

Additional file 1 — Solution of γ angle

The γ angle is the necessary angle that the matrix ${}^{acc_0}\tilde{R}_{acc}$ must rotate around the gravity vector in order to estimate the elbow joint position. Thus, the plane formed by the X axis and Y axis (plane Π in Fig. 3 of the paper) must include the known wrist (W) and shoulder (S) points. The γ angle can be also computed by translating W to Π around the gravity vector placed in S . Hence, the new wrist point (\hat{W}) can be computed as

$$\tilde{W} = (g \cdot \hat{W})g + \cos(\gamma) \left(\hat{W} - (g \cdot \hat{W})g \right) - \sin(\gamma) (g \times \hat{W}),$$

where

$$\hat{W} = \frac{(W - S)}{\|(W - S)\|},$$

$$g = [0 \ 0 \ -1]^T.$$

Therefore, the distance between \tilde{W} and Π should be zero and is expressed as

$$d(\tilde{W}, \Pi) = \frac{|A_{\Pi}\tilde{W}_x + B_{\Pi}\tilde{W}_y + C_{\Pi}\tilde{W}_z + D_{\Pi}|}{\sqrt{A_{\Pi}^2 + B_{\Pi}^2 + C_{\Pi}^2}} = 0; \quad (1)$$

where

$$\begin{bmatrix} A_{\Pi} \\ B_{\Pi} \\ C_{\Pi} \end{bmatrix} = \overline{S\tilde{P}_{acc}^y} \times \overline{\tilde{P}_{acc}^x \tilde{P}_{acc}^y},$$

$$D_{\Pi} = [A_{\Pi} \ B_{\Pi} \ C_{\Pi}]^T \cdot S;$$

and

$$\tilde{P}_{acc}^x = {}^{acc_0}\tilde{R}_{acc} [1 \ 0 \ 0]^T,$$

$$\tilde{P}_{acc}^y = {}^{acc_0}\tilde{R}_{acc} [0 \ 1 \ 0]^T,$$

$$\overline{S\tilde{P}_{acc}^y} = (\tilde{P}_{acc}^y - S),$$

$$\overline{\tilde{P}_{acc}^x \tilde{P}_{acc}^y} = (\tilde{P}_{acc}^y - \tilde{P}_{acc}^x),$$

From this point, the resolution of γ is computationally expensive and a simplification is needed in order to estimate the upper limb joints in real time.

Thereby, by defining

$$\begin{aligned} a &= A_{\Pi} \cdot \tilde{W}_x + B_{\Pi} \cdot \tilde{W}_y, \\ b &= A_{\Pi} \cdot \tilde{W}_y + B_{\Pi} \cdot \tilde{W}_x, \\ c &= A_{\Pi} \cdot S_x + B_{\Pi} \cdot S_y + C_{\Pi} \cdot (\tilde{W}_z + S_z) + D_{\Pi}, \end{aligned}$$

and (1) is rewritten as

$$a \cdot \cos(\gamma) - b \cdot \sin(\gamma) + c = 0. \quad (2)$$

Then, multiplying and dividing by the norm of vector \vec{ab} , (2) yields

$$\sqrt{a^2 + b^2} \left[\frac{a}{\sqrt{a^2 + b^2}} \cdot \cos(\gamma) + \frac{-b}{\sqrt{a^2 + b^2}} \cdot \sin(\gamma) \right] + c = 0. \quad (3)$$

On the other hand, as the component of a vector divided by its norm varies between ± 1 , the following approximation can be assumed

$$\begin{aligned} \cos(\eta) &= \frac{a}{\sqrt{a^2 + b^2}}, \\ \sin(\eta) &= \frac{-b}{\sqrt{a^2 + b^2}}. \end{aligned}$$

Hence, (3) can be rewritten as

$$\begin{aligned} m [\cos(\eta) \cdot \cos(\gamma) + \sin(\eta) \cdot \sin(\gamma)] + c &= 0; \\ m \cdot \cos(\eta - \gamma) + c &= 0. \end{aligned}$$

being $m = \sqrt{a^2 + b^2}$. Thus, the desired γ value remains solved as

$$\gamma = \eta - \arccos\left(\frac{-c}{m}\right);$$

where

$$\eta = \text{artg}\left(\frac{-b}{a}\right).$$

Finally, two possible γ values are obtained as

$$\begin{aligned} \gamma_1 &= \text{artg}\left(\frac{-b}{a}\right) - \arccos\left(\frac{-c}{\sqrt{a^2 + b^2}}\right), \\ \gamma_2 &= \text{artg}\left(\frac{-b}{a}\right) - \arccos\left(\frac{-c}{-\sqrt{a^2 + b^2}}\right) - \pi. \end{aligned}$$

These solutions allows us to perform the kinematic reconstruction in real time as the solutions are computed through simple mathematical operations.