The Hayward Tuning Vine: an interface for Just Intonation

Robin Hayward
Audio Communication Group
Technical University Berlin
Einsteinufer 17, Berlin, Germany
robin.hayward@mailbox.tu-berlin.de

ABSTRACT
The Hayward Tuning Vine is a software interface for exploring the system of microtonal tuning known as Just Intonation. Based ultimately on prime number relationships, harmonic space in Just Intonation is inherently multidimensional, with each prime number tracing a unique path in space. Taking this multidimensionality as its point of departure, the Tuning Vine interface assigns a unique angle and colour to each prime number, along with aligning melodic pitch height to vertical height on the computer screen. These features allow direct and intuitive interaction with Just Intonation. The inclusion of a transposition function along each prime number axis also enables potentially unlimited exploration of harmonic space within prime limit 23. Currently available as desktop software, a prototype for a hardware version has also been constructed, and future tablet app and hardware versions of the Tuning Vine are planned that will allow tangible as well as audiovisual interaction with microtonal harmonic space.

Author Keywords
microtonality, musical interfaces, Just Intonation, tuning software, graphic displays

ACM Classification
1998: H.5.5 [Information Interfaces and Presentation] Sound and Music Computing; H.5.2 [Information Interfaces and Presentation] User Interfaces– Graphic User Interfaces (GUI), Input Devices and Strategies, Screen Design; J.5 [Arts and Humanities] – Performing Arts (e.g. dance, music)

1. INTRODUCTION
Just Intonation is sometimes referred to as 'natural' tuning, as distinct from tempered tuning systems. Whereas the musical intervals contained within Just Intonation are always based on whole number frequency ratios, most of the intervals contained within a system of tempered tuning are altered, and therefore deviate from these whole number ratios to a greater or lesser extent. The current dominant tuning in western culture, based on dividing the octave into 12 equal parts, is known as '12-tone equal temperament'. The harmonic space implicit within this system is essentially one-dimensional, as all intervals are restricted to the 12 equidistant points contained within the octave. In contrast to this, the harmonic space opened up by Just Intonation is inherently multidimensional. What counts is relationships between prime factors contained within whole number ratios, and each different prime number opens up a new dimension in harmonic space [5] [6]. An explanation of the significance of prime numbers for Just Intonation is given below in Section 2.

In conceptualizing harmonic space in Just Intonation it therefore no longer makes sense to think of intervals extending only along a single line, rising from low to high, but rather as extending along multiple sets of parallel lines, each set formed by a different prime number. Pitches and their associated ratios and intervals may then appear at equidistant points along these lines. This paper describes how an examination of the full consequences of this conceptualization of harmonic space led to the development of the Hayward Tuning Vine software interface, and how it allows direct and intuitive audiovisual interaction with harmonic space in Just Intonation.

2. THE HARMONIC LATTICE
2.1 Background
Using lattices to visualize two- and three-dimensional harmonic space is fairly commonplace within the field of Just Intonation. This section reviews how such lattices are constructed, and explains how a consideration of their limitations led to the development of the multidimensional Tuning Vine.

2.2 Dimension zero: the 1/1 ratio
The most basic interval in Just Intonation is absolute consonance, from which all other intervals are derived [3]. It is described mathematically as the ratio 1/1 (pronounced 'one to one'), which may be represented as a single point in space, as shown in Figure 1. Such a single point is described geometrically as having zero dimensions.

Figure 1. The ratio 1/1 represented as a single point in space.

2.3 Dimension one formed by prime number two
If the frequency of any given pitch is multiplied by two, the relationship between the resulting and initial frequencies is described by the ratio 2/1, which in the language of conventional music theory is equivalent to the interval of an octave. If for example the initial frequency were A440, then multiplying it by two would result in the frequency A880. The relationship between the resulting and initial pitch would therefore be 880 Hz / 440 Hz, forming the ratio 2/1.

This multiplication by two opens up the first dimension in the geometric representation of harmonic space in Just Intonation. Because the octave extends above the 1/1 unison, it seems logical to display it vertically above it, as shown in Figure 2.

Figure 2. The ratio 2/1 extends an octave above the 1/1.

If the 1/1 frequency is divided rather than multiplied by two, the relationship between the resulting and initial frequencies is described by the ratio 1/2. It would therefore extend an octave below the initial 1/1, as shown in Figure 3.

Figure 3. The ratio 1/2 extends an octave below the 1/1.

2.4 Dimension two formed by prime number three

If the frequency of A440 is multiplied by three, the resulting pitch will be E1320, extending an octave and a Just perfect fifth above it. The ratio between E1320 and A440 will then be 3/1. The multiplication by three opens up the second dimension in the geometric representation of harmonic space in Just Intonation. As the first dimension, based on prime number two, has already been assigned to the vertical axis, it seems logical to assign to prime number three the horizontal axis, opening up a two-dimensional plane between prime numbers two and three. The 3/1 may now be represented geometrically as extending to the right of the 1/1, as shown in Figure 4.

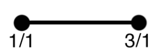

Figure 4. The ratio 3/1, representing an octave and a fifth above the 1/1, is displayed as extending to the right of it.

The ratio 1/3, extending an octave and a fifth below the 1/1, may now be displayed as extending to the left of it. The harmonic space extending along the horizontal axis from 1/3 through 1/1 to 3/1 is shown in Figure 5.

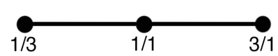

Figure 5. Harmonic space extending along the horizontal axis from 1/3 through 1/1 to 3/1.

Figures 3 and 5 may now be combined to create a two-dimensional model of harmonic space, as shown in Figure 6. The distance between the points is proportional to the size of the musical intervals.

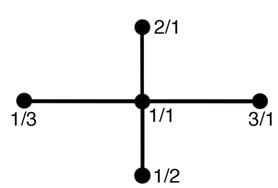

Figure 6. Basic model of two-dimensional harmonic space.

Once the two dimensions have been established, they may be extended indefinitely in any direction: up / down in the case of prime number two, and right / left in the case of prime number three. Figure 7 shows the basic model of two-dimensional harmonic space extended by one step in each direction.

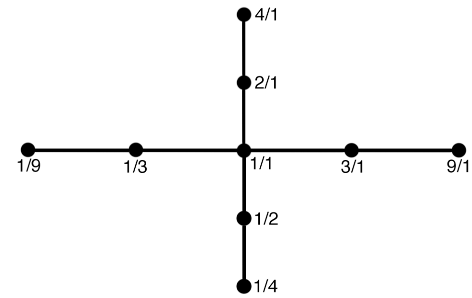

Figure 7. Extended model of two-dimensional harmonic space.

Based on multiplying ratios, this extended model is no longer comprised exclusively of prime numbers. 4/1, equivalent to two octaves, is produced by multiplying 2/1 by itself; 1/9 by multiplying 1/3 by itself etc. If the model were to be extended one step further the ratios would be cubed rather than squared, leading to the ratios 8/1 (equivalent to three octaves), 27/1, 1/8 and 1/27. For every step further along a dimension opened up by a prime number, the initial ratio is raised by a higher order power. This is the reason why it is the *prime* numbers that are important. Numbers such as four and nine, which arise from multiplying a specific prime number together, are by definition already contained within the axis of that prime number.

How might a number such as six, the product of two different prime numbers, fit into this scheme? Figure 8 shows the model further extended into a lattice, in order to include ratios that are the product of the first two prime numbers.

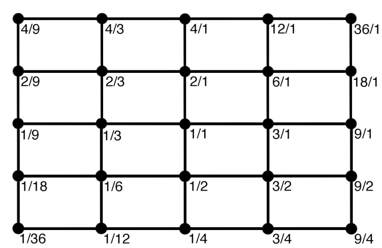

Figure 8. Lattice of two-dimensional harmonic space.

2.5 Dimension three formed by prime number five

The next prime number above three is five, and it is therefore this number that opens up the third dimension in harmonic space. This is illustrated in Figure 9.

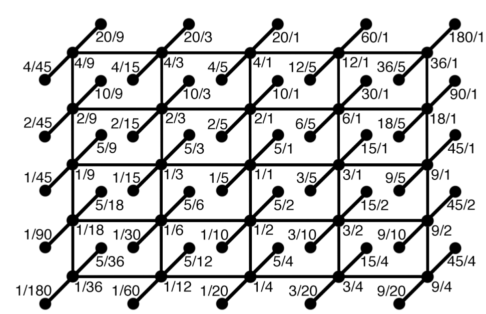

Figure 9. Lattice of three-dimensional harmonic space.

Whilst the cube structure makes the three-dimensionality of the harmonic space relatively easy to follow, such a portrayal remains of fairly limited use, especially when reduced to the two dimensions of a piece of paper or computer screen. Firstly, the relative proportions of the intervals are clearly visible only for the dimensions based on prime numbers two and three. Secondly, it is only the dimension based on prime number two that can be interpreted as accurately reflecting pitch height. It was through considering how to overcome such limitations that the Tuning Vine first came into being.[1]

3. THE DEVELOPMENT OF THE PHYSICAL PROTOTYPE

3.1 The fourth dimension

The three-dimensional lattice that directly preceded the Tuning Vine was slightly different from that shown in Figure 9. There is no reason why the three dimensions need be assigned to the first three prime numbers, and it is fairly common to construct three-dimensional lattices of harmonic space which leave out the octave, allowing prime number seven to be displayed together with prime numbers three and five [1] [7].

The author's *Stained Glass Music*, composed in 2011, takes a subset of such a three, five, seven lattice as the basis for a graphic score, as shown in Figure 10. As it was written for amateur musicians, notating directly by means of the lattice seemed an appropriate way of conveying the idea of microtonal harmonic space to people with widely varying degrees of understanding of conventional music theory. The colour-coding – yellow for prime number three, red for prime number five, and blue for prime number seven – was also introduced primarily for pedagogical reasons.

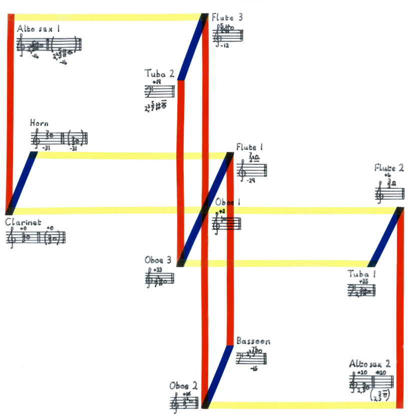

Figure 10. Excerpt from the graphic score *Stained Glass Music*.

As the octave is not present in this lattice, the precise octave positions of the microtonal pitches are not defined by the lattice itself. Instead, the octave positions of the various pitch classes are indicated by the inclusion of conventional music notation.

The first question that arose after the premiere of *Stained Glass Music* was how octaves might be included within this seven-limit lattice. Such a lattice, based on the prime numbers two, three, five and seven, implies four-dimensional space. It therefore pointed in the direction of a four-dimensional hypercube, or tesseract, as depicted in Figure 11.

[1] From an early stage in its development, a further incentive in developing the Tuning Vine was to visualize the harmonic space implicit within a microtonal tuba, developed together with the musical instrument makers B&S in 2009.

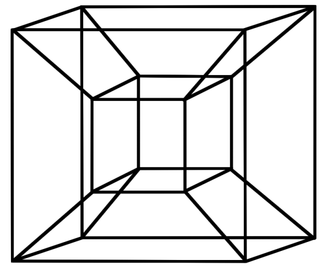

Figure 11. The tesseract could act as a possible model for four-dimensional harmonic space.

3.2 Using Zometool to model harmonic space

The most immediate problem that arises from attempting to model harmonic space on the tesseract is overcrowding. In order to be of much musical use, both the smaller and larger cubes portrayed in Figure 11 would have to be directly connected to neighbouring cubes. This would very quickly result in visual chaos, especially when reduced to the two-dimensional space of the computer screen.

In searching for a solution to this problem, I began experimenting with the children's toy Zometool.[2] Zometool consists of sets of coloured plastic struts of varying lengths that may be connected to each other at a wide variety of angles via plastic balls. It is therefore an ideal tool for the exploration of hyperspace not only in four dimensions, but in higher dimensions as well.

Being able to physically move the lattices around in three dimensions proved to be enormously helpful in exploring possible models for harmonic space. Solutions started suggesting themselves not just to the issue of overcrowding, but also to the other limitations of the three-dimensional lattice outlined above, namely of the correspondence between interval size and geometric distance, and of aligning all pitches (not just those contained within the octave axis) with their corresponding vertical pitch heights.

3.3 Matching strut length to interval size

Starting out by constructing a square using Zometool struts 5.72cm long, it soon became clear that, even with only the third and fourth dimensions added, the lattice would become too full to allow a clear overview of the pitch relationships. The next size up of Zometool strut available – 10.35cm – led to a larger square that helped ameliorate the overcrowding issue, but also to an unwieldily large size for the overall lattice. I therefore started experimenting with a rectangle with the dimensions 7.49 x 12.12cm.[3]

Although these proportions are not identical to those contained within the two-dimensional lattice in Figure 8,[4] as a working model they offered a close approximation. Attempting to open up space for higher primes had therefore inadvertently also suggested a way of incorporating some degree of correspondence between interval size and geometric distance within the Zometool model.

[2] Product details are available at http://www.zometool.com

[3] The Zometool balls have a diameter of 1.77cm, and therefore a radius of 0.885cm. As a ball is placed at each end of the strut, two radius lengths (i.e. one diameter) must be added when calculating the dimensions of the resulting rectangle.

[4] In calculating the correspondence between interval size and geometric length, tempered semitones are used as a grid against which Just tunings are measured. As 7.49cm is defined as being equivalent to 12 semitones, the perfect 12th, comprised of 19 tempered semitones plus two cents, would ideally be equivalent to 7.49cm x 19.02/12 ≈ 11.9cm.

Maintaining such a relationship becomes increasingly difficult for higher prime numbers. Assigning the 2/1 octave ratio to 7.49cm implies a length of c. 11.9cm for the 3/1 ratio, c. 17.3cm for the 5/1 ratio, and c. 21cm for the 7/1 ratio.[5] Prime numbers above seven would of course require even longer lengths.

Although the original intention had been to construct a model of four-dimensional harmonic space following on from the composition of *Stained Glass Music*, the sheer variety of angles and struts made available by Zometool suggested that there was no reason to stop at prime number seven, assuming the problem of overcrowding could be overcome. The issue of the extremely long lengths required for higher prime number ratios therefore either needed to be resolved, or the correspondence between the ratios and the geometric lengths abandoned.

It was through contemplating this problem whilst rotating the two-dimensional lattice in space that a solution first presented itself. Rotated by 90 degrees, with the 10.35cm struts now positioned vertically and the 5.72cm struts positioned horizontally, it became clear that if the vertical axis remained assigned to prime number two, and the horizontal axis to prime number three, then the relative lengths of the struts would closely correspond to the intervals 2/1 (the octave) and 3/2 (the perfect 5th).[6] The 3/1 octave-and-a-fifth intervals at which the pitches are placed along the horizontal axis in Figure 8 thus became reduced to 3/2 perfect fifth intervals, as shown in Figure 12.

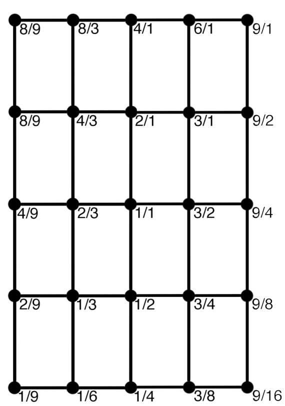

Figure 12. Two-dimensional lattice extending at perfect fifth 3/2 intervals along the horizontal axis (compare Figure 8).

The principle of transposing intervals to within an octave could now be applied to all higher primes, making the need for unrealistically long strut lengths redundant. So for example the 5/1 ratio could be reduced to a 5/4 ratio, and the 7/1 ratio reduced to a 7/4 ratio etc.

3.4 Aligning pitch height to vertical height

In the process of exploring how the struts corresponding to 5/4 and 7/4 could fit into this two-dimensional lattice, I continued rotating it in space. When it was positioned at around 45 degrees anticlockwise, I observed that the vertical position of all the ratios contained within the lattice now corresponded to their relative melodic pitch heights. This is shown in Figure 13.

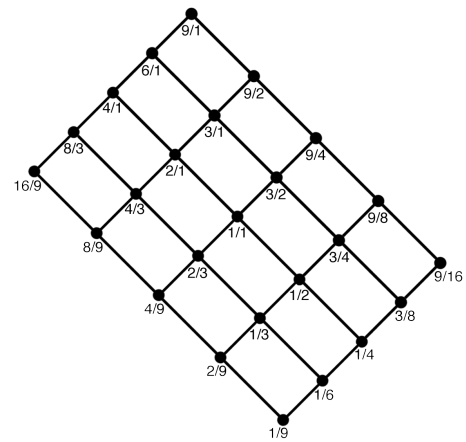

Figure 13. Two-dimensional lattice rotated so as to facilitate vertical alignment of relative pitch heights.

In contrast to Figures 8 and 12, in which only the axis based on prime number two bears any relation to pitch height, both axes are now aligned to reflect the melodic pitch height of the tones contained within them. From this point on it was simply a question of fitting the appropriate struts at the correct angles, in order to construct a multidimensional model that visualized not only harmonic space, but the relative melodic heights of the pitches contained within that space as well. Of all its features, this is the one that makes the Tuning Vine so intuitive to use and distinguishes it most from previous attempts to visualize harmonic space through constructing multidimensional lattices.[7]

Figure 14 shows a subset of the original prototype of the Tuning Vine made from Zometool parts. It includes the first four dimensions of harmonic space based on prime numbers two, three, five and seven.

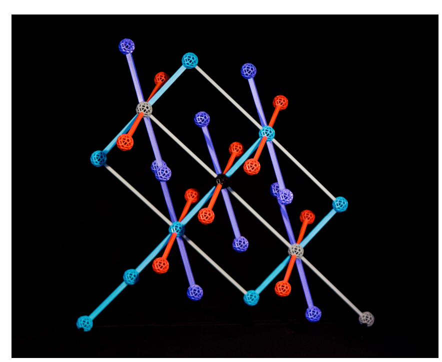

Figure 14. Subset of Tuning Vine showing dimensions based on two, three, five and seven (photograph by Tania Kelley).

[5] The geometric length corresponding to 5/1, two octaves and Just major third, is calculated as 7.49cm x 27.76/12 ≈ 17.3cm, and that corresponding to 7/1, two octaves and a septimal minor seventh, as 7.49cm x 33.69/12 ≈ 21cm.

[6] As the octave would now correspond to 12.12cm, the correct length of the horizontal perfect fifth struts would ideally be 12.12cm x 7.02/12 – 1.77cm ≈ 5.3cm.

[7] The Mexican / American music theorist Erv Wilson has constructed a wide variety of models of harmonic space, but they are generally not colour-coded and, more importantly, do not relate the melodic pitch height of the ratios to vertical height for lattices above two dimensions. Further information on his work may be found at http://anaphoria.com/wilson.html

3.5 Colour-coding

As in the score of *Stained Glass Music* shown in Figure 10, the prime numbers contained within the Tuning Vine are colour-coded. The need for colour-coding as an additional visual aid becomes increasingly acute the more prime numbers are included. Figure 15 shows the first fully developed prototype of the Vine, which includes the first nine prime numbers: 2, 3, 5, 7, 11, 13, 17, 19, and 23.

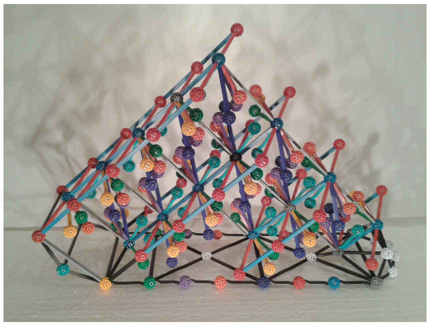

Figure 15. The first complete prototype of the Tuning Vine.

The choice of which colour to link to which prime[8] was made through a process of loose association rather than according to any strict system.[9] The ball corresponding to the central 1/1 is black because it is the origin from which all other intervals are derived.[10] Prime number two is colour-coded grey because of the octave's close association with unison; despite the change in frequency, the pitch class remains unaltered.

Prime number three is coded light blue as it opens up the two-dimensional lattice which may be regarded as the rational foundation upon which the other intervals may be placed. Prime number five is coded red because it contains the intervals of the major and minor thirds, often associated with emotion in music. Dark blue is assigned to prime number seven because it is this number that underlies the harmonies distinct to blues music.

Prime number 11 corresponds on the Vine to 11/8, almost exactly equivalent to a tempered perfect fourth plus quarter-tone. Orange seems an appropriate colour for this interval which might be perceived as being glowing and hot. The next prime number, 13, opens up the intervals of the neutral third 16/13 and neutral sixth 13/8. At the edge of the visible colour spectrum, violet seems to correspond well to intervals lying so far outside the traditional paradigm of western harmony.

Prime numbers 17 and 19 return to more familiar territory, respectively opening up the intervals of the semitone 17/16 and the minor third 19/16, both of which are within five cents of equal tempered tuning. They are therefore assigned the more prosaic colours green and yellow. The highest prime number contained within the current version of the Tuning Vine is 23. Opening up the interval 23/16, the colour turquoise, based on a mixture between two of the colours used for lower primes, seems an appropriate choice for this unfamiliar interval, 28 cents larger than a tempered augmented fourth [2].

3.6 Limitations of the physical prototype

The white balls in Figure 15 are 'function' balls, and the coloured balls along the base of the Vine 'transposition' balls. Their presence is indicative of the fact that the prototype was originally intended as a fully functional hardware interface, to be used in combination with computer software. Each ball in the Vine is fitted with an RFID (radio-frequency identification) tag, which when triggered by an RFID receiver fixed at the end of a 'sound wand', results in the associated microtone being played by the computer software.

Using RFID technology has the advantage that the physical body of the Tuning Vine can remain free from cabling. By using two sound wands, it is possible to play the Tuning Vine from either side, reminiscent of playing a harp, with the undertones operated by the wand held in the left hand and the overtones operated by the wand held in the right hand.

Nevertheless, the resulting physical interface remains far from satisfactory. Perhaps the most serious problem is that it provides no visual clue as to which balls are currently sounding, making it difficult to keep track of which microtones have been activated. Though fitting each ball with LEDs that light up when activated would be theoretically possible, this would mean fitting the entire Vine with cabling, for which the Zometool parts are not designed.

Although the wands are quite effective in sounding the microtones, the feeling when using them remains more one of operating rather than playing an instrument. It is also only possible to turn pitches on and off, allowing no control over the sound while it is actually resonating. Finally, the structure is not nearly robust enough to withstand regular transport and everyday use. The Zometool model provided the playground in which the Tuning Vine came into being, but once built its main purpose would be to act as a prototype, on the basis of which truly interactive versions of the Tuning Vine could be developed.

4. THE SOFTWARE INTERFACE

The first challenge in developing the software was how to further reduce nine-dimensional harmonic space from the three dimensions of the physical prototype to the two dimensions of the computer screen.[11] Displaying the two-dimensional space of the octave / fifth lattice was of course no problem, as is shown in Figure 16.

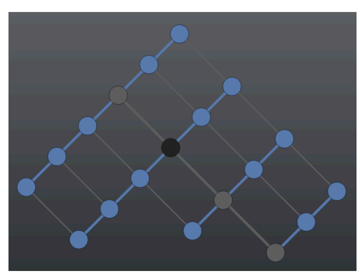

Figure 16. Two-dimensional octave / fifth lattice displayed within the Tuning Vine software.

[8] Harry Partch also uses colours in order to help the player know which ratio each key represents on his Chromelodien. His colour scheme, which he describes as being "purely arbitrary", does not go beyond prime number 11 [4]. I was unaware of Partch's colour scheme when the Tuning Vine was being developed.

[9] A more scientific approach to colour-coding would be to take account of light frequency spectrum to link colours and primes. But as the intention was to develop an interface for intuitive human interaction, it seemed more appropriate to opt for a coding based on psychological association.

[10] Whilst the central 1/1 ball could equally well have been coloured white, this would have been impractical when applying the colour-coding to a system of notation for Just Intonation, white being invisible against a white background.

[11] The software was developed by Robin Hayward, Erik Jälevik and Björn Næsby Nielsen between September 2013 and April 2014. The application code is C++, following the 'Model/View/ Controller' programming pattern. Qt, libpd (Pure Data), and Portaudio are used as open-source library components. Further information may be found at http://www.tuningvine.com

Rather than attempting to portray prime number five as the third dimension, the decision was made to abandon the red struts shown in Figures 14 and 15 and portray only the red balls corresponding to the ratios. This is shown in Figure 17.

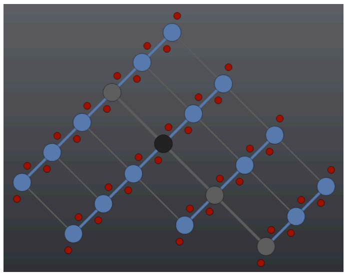

Figure 17. Three-dimensional octave / fifth / third lattice reduced to the two dimensions of the computer screen.

Although reduced to two dimensions, the spatial positioning of the balls in combination with the colour-coding means that this portrayal of three-dimensional harmonic space remains fairly easy to follow. The 5/4 overtone ratios corresponding to the Just major third are now portrayed by the small red balls above and to the right of larger balls, and the 4/5 undertone ratios are below and to the left of them.

The decision not to include connecting struts beyond prime number three also addresses the problem of overcrowding, that would very quickly become an issue once the higher primes are introduced. The balls may be played by clicking on them, and they remain highlighted until clicked off. A card also appears to the right of the lattice containing the pitch's musical and scientific notation, cents deviation from tempered tuning, ratio and hertz number. If multiple pitches within a narrow melodic range are sounded, their corresponding cards will overlap. The user may then bring any desired card to the foreground by hovering the mouse over its associated ball. Figure 18 shows the full virtual Tuning Vine extending to prime number 23 with five microtones sounding.

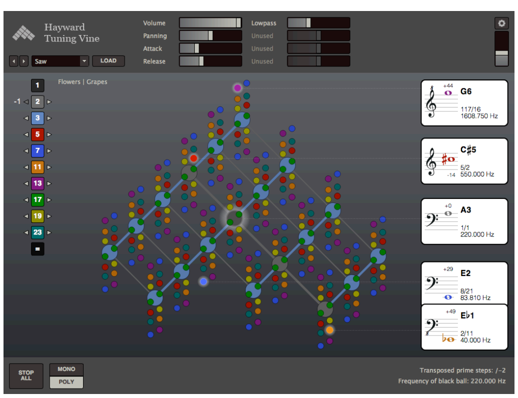

Figure 18. Nine-dimensional harmonic space reduced to two dimensions.

The arrows next to the column of number boxes to the left of the computer screen allow the user to transpose independently along each prime number axis. In Figure 18, for example, the Vine has been transposed down an octave by clicking once on the arrow to the left of the grey number box. This feature of multidimensional transposition makes possible the virtually unlimited exploration of Just Intonation within prime limit 23, throughout the entire audible frequency range from 20Hz to 20kHz.

Toggling between 'MONO' and 'POLY' modes at the bottom left of the screen makes it possible to choose between playing melodies and building chords; in the former case each pitch turns off automatically when the next pitch is sounded, and in the latter case each pitch remains sustained until actively turned off. The remaining functions, placed along the top of the screen, allow the user to select the wave form, load a patch from the open source visual programming language Pure Data, alter the volume, panning, attack and release times, as well as adjust a low-pass filter to alter the overtone spectrum of each individual microtone. By clicking on the 'options' symbol above the main volume slider in the top right-hand corner, it is possible to set the central 1/1 ratio to any frequency. The software is therefore not restricted to the default setting of A440.

The first version of the Hayward Tuning Vine software was released in April 2014, almost exactly two years after the Zometool prototype was first developed.

5. DISCUSSION AND CONCLUSION

Through its incorporation of melodic pitch height within a colour-coded two-dimensional lattice of multidimensional harmonic space, the current software version of the Tuning Vine goes a long way towards allowing direct and intuitive audiovisual interaction with Just Intonation. Along with acting as a tuning interface for musicians and composers, it is also a powerful pedagogical tool for teaching and learning about Just Intonation. Nevertheless there is still much room for improvement, particularly with regard to corporeal interaction. The visual tracking of transposition functions for prime number five and above could also be made easier to follow.

Along with software updates to the current desktop version, a tablet app version is currently being planned that will respond to touch and gesture rather than mouse clicks, allowing the simultaneous sounding of multiple pitches along with a greater degree of corporeal interaction. For example, the transpose function could respond to sweeping gestures to ‘climb’ through the Vine along the axis of the relevant prime number, making it much more intuitive than the current desktop version.

The original idea of making a hardware version that could function as a fully developed musical instrument has by no means been abandoned. Although technically challenging to develop, such an instrument would make the interface not just audiovisual, but tangible as well. Because it takes the multidimensionality of harmonic space as its point of departure, the Tuning Vine could potentially become a standard interface for Just Intonation, much in the same way as the piano keyboard has become a standard interface for 12-tone equal temperament.

6. ACKNOWLEDGMENTS

My thanks to Paul Hildebrandt, Erwin Kotzab and Lyn Taylor from Zometool for their support in developing the physical prototype for the Tuning Vine.

7. REFERENCES

[1] D.B. Doty. *The Just Intonation Primer*, Third Edition. www.dbdoty.com/Words/Primer1.html, 2002, 52.

[2] A.J. Ellis. On the calculation of cents from interval ratios. In H. Helmhotz, *On the Sensations of Tone*, trans. A.J. Ellis, Dover Publications, 1954, 446-451.

[3] H. Partch. *Genesis of a Music*, Second Edition, Enlarged. Da Capo Press, 1974, 87.

[4] H. Partch. *Genesis of a Music*, Second Edition, Enlarged. Da Capo Press, 1974, 214-215.

[5] J. Tenney. The Several Dimensions of Pitch. In C. Barlow (ed.), *The Ratio Book*, Feedback Studio Verlag, 2001, 102-115.

[6] J. Tenney. John Cage and the Theory of Harmony. In P. Garland (ed.), *The Music of James Tenney*, Soundings Press, 1984, 70.

[7] M. Vogel. *On the Relations of Tone*, trans. V.J. Kisselbach. Verlag für systematische Musikwissenschaft, 1993, 125.