Nonlinear Acoustic Synthesis in Augmented Musical Instruments

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ABSTRACT

This paper discusses nonlinear acoustic synthesis in augmented musical instruments via acoustic transduction. Our work expands previous investigations into acoustic amplitude modulation, offering new prototypes that produce intermodulation in several instrumental contexts. Our results show nonlinear intermodulation distortion can be generated and controlled in electromagnetically driven acoustic interfaces that can be deployed in acoustic instruments through augmentation, thus extending the nonlinear acoustic synthesis to a broader range of sonic applications.

Author Keywords

acoustic synthesis, acoustic signal modulation, intermodulation, acoustic sound module, augmented instrument, harmonic oscillators, nonlinear interfaces, acoustic waveshaping

ACM Classification

H.5.5 [Signal analysis, synthesis, and processing]—Sound and Music Computing, H.5.5 [Modeling]—Sound and Music Computing, J.2 [Physics]—Physical Sciences and Engineering

1. INTRODUCTION

Acoustic instruments and speakers exhibit circumstantial nonlinear distortions, generating additional frequency components [11]. In speaker design, for instance, these distortions are seen as undesirable, spurious, and warranting mitigation [8]. Instead of suppressing these nonlinear distortions, we discuss several ways nonlinear distortions can be controlled in simulated and experimental physical systems. Specifically, we define nonlinear acoustic synthesis as the amplification, excitation, and control of frequency content not salient or present in the original signal or excitation sources of any acoustic instrument.

At the center of this investigation is intermodulation (IM), a phenomenon summarized as the high-order sum and difference of frequency harmonics generated by a nonlinear system. Unlike mixing amplifiers and wave-shaping circuits that produce intermodulation in an electrical system, we are primarily interested in producing intermodulation through acoustic transduction via solids, gases, and fluids. One can observe nonlinearities that arise in analog electronics and engineer physical systems as comparable since they obey similar mathematical constructions. With control over the finer mechanics of intermodulation in specific conditions, it is possible to enhance and control rich nonlinear timbre modification in augmented acoustic instruments.

The sections of this paper are structured as follows: We discuss the motivations for nonlinear acoustic synthesis and prior work in Section 2, and we present several implementations of nonlinear acoustic instrument prototypes in Section 3. Section 4 constructs the mathematical framework of intermodulation and demonstrates the machinery required to synthesize and control intermodulation products in terms of frequency and amplitude. Section 5 presents the main experimental methods which are evaluated and discussed in section 6, finishing with conclusions in Section 7.

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2. MOTIVATION AND PRIOR WORK

2.1 Motivation

Due to the easy access to large-scale audio storage, there is reduced demand in music creation for emulation of musical instrument timbres via FM synthesis for instrument timbres [5], voice [6], and other novel digital synthesis techniques, such as Physical Modeling Synthesis [1, 12]. Yet, it is possible to apply sophisticated digital processing techniques in acoustic systems via augmentation to transform nearly all acoustic instruments into expressive modular synthesizers, extending both timbre and control. Bridging the virtual to the physical is thus the primary motivation for this work, offering new ways contextualize electronic music in the acoustic domain. Additionally, the method used to bridge the virtual and physical should be readily applicable to existing instruments and interfaces.

2.2 Prior Work

Modulation is often used in sound synthesis to reduce the number of oscillators needed to generate complex timbres by producing additional signal components prior to the output. For example, FM Synthesis is employed to emulate rich timbres of acoustic instruments [5]. Another method of modulation synthesis is via Intermodulation (IM), a form of amplitude modulation acting on the signal harmonics from two or more injected signals. Harmonics arise from nonlinearities intrinsic to electrical or physical systems.

IM is found in electrical systems such as amplifiers and effects for musical purposes. For example, "power chords" played on electric guitars are an effect resulting from IM within an over-driven mixing amplifier [2]. IM can produce strong subharmonics by injecting two harmonic-rich signals into a nonlinear electrical amplifier. We propose here a mechanical method that generates intermodulation and parametric acoustic timbres in an acoustic system, such as an augmented musical instrument or effect systems.

3. NONLINEAR ACOUSTIC SYNTHESIS

3.1 General Model

We proposed a modular instrument exhibiting intermodulation shown in Figure 3 [3]. Expanding on this work we generalize our approach aimed at designing or modifying musical instruments capable of producing controllable nonlinear acoustic synthesis. Figure 1 illustrates the basic model

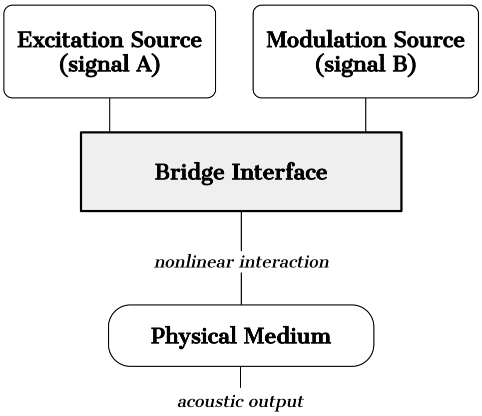

Figure 1: Block Diagram Model for Nonlinear Acoustic Synthesis.

for nonlinear acoustic synthesis, where the primary system components consist of an excitation source and a modulation source that mix in a bridge interface as to produce nonlinear synthesis between a bridge and a physical medium such as a gas, solid, or liquid. Such mediums in musical instruments are included those categorized as idiophones, membranophones, chordophones, and aerophones [13]. The acoustic output is then simply the acoustic mechanism inherent in the modified instrument or design, such as a resonator, waveguide, or vent.

We define the specific elements of the block diagram the following way:

- **Excitation Source:** An arbitrary signal source $\boldsymbol{A}$ acoustically transduced into a bridge interface.
- **Modulation Source:** A signal source $\boldsymbol{B}$ capable of interaction with the excitation source within the bridge interface.
- **Bridge Interface:** The bridge interface provides the critical mixing stage for the two signals $\boldsymbol{A}$ and $\boldsymbol{B}$ respectively, whose coupled behavior with the physical medium produces nonlinear acoustic synthesis
- **Physical Medium:** The matter and materiality in which the signals interact, such as air, metal, wood, plastic, etc.

The exact order and interaction between these components is left undefined, since the context for implementation greatly effects the design. Instead this general model illustrates what we consider to be the minimum number of elements needed to produce nonlinear acoustic synthesis.

3.2 Aerophone Prototype

Figure 2 shows the complete aerophone prototype. In this system, a cabinet houses a Dayton Audio polyimide compression horn driver that generates a continuous signal A, which is focused in a 61cm cylindrical resonator, at the end of which is an articulated damper attached to a voice coil motor (VCM) generating a periodic signal B, detailed in Figure 2.

The basic concept for the instrument is to generate intermodulation through periodic damping of sound propagation, and thus can be generalized and embedded into any wind instrument where the primary port is located at the end of the instrument.

3.3 Idiophone and Membranophone Prototypes

Idiophone and Membranophone instrument prototypes were implemented with equivalent functional design as the SideBand modular acoustic synthesizer shown in Figure 3. The system consists of a tuning fork that behaves as a Harmonic Oscillator (signal A) when driven by an electromagnetic actuator. The tuning fork acoustically transduces a signal into a T-Frame bridge interface. A voice coil motor generates the modulation source (signal B) that effects the interaction between the bridge and soundboard. The entire system and function is further discussed in [3].

To test the membranophone condition, we simply replaced the soundboard with a drumhead, allowing the bridge to interact directly with the membrane at a distance determined through experimentation. Likewise, for the idiophone condition simply required us to use the original soundboard from the SideBand instrument. As an extension of this model, we also replaced the wooden soundboard with low-carbon steel and acrylic soundboards to test the specific material properties in relation to nonlinear acoustic synthesis.

4. THEORY OF INTERMODULATION

IM was first discovered in the context of radio engineering, and subsequently in speaker design along with other engineering contexts [7]. For the sake of generality, we first analyze intermodulation as a mathematical construct between two harmonic oscillators, $A\cos(\omega_a t)$ and $B\cos(\omega_b t)$, or Oscillator A and B, respectively.

When a signal passes through an arbitrary nonlinear system, harmonic multiples appear. For example, if the input signal of a nonlinear system is $\cos\omega t$, then there will be output signals consisting of the harmonics $\cos 2\omega t$, $\cos 3\omega t$, $\cos 4\omega t, \ldots \cos N\omega t$. N will later be defined as the order of intermodulation. In the case where two or more signals are injected into a nonlinear system, as is the case with a harmonic oscillator, then all harmonics will be present. Amplitude modulation then occurs on both the injected signal and their respective harmonics, as defined in Appendix A.1.

When two harmonic oscillator A and B inject signals $A\cos(\omega_a t)$ and $B\cos(\omega_b t)$ respectively into a nonlinear system, the output signal contains IM products. The products are dependent on the order of intermodulation. The IM products and harmonics of signals A and B can be summarized by their order the following way:

Table 1: IM Products

First Order	ω_a, ω_b
Second Order	$2\omega_a, 2\omega_b, \omega_a + \omega_b, \omega_a - \omega_b$
Third Order	$3\omega_a, 3\omega_b, 2\omega_a + \omega_b, 2\omega_a - \omega_b$ $2\omega_a + \omega_b, 2\omega_a\omega_b$

IM products can be up to any order. More generally, let N denote the order of IM. Then the IM products will have frequencies $k_a\omega_a \pm k_b\omega_b$ where $k_a + k_b \leq N$ [3]. Additionally, $k_a\omega_a - k_b\omega_b = -(k_a\omega_b - k_b\omega_a)$, which means IM products with negative frequencies appear as their positive frequency components acoustically.

Figure 2: Aerophone Prototype Instrument (right), voice coil motor attached to an articulated damper (left).

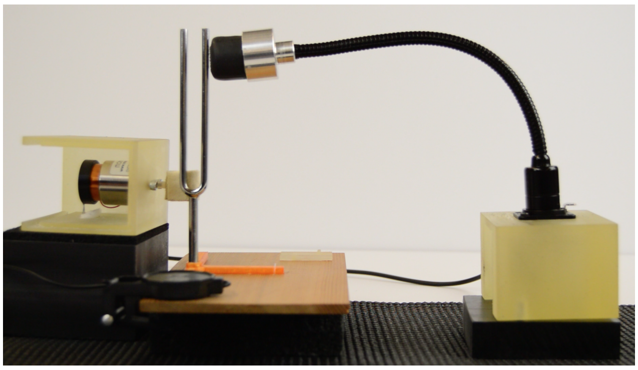

Figure 3: Syrinx Sideband Modular Acoustic Synthesizer [3].

4.1 IM Product Frequency and Amplitudes

4.1.1 Mathematical formulation

Appendix A.2 details the specifics of harmonic frequency interaction. This section discusses how input signals affect output amplitudes, and constructs a framework to discuss results in Section 6. Nonlinear interactions depends on the physical system itself, hence a deterministic model for the amplitudes generated from modulation is complex and system specific.

In general, a system's nonlinearities can be expressed using a polynomial transfer function. If S_{out} and S_{in} are the output and input signals respectively, then the transfer function is written as

$$S_{out} \sim K_1 S_{in} + K_2 S_{in}^2 + K_3 S_{in}^3 + K_4 S_{in}^4 ... = \sum_i^{\infty} K_i S_{in}^i \tag{1}$$

A linear transfer function would mean the output is exactly the input multiplied by scalar coefficient K_1. In other words, $K_2, K_3, K_4, ... = 0$ and $S_{out} = K_1 S_{in}$. In a nonlinear system other coefficients are not uniformly zero. For instance, suppose the input signal is $S_{in} = \cos \omega t$. Then the second term expansion yields

$$K_2 (\cos \omega t)^2 = \frac{K_2}{2}(1 + \cos 2\omega t)$$

The frequency of 2ω accounts for the harmonics produced in the nonlinear system as described in Section 4.

In the scenario of two tone intermodulation, the input signal is a sum of Harmonic Oscillator A and Harmonic Oscillator B, given by

$$S_{in} = A \cos \omega_a t + B \cos \omega_b t$$

Likewise a third-order system would have the following output signal:

$$\begin{aligned} S_{out} \sim K_1(A \cos \omega_a t + B \cos \omega_b t) + K_2(A \cos \omega_a t \\ + B \cos \omega_b t)^2 + K_3(A \cos \omega_a t + B \cos \omega_b t)^3 \end{aligned} \tag{2}$$

The calculations are long and technical [10], and is the sum of components illustrated in Table 4.1.1.

Table 2: Third-Order Transfer Function Expansion

Frequency Components	and their amplitudes
Fundamental term	$(K_1 A + {}^3/_2 K_3 A B^2 + {}^3/_4 A^3) \cos \omega_a t$
	$(K_1 B + {}^3/_2 K_3 A^2 B + {}^3/_4 B^3) \cos \omega_b t$
2^{nd} Harmonic Term	${}^1/_2 K_2 A^2 \cos 2\omega_a t$
	${}^1/_2 K_2 B^2 \cos 2\omega_b t$
2^{nd}-Order IM Products	$K_2 A B \cos(\omega_a - \omega_b)t$
	$K_2 A B \cos(\omega_a + \omega_b)t$
3^{rd} Harmonic Term	${}^1/_4 K_3 A^3 \cos 2\omega_a t$
	${}^1/_4 K_3 B^3 \cos 3\omega_b t$
3^{rd}-Order IM Products	${}^3/_4 K_3 A^2 B \cos(2\omega_a + \omega_b)t$
	${}^3/_4 K_3 A B^2 \cos(2\omega_b + \omega_a)t$
	${}^3/_4 K_3 A^2 B \cos(2\omega_a - \omega_b)t$
	${}^3/_4 K_3 A B^2 \cos(2\omega_b - \omega_a)t$

These are grouped by their frequency components, where their coefficients are the aggregate power of the signal. Harmonic terms are terms where the frequency is an integer multiple of the orginal frequency, while nth-Order terms are a linear combination, known as sum and difference IM Products.

We discuss a specific case using ω_a = 200Hz and ω_b = 201Hz as an example. If we were to only look at the odd-numbered difference IM Products, we obtain Table 3

Table 3: Odd IM Products

1st Order	ω_a	ω_b	200	201
3rd Order	$2\omega_a - \omega_b$	$2\omega_a - \omega_b$	199	202
5th Order	$3\omega_a - 2\omega_b$	$3\omega_a - 2\omega_b$	198	203
7th Order	$4\omega_a - 3\omega_b$	$4\omega_a - 3\omega_b$	197	204

This is compared to the amplitude modulation from Oscillator-B's harmonics when $\omega_b = 1$:

These are the same except for the upper sideband value, which differences by 1Hz. The exact mathematical comparison is included in Appendix A.2 in terms of frequency

Table 4: Amplitude Modulation from Harmonics

ω_a	ω_b	200	1
$\omega_a - \omega_b$	$\omega_a + \omega_b$	199	201
$\omega_a - 2\omega_b$	$3\omega_a + 2\omega_b$	198	202
$\omega_a - 3\omega_b$	$\omega_a + 3\omega_b$	197	203

difference d, and is useful in characterizing the symmetry in sweep tests in Figure 5.

These findings are consistent with simulations. We performed simulations using SimRF in Matlab [9], a robust radio engineering package. The simulated results show 11^{th}-Order Intermodulation with two tarmonic oscillators, with signals $S_A(t) = 2\cos(30t)$ and $S_B(t) = 0.5\cos(20t)$, with a modulation index of $\beta = 0.25$. The modulation index is defined as the modulator signal's peak amplitude over the carrier, where S_A acts as the carrier and S_B the modulator.

$$\beta = \frac{\text{Amplitude of Modulator}}{\text{Amplitude of Carrier}} = \frac{B}{A} \tag{3}$$

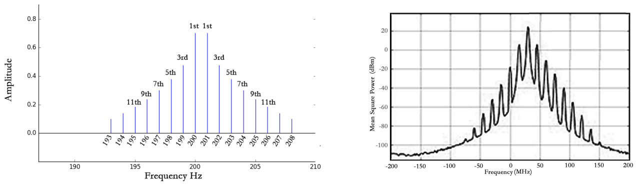

Figure 4: Theoretical & Simulated Odd Difference IM Products.

4.1.2 Additional tones

The same equations are applied when adding an additional signal component, either through another oscillator or more complex signals in Oscillator-B. For instance, if we were to add Oscillator C, the intermodulation products will produce frequencies:

$$k_1\omega_a + k_2\omega_b + k_3\omega_c \quad \text{with } k_1 + k_2 + k_3 \leq N \tag{4}$$

where N is the order of intermodulation. The amplitudes are calculated in a similar fashion, with an input signal of

$$S_{in} = A\cos\omega_a + B\cos\omega_b + C\cos\omega_c$$

The expansion becomes even more complicated using Equation 1, but the frequency components can be shown consistent with Equation 4 via trigonometric manipulation and grouping like terms.

5. EXPERIMENTAL METHODS

5.1 Experimental Setup

During our investigation, we designed an IM prototype based on the SideBand instrument described in [3]. The force-body mechanics of the IM prototype are equivalent to those described in [3, 4]. The change was made primarily to access greater degrees of control in uncovering the source of nonlinearity and achieve this control through different bridge mediums and soundboard materials.

Primary modification of this system was the replacement of the tuning fork with a piezoelectric transducer, which provided a comparable signal to the tuning fork and acts as Oscillator A. To effect the bridge-to-surface damping we activated a sympathetic bridge using a voice-coil motor in an orthogonal position to the sympathetic bridge. The voice-coil motor acts as Oscillator B. The carrier-modulator-bridge unit is the primary component of experimentation, each receiving two signals from Oscillator-A and Oscillator-B

Signals ω_i were generated on a MacBook pro running Max 7 and driven into the system using a SMSL SA-50 Class D 50 Watt amplifier for EM actuators. The output was captured through a Earthworks QTC40 reference microphone placed 5cm from the source of sound, namely the end of the tube for the aerophone prototype and at the point of interaction between the surface and sympathetic bridge in the other instruments. The microphone was connected to a Focusrite Scarlet 2i2 and recorded into Audacity 2.12 using a Lenovo Flex 4. Audio was recorded as 16 bit 44.1 KHz wave files.

5.2 Experiment One: Instrument Sweep Tests

Experiment One aims to show comparable results across three instrument types. The frequency response of aerophones, membranophones and idiophones, were tested using a fixed range frequency sweep. This was performed by keeping ω_a at a constant 1200 Hz and sweeping ω_b from 2 Hz to 700 Hz over 40 seconds to produce varied-frequency intermodulation. Additionally, it tests current prototypes' capacity for control. The sweep test of the aerophone was conducted at $\omega_a = 800$ Hz due to physical limitations of the current prototype. Identical sweep tests were performed on the ideophone prototype with different soundboard materials, namely low-carbon steel, wood and acrylic.

5.3 Experiment Two: Modulation Depth

Control of IM synthesis was demonstrated by controlling the amplitude level of outputs in order to establish a desired dB modulation coefficient β. Only oscillator-A was active and driven at 800Hz in the aerophone prototype, and 1200Hz in all other cases. The RMS dB level of the system was measured for 10 seconds and recorded and the gain settings on the system driving oscillator-A were held constant. Oscillator-A was was stopped and only oscillator-B was driven at 300Hz in the aerophone prototype, and 800Hz in all other cases and the dB level of the system, when driven solely by oscillator-B, was recorded. The gain settings of the system controlling oscillator-B was adjusted to establish a desired modulation index (β), with the modulation index defined in Equation 3, and then held constant. The modulation index was altered by adjusting the gain of the signal driving oscillator-B.
The modulation indices were chosen to explore cases of no modulation ($\beta = 0$), under modulation ($\beta < 1$), critical modulation ($\beta = 1$) and over modulation ($\beta > 1$). This experiment was repeated for all three instrument family prototypes as well as the low-carbon steel, acrylic and wooden soundboards.

6. RESULTS AND DISCUSSION

6.1 Instrument Prototype Evaluations

Our results from the experiment described in 5 show that nonlinear acoustic synthesis in the form of intermodulation can be generated in the idiophone, membranopone, and aerophone conditions as shown in Figure 5. It is important to emphasize that the response and weights of frequency components is greatly influenced by the physical and mechanical properties inherent in each context.

Absent from these prototypes and evaluations is a chordophone model, or string instrument. It is, however, reasonable to consider this condition to be satisfied by the soundboard model, where instead of a tuning fork or other harmonic oscillator, energy from a string can be transduced into a bridge interface specifically designed for nonlinear acoustic synthesis.

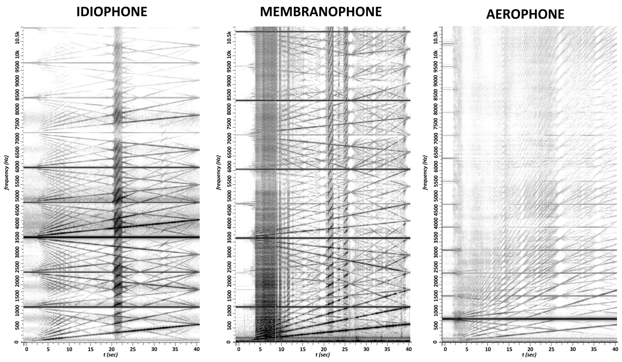

Figure 5: Spectrogram Comparison of Three Instrument Types: Idiophone (left), Membranophone (center), and Aerophone (right).

The horizontal lines are the fundamental tone of Oscillator-A and its harmonics. The diagonal lines spreading from the left tip and converging at the right are the sidebands. The multiple diagonal lines with different slopes are characterized by Equation 6. Vertical striations occur when these slopes converge, particularly When the multiple of the frequency difference d in Equation 6 is equal to a multiple of the modulating signal itself ω_b. Specifically large striations occur when $n_1\omega_b = n_2 d$ where n_1 and n_2 are integers.

Additionally, the spread from the sweep can be seen as further widening or converging gaps in the frequency decomposition. Its behavior has been characterized by radio engineers, where the difference in peak values is $n(\omega_a \pm \omega_b)$ where n is an integer.

6.2 Modulation Depth

6.2.1 FFT Analysis

Control of modulation depth presents another key area in nonlinear acoustic synthesis. When the critically modulated FFT where $\beta = 1$, under-modulated FFT where $\beta = 0.82$ are compared, more modulation products are seen in the critically modulated case. In the under-modulated case 2 distinct intermodulation products can be seen on either side of the of the 9600 Hz peak, where 4 can be seen in the critically modulated case, as theoretically demonstrated in 4.1.1. The FFT of the over-modulated case saturated by distortion. The frequency components greater than -40dB are all harmonic multiples and equal spread from these harmonics.

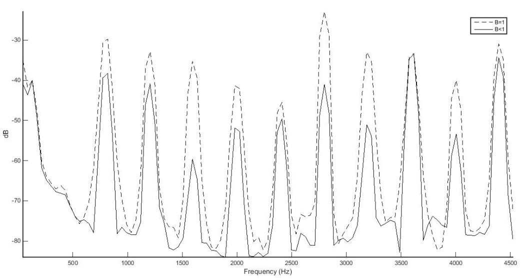

Figure 6: FFT of a Wooden Soundboard Idiophone with $\beta = 1$ and $\beta < 1$.

The FFT was created by taking 200,000 samples with a Hanning window with size 1024. This shows the peak frequencies coincide at intervals of 400Hz. This trend continues above 4500 Hz to well above 20,000Hz. As expected the amplitudes of $\beta < 1$ is bounded above by $\beta = 1$. However, the peaks coincide at irregular frequency bands, which suggests the modulation index directly influences the distribution of IM Products.

6.2.2 Bridge Material

An FFT was conducted by taking 200,000 samples with a Hanning window with size 2048.The -db thresholds were set at -70 and -40dB. An increase in sideband count is observed when increasing the modulation index. In Figure 7 When $\beta = 1$, the number of sidebands generated is approximately the same for different materials. When $\beta < 1$ and $\beta = 1$ the sideband count for metal is greater than that of the wood, but is overtaken by the wood sideband count when $\beta > 1$. With the metal soundboard, there is only a small increase in count between $\beta = 1$ and $\beta > 1$, suggesting that increased modulation does not increase the sideband components significantly. In comparison, the number of peaks for plastic increase significantly when overmodulated. Thus, different materials have different modulation responses suggesting that further variation in nonlinear synthesis techniques can arise from materiality itself. This suggests that when acoustic instruments and acoustic NIMEs are modified with a nonlinear acoustic synthesis device, they will have modulation responses characteristic of their particular instrument type.

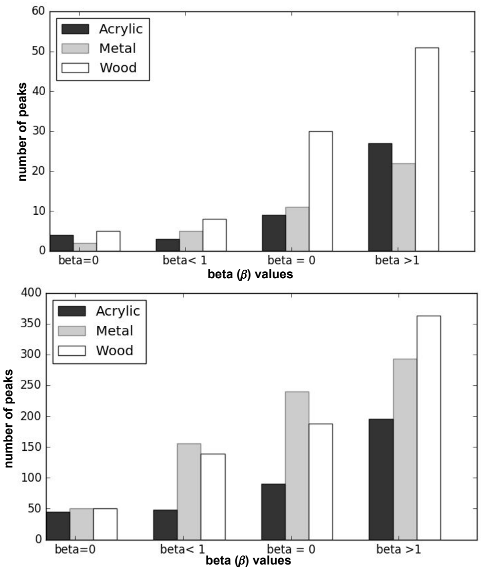

Figure 7: Comparison between Number of Peaks vs. β values for Different Materials with an Amplitude Threshold of -70dB (above), and an Amplitude Threshold of -40dB (below).

In the undermodulated, acrylic soundboard case, in the top graph of Figure 7, there is little to no increase in sideband count, and a decrease in side band count is produced in the bottom graph of Figure 7. This can be accounted to a redistribution of energy across multiple sidebands. The difference in materiality can be attributed to different amplitude responds to the same injected signal. Mathematically, this would appear as the coefficients k_i in Equation 1 for a fixed order of intermodulation N.

6.3 IM Augmentation of Acoustic Instruments

IM synthesis in musical instruments and NIMEs is possible when modifications or attachments can interact directly with the point of acoustic amplification. As shown in Figure 1, a bridge interface generates a nonlinear interaction with a physical medium. One example of such a device is an instrument attachment we call AeroMOD. It is an IM effect that fits into a brass instrument, shown in Figure 8. AeroMOD consists of a thin 1.5mm baffle bridge element attached to a voice-coil motor driven by an arbitrary modulation source, which generates a nonlinear interaction between the baffle and the mouth of the brass instrument bell. It can be used in a similar fashion as a normal brass mute, where the excitation signal is generated from the instrument and the modulation source is generated from the attachment.

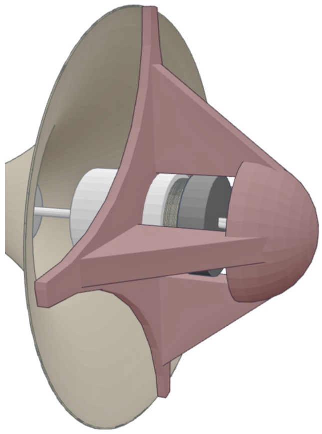

Figure 8: AeroMOD: Brass Instrument Nonlinear Acoustic Synthesis Interface.

7. CONCLUSIONS

In this paper we have detailed and defined a new approach to nonlinear acoustic synthesis through IM. We have shown that it is possible to produce IM components in a variety of instrumental contexts and have shown that by parametrically increasing modulation depth β, more frequency components can be produced in a continuous, controlled fashion. Control over both the number and frequency of sidebands suggests that IM is a powerful method of producing broad timbral synthesis in modified or newly-designed acoustic instruments, capable of bridging the electronic with the acoustic.

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APPENDIX

A. NONLINEAR SIGNAL METHODS

A.1 Amplitude Modulation

Amplitude modulation (AM) describes the convolution of two input signals. The output frequency bands include the sum and difference of the two input signals $C(t)$ and $M(t)$.

Ring modulation is a specific case of amplitude modulation where the modulator is removed and the amplitude can reach zero. If we take $C(t) = C_0 \cos(\omega_C t)$ and $M(t) = M_0 \cos(\omega_M t)$ then

$$A(t) = C(t) \times M(t) = \frac{C_0 M_0}{2} \cos(\omega_C t \pm \omega_M t) \quad (5)$$

Since the modulator is not present in the case of ring modulation, this makes ring modulation desirable for suppressing the modulator signal.

A.2 Frequency Control

Difference IM products whose coefficients sum to its order. Secondly, the change in IM products occur at intervals of 1. Given $\omega_a = \omega_b + 1$ and an IM product with order $N = 2K + 1$, the lower and upper difference IM product is

$$\begin{aligned} \text{IMP}_{lower} &= (K+1)(\omega_a) - K(\omega_a + 1) \\ &= \omega_a K + \omega_a - \omega_a K + K = \omega_a - K \\ \text{IMP}_{Upper} &= -(K+1)(\omega_a + 1) - K(\omega_a) \\ &= \omega_a K + K + \omega_a + 1 - \omega_a K = \omega_a + K + 1 \end{aligned} \quad (6)$$

For instance in the case of Order 7, are value of K is 3. The difference IM products are verified

$$\omega_c - K = 200 - 3 = 197 \qquad \omega_c + K + 1 = 200 + 4 = 204$$

In general, when $\omega_a = \omega_b + d$ where d is the difference between the two input signals, the two products are given as $\omega_a + (K+1)d$ and $\omega_a - dK$. These results are comparable to the Amplitude modulation from the harmonics of ω_2, given as:

$$\omega_a \pm n\omega_b \qquad \forall n < K \quad (7)$$

with K the order of intermodulation.