productivity and the separation rate. I present the case of productivity shocks, the environment with a stochastic separation rate being symmetric. All the endogenous variables that depend on the current value of productivity are henceforth denoted by the subscript p.

The economy is composed by a measure L of risk-neutral workers. Time is continuous and the discount rate is denoted by r. Unemployed workers search for a job, whereas every firm can post only one vacancy. All jobs are identical and every firm-worker pair produces the unique consumption good at the flow rate of p. Autocorrelated shocks affect the value of p. More precisely, the time sequence $\{p_t\}$ is a jump process with an arrival rate λ and a conditional distribution of new values represented by the c.d.f $F_p : P \times P \rightarrow [0,1]$, P being the support of the process.

The flow of new matches is denoted by $m(v, u)$ and the job finding

rate is given by $f(\theta) \equiv m(v,u)/u$; the rate at which vacancies are filled is $m(v,u))/v = f(\theta)/\theta$, positive, decreasing and convex function of θ. At an exogenous rate a firm-worker pair is destroyed. The law of motion of unemployment is equal to:

$$\dot{u}_p = s\left(L - u_p\right) - f(\theta_p)u_p.$$

In steady-state, we have:

$$u_p = \frac{s\,L}{s + f(\theta_p)}. \tag{1}$$

The downward-sloping relationship in v/u space obtained from equation 1 is called the Beveridge curve.

Once a worker finds a firm with a vacant job, a surplus of the match arises. It is given by the difference between the expected discounted value that the two parties will receive by forming a match and the expected discounted value they renounce by being employed. A zero profit condition implies that the expected discounted value of a vacancy unfilled is equal to zero. So, the surplus is defined as: $S_p \equiv J_p + W_p - U_p$, in which J_p is the value of a filled vacancy, W_p is the value of being employed for a worker, U_p is the value of unemployment. Denoting by E_p the expectation operator conditional on the current state p, the Bellman equations take the following form:

$$rU_p = z + f(\theta_p)\left(W_p - U_p\right) + \lambda(E_pU_{p'} - U_p), \tag{2}$$
$$rW_p = w_p + s\left(U_p - W_p\right) + \lambda(E_pW_{p'} - W_p), \tag{3}$$
$$rJ_p = p - w_p - sJ_p + \lambda(E_pJ_{p'} - J_p), \tag{4}$$

in which z denotes the instantaneous utility enjoyed by the unemployed worker, while w_p is the wage. Hence:

$$rS_p = p - z - f(\theta_p)\left(W_p - U_p\right) - sS_p + \lambda(E_pS_{p'} - S_p), \tag{5}$$

To determine the wage allocation, Shimer considers a Nash bargaining solution: the wage w_p is chosen in order to maximize the product $(W_p - U_p)^{\beta}\,(J_p)^{1-\beta}$. Parameter $\beta \in [0,1]$ represents workers' bargaining power. The unique solution to this maximization problem is

$$(1 - \beta)\left(W_p - U_p\right) = \beta\,J_p. \tag{6}$$

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Notice that $W_p - U_p = \beta S_p$ and $J_p = (1 - \beta) S_p$. The free-entry zero profit condition implies that the expected cost of filling a vacancy (given by the expected duration of finding a worker multiplied by the flow cost of keeping a vacancy opened, denoted by c) must be equal to the value of a match to the employer:

$$\frac{c\,\theta_p}{f(\theta_p)} = J_p \tag{7}$$

Using (2), (3), (4), (6), and (7), a standard wage equation can be derived even in this stochastic set-up:

$$w_p = \beta(p - rU_p) + rU_p = \beta\,(p + c\theta_p) + (1 - \beta)\,z. \tag{8}$$

Finally, the match surplus (5) can also be rewritten using $W_p - U_p = \beta c\theta_p / [(1 - \beta)f(\theta_p)]$:

$$\frac{c\theta_p}{f(\theta_p)} = \frac{(1 - \beta)(p - z) - \beta c\theta_p + (1 - \beta)\lambda E_p S_{p'}}{r + s + \lambda} \tag{9}$$

Equation (9) is the equilibrium condition for labour market tightness, with the other endogenous variable being the expectation of future surplus $E_p S_{p'}$.[9]

2.4.1 Elasticities

In his paper, Shimer shows that the elasticity of tightness with respect to productivity when there are no shocks, obtained by differentiating (9) with $\lambda = 0$, is a useful approximation of the volatility of tightness in the dynamic stochastic set-up. Indeed, Mortensen and Nagypál (2007) prove that the two outcomes coincide in the limit when the arrival rate λ is close to 0 or the change in productivity is small (see Proposition 2 of their paper) [10]. This result is extremely useful, since it allows to compare different set-ups without the need of computing numerical simulations.

The elasticity of tightness with respect to productivity when $\lambda = 0$ is equal to:

$$\eta_{\theta p} \equiv \frac{\partial \ln \theta}{\partial \ln p} = \frac{r + s + \beta f(\theta)}{\alpha(r + s) + \beta f(\theta)} \cdot \frac{p}{p - z}, \tag{10}$$

where $\alpha = 1 - f'(\theta)\theta/f(\theta)$ is the elasticity of the expected duration of filling a vacancy with respect to tightness. Recalling that $f(\theta) = \mu\theta^{1-\alpha}$,

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the elasticity of the job-finding rate with respect to productivity is given by $\eta_{f,p} = (1 - \alpha)\eta_{\theta p}$. Using (1), I also get the elasticity of unemployment with respect to productivity, $\eta_{u,p} = (1 - \alpha)(1 - u)\eta_{\theta p}$.

To deal with the labour policy implications of the model and the volatility of the wage, I compute other two related elasticities. Parameter z is the sum of the level of unemployment benefits b and the value of leisure. The semi-elasticity[11] of u with respect to b is equal to:

$$\zeta_{u,b} \equiv \frac{\partial \ln u}{\partial b} = \frac{r + s + \beta f(\theta)}{\alpha(r + s) + \beta f(\theta)} \cdot \frac{(1 - \alpha)(1 - u)}{p - z} = \frac{\eta_{u,p}}{p}. \quad (11)$$

Finally, the elasticity of the wage with respect to productivity is given by:

$$\eta_{wp} \equiv \frac{\partial \ln w}{\partial \ln p} = \left(\frac{\beta}{w/p}\right) \cdot \left[\frac{\alpha(r + s) + f(\theta)}{\alpha(r + s) + \beta f(\theta)}\right]. \quad (12)$$

3 Comparing the Model with the Data

The business cycle data to which scholars have paid more attention concern the volatility of unemployment, vacancies and the job-finding rate. The dynamic correlation between productivity and the labour market variables, as well as the autocorrelation of vacancies, has received less attention. Some papers have also focused on the wage statistics. As regards the policy analysis estimates, the performance of the model is mainly evaluated looking at the (semi)elasticity of the unemployment with respect to the UBs.

3.1 Business Cycle Viewpoint

Shimer's calibrated values are the following: $r = 0.012$, p normalized to 1, $s = 0.10$, $f = 1.355$ (recall he obtained a monthly separation rate of 0.034 and a monthly job finding rate of 0.45), and $\alpha = 0.72$. Moreover, Shimer considers z only as the level of unemployment benefits, ignoring the value of leisure. He set it to $z = 0.4$. Since mean labour income in the stochastic model is equal to 0.993, that value of z belongs to the upper end of the range of replacement ratios in the United States.

Substituting in (10) these values and imposing the Hosios (1990) condition $\beta = \alpha$, that ensures the efficiency of the decentralized equilibrium, one gets

Comparative Statics	Model with shock on p	Data	
$\eta_{\theta p} = 1.72$	$\sigma_\theta/\sigma_p = 1.75$	$\sigma_\theta/\sigma_p = 19.1$	$\rho_{\theta p} \cdot \sigma_\theta/\sigma_p = 7.56$
-	$\rho_{\theta p} = 0.999$	$\rho_{\theta p} = 0.396$	
$\eta_{fp} = 0.481$	$\sigma_f/\sigma_p = 0.5$	$\sigma_f/\sigma_p = 5.9$	$\rho_{fp} \cdot \sigma_f/\sigma_p = 2.34$
-	$\rho_{fp} = 0.999$	$\rho_{fp} = 0.396$	
Comparative Statics	**Model with shock on s**	**Data**	
-	$\rho_{uv} = 0.999$	$\rho_{uv} = -0.894$	

Table 2. A comparison of the model with the U.S. data (period 1951-2003). Source: Shimer (2005).

$\eta_{\theta p} = 1.72$ and $\eta_{fp} = 0.481$. A comparison between the the data and the model is presented in Table 2. Assuming, as Shimer does, that shocks on productivity are the only source of fluctuations in labour market tightness, we can compare the elasticities $\eta_{\theta p}$ and η_{fp} with the corresponding ratios σ_θ/σ_p and σ_f/σ_p found in the data. The difference is striking: σ_θ/σ_p is eleven times larger that $\eta_{\theta p}$, while σ_f/σ_p is more than twelve times larger than η_{fp}. This is Shimer's main point: a standard matching model can explain less than 10 per cent of the observed fluctuations in the vacancy/unemployment ratio.

The Propagation problem

The other deficiency of the model is the absence of propagation of the labour productivity shock. This can be seen by comparing the contemporaneous correlations between productivity and all the labour market variables of interest. In the model, these moments are equal to 1, in the data they are close to 0.4 in absolute value. Other shortcomings concern the autocorrelation of vacancies and the time response of tightness to productivity. These problems - as well as the various solutions proposed by the literature - will be discussed in section 7.

Conditional vs. unconditional moments

Not all the papers analysing the amplification puzzle target the ratio of standard deviations as Shimer does. The reason is explained by Mortensen and Nagypál (2007) that argue that an empirical correlation between productivity and tightness equal to 0.396 makes questionable the assumption of productivity as the unique explanation for tightness fluctuations. Rather, such a

value suggests the coexistence of more driving forces behind the volatility of tightness and unemployment. Thus, instead of considering the ratios of standard deviations, they gauge the consistency of the model by comparing the empirical OLS regression coefficients $\rho_{\theta p} \cdot \frac{\sigma_\theta}{\sigma_p}$ and $\rho_{fp} \cdot \frac{\sigma_f}{\sigma_p}$ with the simulated ones. Data show that the latter coefficient is equal to 7.56 and the former is equal to 2.34, while the simulated counterparts are close to the corresponding ratios of standard deviations because of the unit correlations delivered by the model. The lack of amplification highlighted by Shimer is still present, but in less dramatic terms.

[FIGURE 1 ABOUT HERE]

Separation rate shock

Shimer considers also the effects of a shock on the separation rate. The main results is that it delivers a positive counterfactual correlation between unemployment and vacancies. To understand why, consider equation (9): a higher separation rate lowers θ, since it makes entry less profitable for firms. A decrease in labour market tightness is depicted in the $v - u$ space by a less steep ray starting from the origin; yet, the Beveridge curve (1) moves to the right, so it is not possible to discern the behaviour of the vacancies (see Figure 1). With a Cobb-Douglas matching technology, the shift of the Beveridge curve *may be* large enough to make both unemployment and vacancies increase, explaining why in the stochastic simulation it is observed a positive correlation between these two variables.

A framework with both productivity and separation rate shocks does not deliver significant improvements. Unemployment appears to be more cyclical, but both tightness and the job finding rate are still less than 10 per cent as volatile as expected.

Wage share and wage volatility

Two distinctive features of Shimer's calibrated model are the large value obtained for the wage share and the high volatility of the wage.

As regards the former, inserting the calibrated values in equation (8) gives a wage ratio $w/p = 0.973$. As Hornstein, Krusell, and Violante (2005) clearly show, the wage share is large in Shimer's calibrated model because both the job finding rate $f(\theta)$ and the workers' bargaining power β are much

greater than the job separation rate and the discount rate [12].

From (12), it is also clear that $\eta_{wp} \approx p/w$ if β is close to 1. So, in Shimer's calibrated model, the wage elasticity is around 1. This value is far beyond the range of estimates obtained by examining aggregate wage time series data, but is consistent with the results of Pissarides (2007) and Haefke, Sonntag, and van Rens (2007) on the volatility of wages for the new employees. So a natural question is which of the two statistics is more relevant to the search and matching model. Haefke *et al.* contend that it is the latter. In a matching framework, firms decide to post a vacancy on the basis of the expected present values of productivity, wages, and search costs. Hence, the variable η_{wp} represents the elasticity of the expected present value of wage payments with respect to the expected present value of productivity, not observable in the data. Haefke *et al.* show that, over a plausible range of parameters, the elasticity of the current period wage of newly hired workers with respect to productivity is an accurate proxy for η_{wp}.

3.2   Policy Evaluation Viewpoint

Now I compare the semi-elasticity of unemployment with respect to b predicted by the model with its empirical counterpart. Substituting the calibrated values chosen by Shimer in (11), I get $\zeta_{u,b} = 0.45$, lower than the values reported in section 2.3.[13] But the crucial point here is another. Since p is set equal to 1, the semi-elasticity of unemployment with respect to the unemployment benefits, $\zeta_{u,b}$ is equal to the elasticity of unemployment with respect to productivity $\eta_{u,p}$. Data show $\sigma_u/\sigma_p = 9.5$. The trade-off is clear: either the parametrization is constructed to match the business cycle volatility of unemployment, or it is constructed to match the policy estimates cited above. No calibration can attain both tasks.

4   First Approach: Changes in the Wage Formation

Why is the matching model unable to replicate the observed fluctuations in unemployment and vacancies? Shimer argues that the main culprit is the

Nash bargaining solution. "An alternative wage determination mechanism that generates more rigid wages in new jobs, measured in present value terms, will amplify the effects of productivity shocks on the v-u ratio, helping to reconcile theory and evidence." (Shimer, 2005).

4.1 Exogenous Wage Rigidity

The most straightforward way to break the link between wages and productivity is to impose wage rigidity.

In this respect, an important point deserves to be stressed. Assuming wage rigidity only in the existing matches (meaning that firms and workers negotiate the wage only the first time they match and then it never changes following subsequent shocks) has no impact on the vacancy/unemployment ratio. The reason is that this kind of rigidity does not affect the discounted expected profits of the firms, but simply the timing of the wage payments. Since in a matching model a firm decides to post a vacancy only on the basis of its future expected profits, the level of tightness takes the same value as in the flexible wage set-up.

As Shimer (2004) show, the results change dramatically in the case of rigid wages also in new matches. Under this hypothesis, firms and workers never bargain and take the wage as an exogenous variable. The new equilibrium equation when $w = \bar{w}$ and $\lambda = 0$ becomes:

$$\frac{c\theta_p}{f(\theta_p)} = \frac{p - \bar{w}}{r + s}$$

With $\bar{w} = 0.967$ (chosen to have an average U.S. unemployment rate of 5.7), the elasticity of tightness with respect to productivity is equal to $\eta_{\theta p} = p/\left[\alpha\left(p - \bar{w}\right)\right] = 42.08$, a value more than twice larger as the ratio of standard deviations found in the data[14].

However, the success of the rigid wage model can be questioned on different grounds:

1. The large values for the elasticities mainly depend on the value $\bar{w} = 0.967$. If $\overline{w} = 0.5$, then $\eta_{\theta p} = 2.7$, higher than in the flexible wage set-up but inconsistent with the data. Wage rigidity is not sufficient

to improve the business cycle consistency of the model. A small profit share $(p-w)/p$ is also needed, so that the percentage increase in profits is large for a given percentage increase in productivity.

2. As we have seen a the end of section 2.1, imposing wage rigidity for new hires is not empirically grounded, Haefke *et al.* (2007) and Pissarides (2007) having shown that the corresponding elasticity $\eta_{w,p}$ for this subset of wages is close to unity.

3. With perfect wage rigidity, the level of unemployment benefits does not affect tightness and employment. The semi-elasticity ζ_{ub} is equal to zero. So, the price of the business cycle consistency of the model is to make it useless for a policy evaluation analysis[15].

4.1.1 Sticky wages

"Milder" forms of wage rigidity may be also introduced. Farmer and Hollenhorst (2006) build up a fully blown DSGE model, in which households take consumption-saving decisions and search effort is endogenous, and assume that only the 19% of the wage is bargained while the remaining fraction is unaffected by productivity shocks. Even their frameworks fits particularly well U.S. data. The sticky wage economy matches the unemployment standard deviation and slightly overshoots on vacancy standard deviation. As the authors stress, two parameters are decisive: the disutility of effort parameter and the degree of wage stickiness. The latter in particular is key in replicating both a positive (negative) correlation between output and vacancies (unemployment).

4.1.2 Staggered wage contracts

Staggered wage contracts constitute a middle way between perfectly rigid and flexible payments. Firms and workers bargain over the wage at an exogenous Poisson rate $1 - \varphi$. The expected duration of a contract is therefore equal to $1/(1 - \varphi)$. The aggregate wage in the economy is $w_t = (1 - \varphi)w_t^* + \varphi w_{t-1}$, with w_t^* being the payment negotiated at time t. Gertler and Trigari (2006) and Bodart, Pierrard, and Sneessens (2006) pursue this approach.

The data Gertler and Trigari want to target are the following: $\sigma_\theta/\sigma_p = 12.10$, $\sigma_u/\sigma_p = 5.81$, and $\sigma_w/\sigma_p = 0.46$. Assuming three or four quarters as average length of the contract, their model explains 81% of the unemployment volatility, 89% of tightness volatility, and 95% of wage volatility. Yet, the success of both papers hinges on the assumption of wages stickiness also for new matches (i.e. newly hired workers receive the same wage paid to the other employees[16]), so incurring the same critique addressed to the rigid wage hypothesis by Pissarides and Haefke *et al.* .

Moreover, as emphasized by Mortensen and Nagypál (2007), staggered contracts imply that labour market tightness increases more than in the flexible set-up after a positive shock on p because only a fraction $1 - \varphi$ of the new employees bargain the wage at time t. But when the wages are finally renegotiated in accordance with the new productivity value, tightness decreases to a level *below* its initial response. This is at odds with data.

4.2 Endogenous Wage Stickiness

4.2.1 Social Norm

Imposing exogenous sticky wages in the model implies that the agents are not fully rational, since they are not exploiting all the advantages of the negotiation. Hall (2003, 2005) overcomes this critique, by imposing a wage norm that never lies outside the bargaining set. Adopting the same notation used in the previous sections, worker's reservation wage is equivalent to rU, whereas p is the highest level of wage that an employer is willing to pay. Suppose an idiosyncratic random shock, ϵ, normally distributed with zero mean and standard deviation σ, that shifts the bargaining set so that it becomes $[rU + \epsilon, p + \epsilon]$. Then the current wage takes the following form:

$$\begin{cases} w_t = rU_t + \epsilon & \text{if } w_{t-1} < rU_t + \epsilon \\ w_t = p_t + \epsilon & \text{if } w_{t-1} > p_t + \epsilon \\ w_t = w_{t-1} & \text{otherwise} \end{cases}$$

The wage does not change if it remains in the bargaining set, otherwise it takes the value of the nearest boundary. Hall imposes the norm $w_t = E(w_t(\epsilon))$. The average wage at time t is a function of w_{t-1}, rU_t, and p_t. It

is worth noticing that such a norm has an impact also on the wages of new matches, that become stickier.

Hall's analysis differ from Shimer's one in that he considers a permanent price shock. Productivity jumps from 1 to $1 + \Delta$ and then remains at that level. His model succeeds in replicating the behaviour of key labour market variables in the U.S. economy. A reduction in productivity by 1 per cent produces the classical hump-shaped form for the dynamics of the unemployment rate: it starts from 5.6%, reaches the maximum value of 6.7% after seven months, and then it starts to decline.

4.2.2 Long Term Wage Contracts

In the paper of Rudanko (2007), wage rigidity is the consequence of long term wage contracts proposed by risk-neutral firms to risk-neutral workers in order to smooth income in response to labour productivity shocks. Rudanko consider three types of contracts: full commitment, where both the firm and the employee commit not to quitting even if such choice may not be the optimal one *ex post*, 1-side limited commitment, where only firms commit to contracts, and 2-side limited commitment, in which the two parties are able to break the contract.

The first type of contract features perfect wage rigidity, the firm bearing all the risk caused by fluctuations in productivity. Under the second type of contract, should a positive shock on productivity make more valuable for the workers to quit the job, wages would adjust up to the new value of the opportunity cost of employment to prevent it. Under 2-sided commitment, the contract also foresees a wage decrease during troughs in order to keep firms from firing workers.

The similarity between the last contract and Hall's model is evident. However, the simulation results show that none of these contracts amplify the fluctuations in unemployment and vacancies[17]. Moreover, the first two types of contract feature a volatility of the aggregate wage lower than the targeted ratio $\sigma_w/\sigma_p \approx 0.5$.

The reason of this twofold failure is that the wage contracts studied by Rudanko augment the rigidity of the wages in ongoing matches - so that the

volatility of the aggregate wage results lower than in the data - but do not dampen the cyclicality of the wages for new hires. Indeed, under each of the three types of contracts, the firm makes a wage offer at the moment of the matching that depends on the present conditions in the economy. As Rudanko observes, in her model wage rigidity fails to fit both tightness and the wage fluctuations observed in the data.

4.3 Changing the bargaining threat points

A productivity shock affects the wage both directly and via an increase in the opportunity cost of employment, rU. The latter enters in the wage equation (8) because in a standard Nash solution the threat points are constituted by the utility for firms and workers of being unmatched (the so-called outside options, U and $V = 0$ respectively for workers and firms). By changing the threats points, Hall and Milgrom (2008) avoid that a higher opportunity cost of employment rU translates into higher wages.

Following the non-cooperative bargaining approach of Binmore, Rubinstein, and Wolinsky (1986), Hall and Milgrom distinguish between outside option and disagreement payoff. The latter is what the part gets by prolonging the bargaining period - refusing the counterpart's offer and making a counterproposal - and is independent on the outside conditions in the labour market (namely, market tightness). It is assumed to be a flow cost for the firm and a flow benefit for the worker. So, in Hall and Milgrom's setup the threats points are given by a weighted average between the outside option and the disagreement payoff, the weights being respectively the exogenous probability that the match is broken during bargaining and its complement.

Such probability is crucial in Hall and Milgrom's analysis; the less likely is a separation during bargaining, the weaker the impact of productivity on the wage via U. On the other hand, if the probability is equal to 1, the bargaining process coincides with the canonical Nash solution.

By setting the daily separation probability during the bargaining process equal to 0.0055, Hall and Milgrom succeed in matching the U.S. data on vacancies and unemployment fluctuations[18]. The elasticity of the wage with respect to productivity is equal to 0.69, a result in a middle way between

the unit value suggested by Haefke, Sonntag, and van Rens (2007) as a good target for the wage elasticity in matching models and the estimates obtained by examining aggregate wage statistics.

4.4 Asymmetric Information

As a possible solution to the poor quantitative performance of the matching model, Shimer (2005) also suggests the introduction of asymmetric information in the wage process. The results summarized in this section show that asymmetric information is not sufficient *per se*; in order to augment the volatility in the model, the amount of private information hold by the employer must be also increasing in booms.

4.4.1 Acyclical informational rents

Brügemann and Moscarini (2007) explain the limited impact of asymmetric information on amplifying fluctuations by examining the properties of the wage equation. When the matching between a worker and a firm gives rise to quasi-rents, the wage can be divided in two parts: a fraction of the total rent generated by the match and the opportunity cost of employment, that is always procyclical. For instance, in the Nash solution, the former is given by $\beta (p - rU)$ and the latter by rU.

Brügemann and Moscarini first show that in many models with asymmetric information in the bargaining process workers' rents are at most acyclical, but never countercyclical. Then they prove that an acyclical worker's rent is not sufficient to generate the observed business cycle fluctuations[19]. In their baseline framework, the output of an employer-worker match is the sum of an aggregate productivity component and a match-specific one, the latter being private information of the employer. Similarly, a match specific amenity value of the job, known only by the worker, adds to the wage to determine his utility in employment. Nesting in such set-up three different wage determination schemes (a take-it-or-leave-it offer, a sequential bargaining with one-sided asymmetric information, and a bilateral asymmetric information scheme), they show that none of these set-ups is able to amplify unemployment and vacancy fluctuations.

Their conclusion is corroborated by the results of Guerrieri (2008). She constructs a competitive search model in which workers privately observe their type[20]. Her numerical exercises show that asymmetric information does not help in replicating the amplification in vacancy and unemployment observed in the data.

4.4.2 Procyclical informational rents

Brügemann and Moscarini analyse a symmetric set-up, in which both the employer and the employee hide some information to the counterpart and the outside economic conditions do not affect such informational rents. Two papers show that the performance of the model is bettered if it is assumed that the gain that firms obtain by being more informed than workers is increasing in booms.

Kennan (2007) develops this idea by considering two aggregate states in the economy (1, the bad state and 2, the good state) and two different idiosyncratic values for the productivity of a job (high or low). After matched with a worker, only the firm knows the idiosyncratic value of a job. So, if the firm makes the offer, it will get all the surplus and the worker will receive his opportunity cost of employment. On the other hand, if the worker makes the offer, he faces a potential trade-off between demanding a higher wage and being sure to reach an agreement. Kennan imposes two crucial assumptions: 1) workers always demand the low surplus; 2) when the aggregate state of the economy is good, there are more jobs with high idiosyncratic productivity.

Given these two assumptions, an average firm will earn more profits during booms also because the higher idiosyncratic value of the match does not translate into higher wages. Kennan shows that such procyclical informational rent enjoyed by the firm magnify the fluctuations in unemployment and vacancies[21].

A similar approach is pursued by Menzio (2005) in a wage-posting model with intra-firm bargaining. Firms have private information about their productivity type (composed by a permanent and a transitory part) and every period advance a wage offer both to its employees and to the fraction of unemployed workers contacted. Once the worker has observed all the offers re-

ceived, he chooses a trading partner and the firm-worker negotiation begins. In the stable equilibrium of the extensive form bargaining game, identical workers employed in the same firm cannot be payed differently; otherwise, a discriminated worker could ask for a renegotiation of his payment conditions. Because of this non-discrimination constraint, the cost for a firm willing to adjust his wage bill in response to positive productivity shock is proportional to the measure of its employees. On the other hand, the benefit of raising the wage is given by a higher acceptance rate of contacted workers and a lower separation rate of the employees searching on-the-job. The shorter the expected duration of the shock, the smaller will be the marginal benefit of raising the wage. So, the wage is independent of the realization of productivity shocks if its persistence is below a certain threshold. This kind of wage rigidity, together with a low calibrated profit share, amplifies the volatility of unemployment and vacancies fluctuations (Menzio's numerical results are $\sigma_v/\sigma_p = 7.83$ and $\sigma_v/\sigma_p = 6.45$)[22].

4.5 Final remarks on the first approach

It is undeniable that the wage flexibility implied by the Nash solution plays a role in weakening the volatility of unemployment and vacancies in response to a productivity shock. However, Shimer (2005) and the subsequent literature summarized in this section have probably put too much emphasis on that aspect. Indeed, loosening or even breaking the link between wages and productivity in the model is not *per se* sufficient to replicate business cycle facts. A low profit share is also needed. In addition, the critique raised by Pissarides and Haefke, Sonntag and van Rens concern all the sticky wages model presented din this section barring Hall and Milgrom (2008).

Models with asymmetric information deliver conflicting results. The business cycle consistency of the model is improved only by imposing procyclical information rents. Such assumption is however difficult to test empirically.

5 Second Approach: Changes in the Calibration

The papers examined so far are based on the belief that Shimer is right both when he denounces the quantitative inconsistency of a standard matching model and when he identifies the Nash wage bargain as the main culprit. Hagedorn and Manovskii (2007a) take a different route and focus on the calibration of the model.

Their model is substantially similar to the one presented in section 2.4, the only difference being the introduction of capital in the production technology in order to measure the capital cost of vacancy creation. Some calibrated values are the same as in Shimer's paper: $r = 0.012$, $f = 1.355$, and $s = 0.10$. Mean labour productivity is normalized to 1.

Hagedorn and Manovskii's calibration method essentially differs in computing β and z.[23] To pin down these two variables and the matching function parameter, they use as targets $f = 1.35$, $\theta = 0.634$ (obtained by choosing a quarterly job filling rate $f/\theta = 2.13$), and, more importantly, the elasticity $\eta_{w,p} = 0.449$. They obtain $z = 0.955$ and $\beta = 0.052$, in a striking contrast with the values chosen by Shimer ($z = 0.4$ and $\beta = \alpha = 0.72$). Hagedorn and Manovskii claim that such a discrepancy can be explained by focusing on two aspects: 1) In the model, the profit share $(p - w)/p$ is small and 2) The wages are moderately procyclical in the data.

The first fact has been already emphasized in the previous sections. A low profit share is necessary (but not sufficient) to amplify the business cycle fluctuations in unemployment and vacancies. As noticed in section 3.1, Shimer obtains a large value for w/p because he sets $\beta = 0.72$, so $\beta f(\theta)$ is large relative to s and r. But we can have a high labour share also by choosing a high value for z. By manipulating equations (5) and (8), one gets:

$$\frac{w}{p} = \frac{(r + s)\left[\beta + (1 - \beta)\frac{z}{p}\right] + \beta f(\theta)}{r + s + \beta f(\theta)} \tag{13}$$

The labour share is close to one if the term inside the square brackets is close to one. This can be obtained either with a large β, or with a high fraction z/p. But a high β implies an elasticity η_{wp} close to unity (see equation

12). Since Hagedorn and Manovskii target $\eta_{wp} = 0.449$, they opt for a high fraction z/p and a low β.[24]

Hagedorn and Manovskii show that their results are not sensitive to the kind of matching technology assumed. So, keeping the usual Cobb-Douglas matching function with $\alpha = 0.72$, one gets the following elasticities values: $\eta_{\theta p} = 26.8$ and $\eta_{fp} = 7.5$. Compared to the results presented in Table 2, their calibrated model succeeds in amplifying the fluctuations in tightness and in unemployment. The reason stems from the high value of z, that raises the values of the elasticities $\eta_{\theta p}$ and η_{fp}.

5.1 Pros and cons of the second approach

The parametrization performed by Hagedorn and Manovskii can be questioned on three different grounds.

First, with $z = 0.955$ the gap between the utility of being employed and the utility of being unemployed becomes extremely small (the difference $w - z$ is only 0.022). Is it realistic to think that employees work for a 2.2% surplus? Hagedorn and Manovskii list a series of reason in favour of a high z. They claim for instance that since in the model the expected duration of finding a work is low, it is plausible to imagine that for people remaining without a job for 2.5 months on average the utility gap is almost zero. Further, a standard RBC set-up with indivisibility of labour and without search frictions would imply a difference $w - z$ equal to zero, so their calibrated model can be viewed as a linear approximation of a richer framework in which workers take consumption-saving decisions and firms face a downward-sloping demand.

Second, targeting an elasticity $\eta_{wp} = 0.449$ is in contrast with the conclusions of Haefke, Sonntag, and van Rens (2007) that consider the elasticity of the wage for the new matches (close to unity, according to their estimates) as a correct approximation in a canonical matching model with Nash bargain.

Third, using Hagedorn and Manovskii's calibration, one gets $\zeta_{ub} = 6.96$, three times larger than the value reported in section 2.3. This confirms the existence of a business cycle / policy analysis trade-off[25].

6 Third approach: Enriching the Standard Model

Papers following the third approach agree with Shimer's diagnosis about the business cycle inconsistency of a standard matching model, but doubt that the Nash wage bargain is the main culprit for the quantitative failings of the model. The underlying motivation of these papers is that a text-book matching model is a useful tool to look at the qualitative effects of a policy change or a shock, but it is too stylized to be also consistent with data. Therefore, embedding in the standard setting other realistic features like on-the-job search, hiring and firing costs, imperfectly competitive product markets, firms heterogeneity, should make the model more suitable for a quantitative scrutiny.

6.1 Turnover costs

Silva and Toledo (2007) insert two different kinds of turnover costs in a standard matching model: training costs that firms spend for new entrants and separation costs suffered by employers when a job is destroyed.

Both the model and their calibration do not stray too much from Shimer (2005). It is worth stressing that they set $\beta = 0.34$ and $z = 0.715$ in order to obtain ζ_{ub} equal to 2.0, in line with the policy evaluation estimates presented in section 2.3 and so avoiding the criticisms addressed to Hagedorn and Manovskii's (2007a) setup.

Inserting Silva and Toledo's calibrated values in their equations for the elasticities, one gets: $\eta_{\theta p} = 6.62$, almost four times larger than in Shimer's setting, $\eta_{fp} = 1.85$, $\eta_{u,p} = 1.747$, and $\eta_{w,p} = 1$. Recall that in U.S. data, $\sigma_f/\sigma_p = 5.9$ and $\sigma_u/\sigma_p = 9.5$, while Haefke, Sonntag, and van Rens (2007) suggest that a unit wage elasticity is a good proxy in a standard matching framework. Silva and Toledo's paper improves upon the business cycle consistency of the model while keeping it consistent with the microeconometric estimates presented in section 2.3.[26]

Why do turnover costs better the performance of the model? The mechanism is analogous to that obtained by a low profit share. Turnover costs

lower the value of a filled job, so a higher p yields a large percentage increase in profits. More vacancies are posted in order to restore the free-entry equilibrium, enhancing the tightness elasticity.

A similar effect is present in Garibaldi (2006). He considers two types of large (multiple jobs) firms in his model: those that, having a high productivity value, react to adverse shocks by simply posting less vacancies, and those "at the margin", with a very low profit share. When a productivity shock hits the economy, this second kind of firms either declares bankruptcy, firing all the employees, or freezes its hirings, not replacing the workers who quit. If the expected losses are greater in absolute value than firing costs, the firm declares bankruptcy, otherwise it experiences a "hiring freeze". For firms "at the margin" a small change in productivity can make a great difference, and the number of vacancies they post in good times is much higher than the number of vacancies (not) posted in bad times. The volatility of such variable greatly increases. Accounting for hiring freeze and bankruptcy allows Garibaldi to explain up to 35% of the tightness volatitly displayed in the data.

6.2   On-the-job search

6.2.1   Hiring costs and on-the-job search

Nagypál (2005) shows that a combination of hiring costs and on-the-job search may reconcile the model with the empirical evidence.

In her set-up, firms incur hiring costs - i.e. they pay a fixed amount of resources upon the matching - and employees search for jobs with a higher idiosyncratic payoff. Hence, some matches have a negative pay-off for firms, for an employee working in a job with a low idiosyncratic payoff can quit before the employer has recouped his initial investment. Firms do not know the idiosyncratic value of the match, but they realize that unemployed searchers are more likely to accept matches with a low match quality than employed searchers. This is due to a positive selection effect that shifts workers into match qualities towards the top of the distribution. So, with high hiring costs firms have a lower expected pay-off from contacting an unemployed searcher than an employed one. After a positive shock on p, the fraction of employed

searchers out of the total number of searchers increases, making firms even more willing to post vacancies.

Nagypáls compute the elasticity of the job finding rate in response to a positive shock on productivity and a negative one on the separation rate, in order to amplify the effects in terms of vacancy creation. If hiring costs are imposed to be 2 or 3 times the quarterly profit flow $p-w$ this implies a σ_f/σ_p respectively of 3.086 and 7.168, close to the value of 5.9 found by Shimer in the data.

6.2.2 Wage heterogeneity

An increase in the number of job-seekers in response to a positive shock on productivity may enhance the volatility of vacancies and unemployment in the model, because it increases the job filling rate, boosting vacancy creation. This is the mechanism at work in the paper of Krause and Lubik (2006). They consider an economy with an high-paid sector and a low-paid one. The employees that work in the low-paid sector exert some effort on searching on-the-job. Following an economy-wide positive productivity shock, firms post more vacancies both in the high-paid and in the low-paid sector. A higher tightness in the high-paid sector raises the search effort of the low-paid employees, implying a higher number of job seekers in terms search units. The rate at which a high-paid job is filled goes up, so making firms even more willing to post vacancies there. The process ends because of a convexity assumption of the search cost, delivering a larger amplification in vacancy posting in response to a productivity shock. A complementarity between sectors also arises. If search effort of low-paid employees goes up, congestion effects in the matching technology will make more difficult for unemployed workers to find a high-paid job. Then, they will direct their search toward the low-paid sector. This in turn will boost vacancy creation in that sector. The model succeeds in replicating almost the 90% of tightness fluctuations but it predicts a too small wage volatility ($\sigma_w/\sigma_p = 0.117$).

6.3 Embodied Technical Change

Shimer assumes that the productivity shock hits in the same way all the matches in the economy. Costain and Reiter (2007) and Reiter (2008) argue that a model with embodied technical change helps to solve the amplification puzzle. In their model, the productivity of a match is denoted by Y and takes the following form:

$$Y \;=\; (1 - \xi_p) \cdot p \,+\, \xi_p \cdot p_m, \quad 0 \leq \xi_p \leq 1$$

in which p is the current level of productivity and p_m is the level of aggregate productivity at the time the match was formed. The higher the value of the parameter ξ_p, the more embodied in the match is the technical change. When $\xi_p = 0$, the model is identical to the standard one studied in section 2.4.

Why should embodied technical change amplify fluctuations? The reason is twofold. First, if the observed productivity is an average of past vintages, then the productivity for new matches, that is the source of unemployment and vacancies fluctuations, results underestimated. The calibrated model must therefore account for a higher variability of current productivity.

Second, embodied technical change makes employers's surplus more pro-cyclical. The productivity of a match is only partially affected by current shocks. Anticipating that, firms will post many vacancies when the shock is positive. Fluctuations in vacancies and unemployment are bigger as in the disembodied productivity case. With $\xi_p = 0.302$ or $\xi_p = 0.576$, Reiter (2008) gets a ratio σ_θ / σ_p equal respectively to 7.9 or 18.65, while the same set-up with $\xi_p = 0$, delivers a relative standard deviation equal to 4.15.

Costain and Reiter (2007) highlight two shortcomings of the model. First, endogenous separation rate should be taken into account, because workers could find optimal to quit their job when new matches with higher productivity are created in the economy. This kind of separations raises the number of unemployed people in upturns, destroying the Beveridge curve. The second shortcoming concerns the excessive wage volatility delivered by the model[27].

To overcome these problems, Reiter (2008) introduces two features: a long-term wage contract *à la* Rudanko (2007) and turnover costs *à la* Silva and Toledo (2007). The former reduces the wage volatility, while post-match

training costs paid by the workers make more costly for them to quit a job and search for another opportunity. Reiter's results seem to go in the right direction.[28]

6.4   The role of the separation rate

What is the role of the separation rate in amplifying unemployment and vacancies fluctuations? Several scholars have addressed this question. To clarify their conclusions, it is important to distinguish between models that consider an exogenous separation rate and models in which it is endogenous.

In the former case, the idea consists on incorporating a countercyclical shock on the separation rate to the standard model with a productivity shock. Recall from section 3.1 that Shimer (2005) has already performed such exercise, reaching ambiguous results. Fluctuations in unemployment are amplified, because the level of this variable along the cycle is affected not only by procyclical job creation but also by countercyclical separation rates. The problems concern the behaviour of the vacancies. An increase in the separation rate has two effects opposite in sign. On one hand, it raises the factor to which profits are discounted, stifling vacancy creation (job creation effect). On the other hand, it shifts the Beveridge curve to the right, so more vacancies must be posted for the equality of labour market flows to hold; this is obtained by an increase in the job filling rate $f(\theta)/\theta$ that induces more firms to enter the market (Beveridge curve effect). As a result, a positive shock on the separation rate may deliver a counterfactual increase in vacancies. Figure 1 illustrates the two mechanisms at work. The fluctuations in tightness and the job finding rate, that are bigger the more procyclical is the level of vacancies, remain insufficiently low.

Such discouraging conclusions are in part reversed by Mortensen and Nagypál (2007). Mortensen and Nagypál show that the counterfactual increase in vacancies during troughs may be ruled out once a wage bargaining *à la* Hall and Milgrom (2008) is introduced in the model. Loosening the link between wage and tightness amplifies the negative effect of the separation rate on vacancy creation, for the decrease in expected profits is not offset by a lower wage. So, the equality of labour market flows is reached at a level

of vacancies lower than before. In terms of Figure 1, the job creation 0-ray shifts down more than under the canonical Nash solution.[29].

More recent papers have sought to answer Shimer's puzzle by introducing endogenous separation rates. Despite its theoretical appeal, such a try does not seem to reach satisfactory results. Either analysing the standard Mortensen and Pissarides (1994) model (Pissarides, 2007) or considering an extended version of it (Mortensen and Nagypál, 2008), the conclusions are similar: endogenous separations have a scarce impact on job creation and the volatility of tightness. The reason is well explained by Pissarides. When the separation rate is exogenous, all jobs are equally likely to be destroyed. Job creation is dampened because a positive shock on s reduces the expected profits of any match, regardless of their productivity value. When the separation rate is endogenous, only matches with a value close to the reservation productivity are destroyed. Job creation is scarcely affected because the expected returns of such jobs is close to zero and firms and workers are indifferent in continuing the match or separating.

Elsby and Michaels and Andrés, Doménech, and Ferri (2006) are able to match the data in a model with endogenous separations, but their models present several mechanisms that may potentially amplify vacancy fluctuations. In Elsby and Michaels's paper, it is key the assumption of decreasing marginal productivity and intra-firm bargaining. Firms hire more than one worker and, under Nash bargaining, the wage is a constant fraction of firms' marginal revenues. The employers take into account the marginal decrease in the wage bill caused by hiring one additional worker, so, *ceteris paribus*, they will post more vacancies than in the standard linear case.[30] The increase in job creation caused by a positive shock on productivity outweighs the Beveridge curve effect, and vacancies are strongly procyclical.

6.5 Price rigidity

Andrés, Doménech, and Ferri (2006) construct a DSGE model in which several mechanisms affect vacancy behaviour: endogenous separation rates, capital, taxes, intertemporal substitution, and price rigidity. The latter gives the most decisive contribution in terms of tightness amplification.

Their model features a two-tier productive scheme: wholesale firms operate in a competitive market using labour and capital. Retail firms adopt as only input the good bought by wholesale firms and are monopolistically competitive. The final consumption good is a composite of different varieties produced in the retail sectors. As for the wage contracts in Gertler and Trigari (2006), the price decisions of the retail firms are staggered over time.

When a positive shock affects the productivity of the wholesale firms, the relative price of their good immediately go down, due to the price rigidities in the retail sector. Yet, it soars in the following periods because of the downward adjustments of the prices in the retail sectors. Such a jump is less smooth than in the case of flexible prices, so that the value of match for the wholesale firms varies more and vacancy creation is boosted.

6.6 Other sources of shocks

If changes in labour productivity are tiny in comparison with the variations in the labour market, maybe it is because the former is not the (only) exogenous driving force in the economy. As a result, the failure of the model does not depend on the lack of an amplification mechanism, but on the misidentification of the correct shock hitting the economy.

Such a reading of the Shimer puzzle is gaining the attention of several scholars. The papers following this route present a rich framework that encompasses several departures from the standard search and matching model. While the results in terms of amplifications and propagation mechanism are often promising, it is difficult to disentangle all the effects at work in these set-ups.

6.6.1 A VAR approach

Yashiv (2005; 2006) considers a reduced-form VAR of the actual data to specify the driving shocks. Assuming that three variables (the rate of productivity growth, the separation rate, and the interest rate) follow a first-order VAR, his model captures the persistence, the volatility and some co-movements of the main labour market variable in the data. The high persistence of vacancies is obtained by imposing convex hiring costs that make vacancy

creation more sluggish. Yet, convex costs also tend to reduce the volatility of the vacancies. To amplify fluctuations, according to Yashiv, the stochastic properties of the separation rate play a crucial role. He rightly argues that the separation rate is key in evaluating the expected discounted value of a match. Nevertheless, it is not clear why in Yashiv's VAR approach it is so essential in engendering the correct volatility, while Shimer's setting with contemporaneous shocks both in p and in s does not deliver analogous outcomes.

6.6.2 Market power

While in Shimer's model a technological shock hits all the jobs, Rotemberg (2006) considers a change in firms' market power as driving force in the economy. The key is that in the former set-up the marginal productivity of labour and, in turn, the wage increase. In the latter, firms react to fiercer competition by producing more, and labour productivity goes down. Workers have to moderate their wage demand, while employment goes up[31].

Rotemberg's main objective is to match data on wage volatility. In his model $\sigma_w/\sigma_p = 0.56$, a value that fits the data on the aggregate wage targeted by the author, but that is inconsistent with the conclusions of Haefke *et al.* (2007) about a unit wage elasticity (so, an even higher ratio of standard deviations) as the right target in a matching model with Nash bargaining. Further, the model overshoots on vacancies volatility, whereas it captures one third of the employment fluctuations and half of unemployment volatility.

However, as Trigari (2006) documents in a detailed comment, the success of the paper depends less on the imperfect competition, and diminishing returns of labour than on the high value assigned to the instantaneous utility in unemployment (Rotemberg sets $z/w = 0.9$), confirming once again the importance of a low profit share in order to amplify fluctuations.

6.6.3 Establishment level shocks

Cooper, Haltiwanger, and Willis (2007) build up a matching model with an establishment level profitability shocks and succeed in fitting cyclical the behaviour of unemployment, vacancies and wages. In their framework, firms

can adjust both the intensive and the extensive margin (hours worked and level of employment), have all the bargaining power in the wage negotiation, and face fixed and variable costs when hiring or firing workers. Firms' production function is subject both to aggregate and idiosyncratic (establishment levels) shocks. The objective of their paper is not only to solve Shimer's puzzle but also to match some establishment observations, such as the negative correlation between hours growth and employment growth. It is difficult to isolate all the effects that are at work in such a rich setting. The authors argue that their set-up is able to fit the aggregate data partly because of the distribution of the idiosyncratic shocks that are not smoothed out by aggregation, while another potentially interesting feature of the model - the possibility for firms to fire workers as in Garibaldi (2006) - is unexplained.

6.6.4 Monetary shocks

A recent strand of the literature has investigated the contribution of demand (monetary and not monetary) shocks to the labour market. In this respect, these papers belong to the vein of the new-Keynesian models, in which several departures from a standard RBC framework contribute to the response of the labour market variables.

The paper that most convincingly stresses the importance of monetary shocks in order to understand the Shimer puzzle is Barnichon (2007). He argues that the cyclical changes in labour productivity, considered by Shimer as the only determinant of unemployment and tightness fluctuations, are actually caused by exogenous shocks on monetary policy. Productivity and tightness comove in response to the same shock, but there is not causal relationship between the former and the latter. In his model a shock that increases the supply of money is able to explain about 50% of the tightness volatility.

Braun (2005)'s model presents many features that may potentially amplify the effects of a monetary policy shocks: wage stickiness, price rigidity in the intermediate sectors, a high opportunity cost of employment, and training costs. After having estimated an identified VAR on US data, he finds that wage rigidity and a high opportunity cost of employment contribute

more decisively in matching the empirical responses.

Braun, de Bock, and Di Cecio (2007) also adopt a VAR approach to ask which shock is the most crucial in driving labour market fluctuations. They identify three kinds of shocks: supply shocks, that are required to have opposite effects on output and the price level, demand shocks, that move output and prices in the same direction, and monetary shocks, that also push down the interest rate. Their conclusion is that, although the response of hours worker, unemployment, and vacancies is qualitatively the same regardless of the shock considered, demand shocks may be more important in driving labour market fluctuations.

7   The Propagation of Shocks

A standard matching model not only falls short of replicating labour market fluctuations but also exhibits no propagation of productivity shocks. Three facts are worth stressing: 1) Data show that the maximum correlation between vacancies and current productivity is observed when vacancies are one or two quarters ahead, while in the model the peak is reached at zero lags. 2) In the data, labour market tightness follows productivity by one year and the contemporaneous correlation between these two variables is 0.40. Simulation results, on the contrary, predicts a correlation $\rho_{\theta p}$ equal to 0.999 . The correlation between productivity and the other labour market variables are also greatly overshooted. 3) The autocorrelation of vacancies is lower in the model than in the data.

The absence of propagation in a matching model depends on how vacancy behaviour is modeled. As Pissarides (2000, p.26-31) stresses, vacancies - and consequently market tightness - are a "jump variable", so they adjust too rapidly in response to a productivity shock. Thus, any mechanism allowing for a more sluggish vacancy behaviour should yield a more realistic dynamics.

Fujita (2003), Fujita and Ramey (2007b), and Hagedorn and Manovskii (2007b) address this issue. Hagedorn and Manovskii introduce time to build in job creation. Any vacancy created at a certain moment enters the market with some delay. Their model can match the contemporaneous correlation of tightness and productivity, but still fails to account for the correct auto-

correlation of vacancies.

In the model of Fujita and Ramey (2007b), a firm-pair can be destroyed for two reasons: obsolescence, meaning that the worker becomes unemployed and the position disappears, or "normal" separation, meaning that both the worker and the job position enters the matching pool the following period. Moreover, the zero profit equilibrium equation is assumed to be equal to $V_t = K \cdot n_t$. The value of a vacancy at time t is equal to a sunk cost, that Fujita and Ramey assume increasing in the number of positions created in the economy at t, n_t. These changes has two effects.

First, marginal costs that increase with the number of positions induce firms to spread out vacancy creation. In turn, this translates into a more realistic value for the autocorrelation of vacancies, like in the Yashiv (2006) framework with convex hiring costs. Second, since the value of a vacancy is strictly positive at the equilibrium, firms are willing to keep a position open after a separation. Vacancies become a predetermined variable. Differently from Shimer (2005) and Hagedorn and Manovskii (2007b), Fujita and Ramey first estimate a reduced-form VAR to study the dynamics of employment, tightness and productivity. Then they compare their simulation results with *conditional* empirical correlations. Their model delivers a more realistic propagation mechanism, both in terms of cross correlations and in terms of impulse response.

8 Concluding Remarks

Shimer (2005) and Costain and Reiter (2007) have called in question the quantitative consistency of the matching models. Considering the relative short period of time elapsed from the publications of these papers, many scholars have reacted to their findings, and with competing approaches.

Shimer's results concern the inability of the model to reproduce realistic fluctuations and propagations of shocks. As far as the propagation problem is concerned, Fujita and Ramey (2007b) and Hagedorn and Manovskii (2007b) are able to ameliorate the dynamics of the model by making vacancies response more sluggish.

As regards the amplification puzzle, scholars have followed three different

routes, that Table 4 summarizes. Unfortunately, it is not possible to adopt a unique criterion in evaluating all the papers: some of them ignore policy analysis issues, others do not report statistics on the real wage, some others present their findings in terms of impulse response functions and not in terms of elasticities or relative standard deviations. Table 4 therefore can give only an idea of the results presented in the previous sections.

However, the general conclusion that may be drawn is that the third approach appears the most effective. A model with turnover costs improve the business cycle fluctuations and fit the policy analysis long-run estimates. Cohort-specific shocks on productivity match data quite well and are empirically plausible. The combination of hiring costs and on-the-job search betters the performance of the matching model also in response to shocks on the separation-rate.

Nevertheless, some questions remain open. To which extent is job separation important in explaining unemployment fluctuations? Can a standard matching model match business cycle facts in other countries, in Europe for instance? The macroeconomic performance of matching models is far from being an exhausted research area.

FIRST APPROACH		
Completely rigid wages	Shimer (2004)	Fit the data if profit share is low. Wage elasticity empirically inconsistent (see Pissarides, 2007; Haefke et al, 2007).
Sticky wages	Farmer and Hollenhorst (2006)	Amplify fluctuations. Decisive the calibrated value for the leisure disutility parameter
Staggered contracts	Gertler and Trigari (2006); Bodart et al. (2006)	Improve the model. Wage elasticity empirically inconsistent (see Pissarides, 2007; Haefke et al, 2007).
Social norm	Hall (2003;2005)	Improve the model. Wages of new hires too sticky (see Pissarides, 2007; Haefke et al, 2007).
Long-term contracts	Rudanko (2007)	No amplification. Wages too rigid in two out of three contract types.
Changing the bargaining threats	Hall and Milgrom (2008)	Fit the empirical conditional moments on u and v. Crucial is the separation during bargaining parameter.
Acyclical informational rents	Brugemman and Moscarini (2007); Guerrieri (2008)	No amplification.
Cyclical informational rent	Kennan (2007); Menzio(2005)	Amplify fluctuations. Kennan's model hard to check empirically.
SECOND APPROACH		
Changes in the calibration	Hagedorn and Manovskii (2007a;b)	Fit the business cycle data. However, the policy analysis consistency of the model is deteriorated and the wage elasticity they target is too low (see Pissarides, 2007; Haefke et al, 2007).
THIRD APPROACH		
Turnover costs	Silva and Toledo (2007)	Fit the model both from a business cycle and from a policy analysis viewpoint.
Hiring freeze	Garibaldi (2006)	Amplify fluctuations
Hiring costs +on-the-job search	Nagypál (2005)	Amplify fluctuations
On the job search + job quality	Krause and Lubik (2006)	Amplify fluctuations
Embodied Technical Change	Costain and Reiter (2007); Reiter (2008)	Fit the model. Excessive wage volatility is corrected by long term wage contracts. The Beveridge curve is restored by introducing turnover costs
Shock on the separation rate + shock on p	Mortensen and Nagypál (2007)	If the bargaining process is as in Hall and Milgrom (2008), both unemployment and vacancies fluctuations are fitted and the Beveridge curve is not destroyed
Endogenous separation rate	Mortensen and Nagypál (2008); Pissarides (2007)	The vacancy fluctuations are not amplified as in the case of a shock in the separation rate
Price rigidity	Andrès et al. (2006)	Along with other amplification mechanisms, it succeeds in matching the data.
VAR approach	Yashiv (2005; 2006)	The separation rate is key in matching data on unemployment and vacancies volatility.
Market power	Rotemberg (2006)	No satisfactory results.
Establishment level shocks	Cooper et al. (2007)	Fit the data.
Monetary policy	Barnichon (2007); Braun (2006); Braun et al. (2007)	Barnichon explains 50% of the Shimer puzzle as a misidentification of shocks. Braun (2006) shows that wage rigidity and a low profit share are crucial even in presence of a monetary shock. Braun et al. (2007) argue that demand shocks are at least as important as supply shocks in driving labour market fluctuations.

Figure 1: The Amplification Puzzle

Acknowledgements

Notes

[1]Pioneered by Diamond, Mortensen, and Pissarides. See Mortensen and Pissarides (1999) and Pissarides (2000) for a detailed exposition.

[2]This point has been first spelled out by Hornstein, Krusell, and Violante (2005) and Mortensen and Nagypál (2007).

[3]The same space limitations do not allow me to review some recent DSGE matching models that bear some relationship with the issues raised in this survey, but whose main scope is not to react to Shimer's findings. For instance, Blanchard and Gali (2005) and Walsh (2005) are more oriented on monetary issues, while Veracierto (2002) focuses on the dynamic behaviour of labour force participation.

[4]Statistics on unemployment are constructed by the BLS from the Current Population Survey (CPS), while measures on job vacancies are proxied using the the Conference Board help-wanted index, that computes the number of help-wanted advertisements in 51 major newspapers. The Job Openings and Labor Turnover Survey (JOLTS) would the ideal source for an analysis of job vacancies, but it collects data only from December 2000. However, a comparison of the two measures from 2000 to 2003 shows that the Conference Board help-wanted index does not differ substantially from the JOLTS.

[5]To measure the job-finding rate, Shimer prefers not to use gross worker flow data, for the dataset is available only since 1976 and measurement and classification errors could bias his estimation. Instead, he constructs it by using the monthly number of unemployed people and assuming that workers are homogeneous and neither enter nor exit the labour force. In a subsequent paper, he shows that relaxing such strong assumption does not bias his computations (see Shimer, 2007).

[6]Hagedorn and Manovskii (2007b) have recently pointed out that the results concerning the correlation between productivity and market tightness are strikingly different depending on using CPS data or the Current Employment Statistics. The reason has still to be ascertained. The papers I will survey in the following sections have followed Shimer (2005) and considered the CES data.

[7]Since the variables are expressed in log, OLS estimation tells us that $\eta_{wp} = \rho_{wp} \cdot \sigma_w/\sigma_p$.

[8]Moreover, Davis and Haltiwanger consider only manufacturing establishments in which job separation is more volatile than in the rest of the economy.

[9] Mortensen and Nagypál (2007) prove that under some conditions, an equilibrium solution defined as a vector of functions ($w_p, \theta_p, U_p, W_p, J_p, S_p$) for any possible value of productivity p exists, is unique, and all the functions are increasing in p. For the proof, I refer to them.

[10] Shimer sets $\lambda = 4$, a relatively large value, but also imposes Δ equal to 0.0083, so the approximation can be accepted.

[11] I compute the semi-elasticity of u with respect to b and not the elasticity simply to follow Costain and Reiter (2007) and compare more easily the results of the model with the estimates presented in section 2.3.

[12] The wage share is equal to $\frac{w}{p} = \beta + \beta \frac{c\theta}{p} + (1 - \beta)\frac{z}{p}$. Hornstein, Krusell, and Violante (2005) show that if r and s are much smaller than $\beta f(\theta)$, $\frac{c\theta}{p} \approx \frac{1-\beta}{\beta}\left(1 - \frac{z}{p}\right)$. So $\frac{w}{p} \approx 1$.

[13] Actually, Layard and Nickell consider the semi-elasticity of unemployment with respect to the replacement ratio. So one would have $b = v \cdot w$ with v being the replacement ratio. However, in the present calibration, $w \approx p = 1$. So $\zeta_{u,b} \approx \zeta_{u,v}$.

[14] Hornstein, Krusell and Violante (2005) show that when wages are rigid the comparative statics elasticity $\eta_{\theta p}$ overestimates the response of θ to a productivity shock. For simplicity, I still compute $\eta_{\theta p}$, considering it as an upper-bound for volatility. Shimer (2004) sets $\alpha = 0.5$. In this case, $\eta_{\theta p} = 60.6$.

[15] Another shortcoming of the rigid wage model is the lack of persistence in vacancies: their autocorrelation is 0.715, while in the data it is 0.930. I will discuss this issue in section 7.

[16] This is what Gertler and Trigari assume. Bodart, Pierrard, and Sneessens impose that a fraction of new jobs can have a freely negotiated wage but their sensitivity analysis shows that such a fraction must be close to 0 in order to have realistic unemployment fluctuations.

[17] Actually, the model gets close to the data only by setting the instantaneous utility of unemployment z higher than 0.8. This confirms again the crucial role played by a low profit share.

[18] Following Mortensen and Nagypál (2007), they disentangle the part of volatility captured by fluctuations in productivity from that beyond the reach of a productivity explanation.

[19] The rationale behind this point goes as follows. If worker's fraction of the total rent is rigid and large, this implies that the capital gain from finding a job is also high. Data show that $f(\theta)$ is extremely volatile and procyclical. But a high job finding rate and a large capital gain in booms will enhance the opportunity cost of employment that, in turn, will generate strongly procyclical wages, so dampening the incentives on vacancy creation during booms. Brügemann and Moscarini call such mechanism the feedback effect. On the other hand, if the rent accruing to the worker is low and acyclical, the same problem analyzed in the rigid wage model occurs. A lower rent going to the employee means large profits for the firm, so that a percentage increase in productivity will enhance them only by a small percentage amount. Firms will not open many vacancies in booms. This second mechanism is called by the authors the congestion effect, because it depends on the free-entry condition that links directly the number of vacancies posted with firms' profits. Both the congestion effect and the feedback effect limit the response of tightness to a productivity shock.

[20] Workers are heterogeneous in terms of sunk, training, costs they incur at the beginning of their employment spell.

[21] Kennan compares the steady-state rate of unemployment both in the bad and in the good state of the economy (denoted respectively, u_1 and u_2) with the corresponding values obtained in a model without asymmetric information. The informational rent moves unemployment by about 40%, even though the difference in productivity levels is only 3%: $u_1 = 5.6\%$ and $u_2 = 5.5\%$ in the case of complete information, whereas $u_1 = 7.5\%$ and $u_2 = 5.2\%$ when productivity is observed privately by the employer.

[22]Menzio also introduces concave vacancy costs. This assumption can by itself magnify the volatility of vacancies.

[23]A part from the values of β and z, other departures from Shimer's calibration are: i) a different matching function: $m(u,v) = u \cdot v / \left(u^l + v^l \right)^{1/l}$, with l being the only parameter to be estimated; ii) a quarterly job filling rate $f/\theta = 2.13$, so that $\theta = 0.634$; iii) the total flow cost of opening a vacancy defined as $c = c^K p + c^W p^{\eta_{wp}}$, with $c^K = 0.474$ being a capital cost and the labour cost c^W equal to 4.5% of quarterly wages of a new hire or 11% of labour productivity.

[24]As stressed in section 3.1, the correlation between tightness and productivity $\rho_{\theta p}$ is 0.393 in the data and close to 1 in the model. So the wage elasticity η_{wp} and the ratio of standard deviations σ_w/σ_p are virtually identical in the model, while in the data the the former is about 60% lower than the latter. In a subsequent paper, Hagedorn and Manovskii (2007b) address this question and show that adding an additional shock to non-market activity to the model allows to target both η_{wp} and σ_w/σ_p. The calibration method they employ is the same as shown in this section.

[25]Hagedorn and Manovskii react to this point, raising some doubts about the endogeneity problems that in cross-country regressions like those performed by Costain and Reiter cannot be ruled out.

[26]Actually, in their simulation part, Silva and Toledo get $\sigma_\theta/\sigma_p = 20$, much larger than $\eta_{\theta p}$ and very close to the ratio σ_θ/σ_p found in the data. Such discrepancy probably depends on the choice of the grid step size $\Delta = 0.053$ and the arrival rate $\lambda = 0.4$ made by the authors. These values are not sufficiently close to 0 for the approximation to hold.

[27]Wages are volatile because the worker' outside option, that depends on the current value of productivity and not on the productivity of the match, is strongly procyclical.

[28]With long-term contracts and $\xi_p = 0.302$, the elasticity of the wages for new entrants with respect to productivity is about 1, but the elasticity of the aggregate wage results too small (less than 0.1). Further, turnover costs reduce the amount of endogenous separations.

[29]A similar effect is obtained by Nagypál (2005) by introducing on-the-job search in the standard model. A higher separation rate encourages firms to post more vacancies because it raises the number of unemployed people, who are the only job seekers in the economy. In a framework with on-the-job search, an increase in s has a smaller impact on the total number of searching workers and the incentive for firms to post more vacancies as s goes up is weakened.

[30]On intra-firm wage bargaining within a matching framework see Cahuc and Wasmer (2001).

[31]He also assumes concave vacancy costs that induce firms to post more vacancies in booms, so amplifying the volatility of such variable. So, comparing Yashiv's and Rotemberg's model, one concludes that concave vacancy costs tend to amplify the shocks but worsen the ability of the model to propagate them. Convex costs engender the opposite effects.

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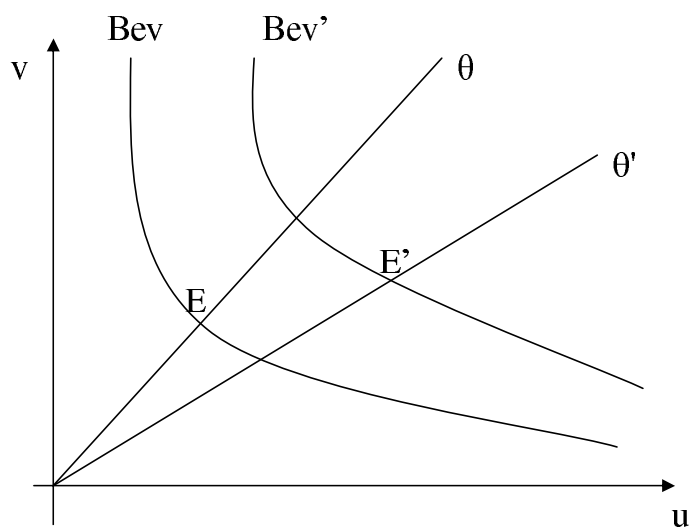——— (2008): "U.S. Labor Market Dynamics Revisited," *Scandinavian Journal of Economics*, 109, 779–806.

Figure 2: An increase in the separation rate. The new equilibrium point is E'.