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The Process of Alpha Decay Vs Principle of Conservation of Energy

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ABSTRACT

Alpha decay is proved to be one of the most puzzling because the α particles appears at energies which, classically, should be inaccessible. An α particle, incident on a heavy nucleus like uranium, experiences a Coulomb barrier in excess of more than 20MeV. Yet α particles are emitted from uranium with energies of less than 5MeV, apparently violating energy conservation in the vicinity of the barrier. The aim of this paper was to verify whether the emission of α- particle through a coulomb barrier actually contradicts with the principle of conservation of energy. However, the calculation of the degree of transmission probability or penetrative factor of α- particle through the barrier height shows that the process of α- decay does not actually violate the principle of conservation of energy in the vicinity of a Coulomb barrier.

Keywords: *Alpha decay, Energy, theory,*

INTRODUCTION

In series of seminal experiments Ernest Rutherford and his collaborators established the important features of alpha decay. The behavior of the radiations from the natural sources of uranium and thorium and their daughters was studied in magnetic and electric fields. In subsequent experiment the α- rays from the needle-like source were collected in a very small concentric discharge tube and the emission spectrum of helium was observed in the trapped volume[1,2,3]. Thus, alpha rays were proven to be energetic helium nuclei[1,4]. The α- particles are the most ionizing radiation emitted by natural sources and spontaneous fission of uranium and another heavy nuclei. These particles are emitted with energies, $E_\alpha = 4 - 9$MeV from these different sources[1].

The process of alpha decay is a nuclear reaction that can be written as[1,5]:

$$ {}^{A}_{Z}(Z)_{N} \rightarrow {}^{A-4}_{Z-2}(X)^{2-}_{N-2} + {}^{4}_{2}H^{2+}_{2} + Q_\alpha \text{ .... (1)} $$

where we have chosen to write out all the superscripts and subscripts. Thus the α- decay of ^{238}U can be written

$$ {}^{238}U \rightarrow {}^{234}Th^{-2} + {}^{4}_{2}He^{+2} + Q_\alpha \text{.................. (2)} $$

The alpha particle, or ^{4}He, is an especially strongly bound particle[1,2,3,4,5]. This combined with the fact that the binding energy per nucleon has a maximum value near A≈ 56 and systematically decreases for the heavier nuclei, creates the situation that nuclei with A>150 have positive Q_α- values for the emission of alpha particles[1,4]. Accordingly, ^{238}U(a mass access,Δ, of +47.3070 MeV) decays by emission to ^{234}Th(Δ= +40.612MeV) giving a Q_α- value of[1]:

$$ Q_\alpha = 47.3070 - (40.612 + 2.4249) = 4.270\text{MeV} \quad (3) $$

The decay energy will be divided between the alpha particle and the heavy recoiling daughter so that the kinetic energy of the alpha particle will be slightly less. (The kinetic energy of the recoiling ^{234}Th nucleus produced in the decay of ^{238}U is ~0.070MeV.) Conservation of momentum and energy in this reaction requires that the kinetic energy of the α- particle ,T_α, is[1,5]:

$$ E\_\alpha = 234/238\ Q\_\alpha = 4.198\text{MeV...... (4)} $$

The kinetic energy of the emitted alpha particle can be measured very precisely so we should be careful to distinguish between the Q_α- value and kinetic energy, T_α.

Even though the energy released by the decay of a ^{238}U into the alpha particle and a ^{234}Th nucleus are quite substantial; the energy is paradoxically small compared to the energy necessary to bring the alpha particle back in to the nuclear contact with the ^{234}Th. The electrostatic potential energy between the two positively charged nuclei, called the Coulomb potential, can be written as[1,4]:

$$ V\_C = 2Z/R\ e\text{^}2/(4\pi\varepsilon\_0) \quad . (5) $$

where Z is the atomic number of the daughter, R is the separation between the centers of the two nuclei and $\frac{e^2}{4\pi\varepsilon_0}$ is 1.440 MeV × fm. To obtain a rough estimate of the Coulomb energy we take R to be $1.2(A^{1/3} + 4^{1/3})$fm, where A is the mass number of the daughter. Therefore, for the decay of ^{238}U[1]:

$$ V_C = \frac{(2)(90)(1.440\text{ MeV f m})}{1.2\left(234^{\frac{1}{3}}+4^{\frac{1}{3}}\right)\text{ fm}} \approx \frac{259\text{ MeV fm}}{9.3\text{ fm}} = 28\text{MeV .... (6)} $$

which is 6 to 7 times the decay energy.

Therefore, alpha decay is proved to be one of the most puzzling because the α particles appears at energies which, classically, should be inaccessible. An α particle, incident on a heavy nucleus like uranium, experiences a Coulomb barrier in excess of more 20MeV. Yet α particles are emitted from uranium with energies of less than 5MeV, apparently violating energy conservation in the vicinity of the barrier[1,4].

In the next sections , I will further summarize theories of alpha decay and subsequently discuss how α- particle does quantum mechanically penetrate Coulomb barrier of 28 MeV by kinetic energy of 4.198MeV. Besides, by defining and mathematically calculating the transmission probability or penetrative factor of α- particle, I will examine what does the estimated amount of transmission probability mean to the principle conservation of energy, whether α- emission actually violates conservation of energy or not.

II. Theories of alpha decay

The explanation of the above general features of α decay was given independently and almost simultaneously in 1928 by G. Gamow and by R. Gurney and E. Condon. In a simple version of the theory, the α particle is assumed to be trapped inside the nucleus by the Coulomb barrier, but has a finite chance of escaping to the outside world by a process known as quantum-mechanical barrier penetration [1,4,8]. The situation is illustrated in figure 1 for spherical nucleus, where V(r), the potential energy between the α particle and the daughter nucleus, is plotted as a function of the distance r between their centers. Clearly, the decay Q value must be positive, otherwise the α is bound and cannot escape at all. Outside the nucleus, V(r) is given by the coulomb potential. This is inversely proportional to r and, asymptotically, the kinetic energy of the α particle is equal to Q, if we neglect the recoil energy of the daughter nucleus [2,3,4]. Close to the nuclear surface, the α experiences the attractive, short-range nuclear force and, once inside the nucleus, acquires a kinetic energy that depends on U, the depth of the well. In figure the below, the nuclear potential is taken to be a square well, radius a, with a sharp edge [4,5]. In practice, the surface will not be sharp and the peak of the Coulomb barrier will have a rounded shape [4].

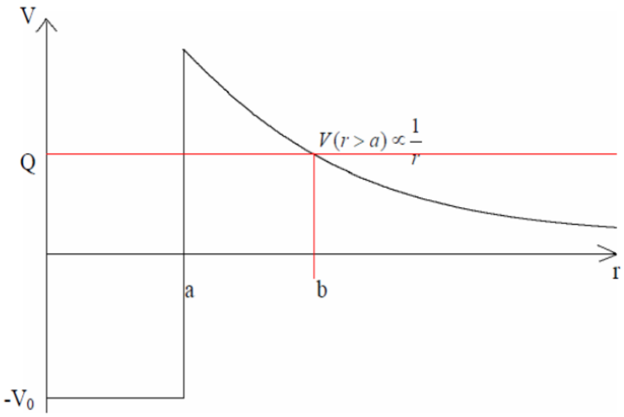

Figure: Schematic view of barrier penetration in α decay. The α particle wave function is oscillatory inside the nucleus(r<a) and at large distances(r>b). Within the barrier region (a≤ r≤ *b*), it is a decreasing function of r[4].

III. Derivation and Calculation of Transmission Probability Through Barrier Height

As it has already been indicated, α- particle would trap inside Uranium-238 and come out to the outside world with energy 4.198 MeV by obviously penetrating wall height of 28 MeV, and this phenomenon apparently violates principle of conservation of energy in the vicinity of the barrier[2,3,4,5]. On the other hand, in a closed system, no energy can leak into the system from the inside (or be introduced into the system from outside) where the nucleus is considered to be a closed system in this case. The amount of energy is fixed and we can't create any more energy inside the system or destroy any part of energy that is already in there, according to principle of conservation of energy. Moreover, Energy does not continuously increase or decrease but it does in quanta[6,7].

The probability with which α- particle would come out by penetrating barrier wall can be estimated by transmission probability or penetrative factor and is defined as the measure of the degree of probability with which α- particle would quantum mechanically penetrate the barrier wall[1,8]. Mathematically, transmission Probability or penetrative factor which is usually symbolized by the letter T is defined as[1]:

$$T=\frac{\text{Transmitted wave of }\alpha\text{−beams through the wall}}{\text{Incident wave of }\alpha\text{−beams on the wall}} = \left(\frac{A_3}{A_1}\right)\left(\frac{A_3}{A_1}\right)^* \quad (7)$$

Quantum mechanically, the transmission probability of the alpha particle is derived and calculated as follows [1,8,9]:

The Schrödinger wave equations for the motion of α- particle inside the nucleus(a<r), within the barrier region(a≤b≤r) and at large distances(r> *b*) can respectively be written as

$$\frac{\partial^2\psi_1}{\partial r^2} + \frac{2m_\alpha}{\hbar^2}E_\alpha\psi_1 = 0, V = 0 \text{ (oscillatory wave)}$$

$$\frac{\partial^2\psi_2}{\partial r^2} - \frac{2m_\alpha}{\hbar^2}(V - E_\alpha)\psi_2 = 0, V \neq 0 \text{ (decreasing wave)}$$

$$\frac{\partial^2\psi_3}{\partial r^2} + \frac{2m_\alpha}{\hbar^2}E_\alpha\psi_3 = 0, V = 0 \text{ (outgoing wave)}$$

Their solutions are also respectively,

$\psi_1 = A_1e^{ik_1r} + B_1e^{-ik_1r}$, where $k_1 = \sqrt{\frac{2m_\alpha E_\alpha}{\hbar^2}}$, m_α is mass of α-particle

$\psi_2 = A_2e^{-k_2r} + B_2e^{k_2r}$, where $k_2 = \sqrt{\frac{2m_\alpha(V-E_\alpha)}{\hbar^2}}$

$\psi_3 = A_1e^{ik_3r} = A_1e^{ik_1r}$ (only outgoing wave), where $k_3 = k_1 = \sqrt{\frac{2mE}{\hbar^2}}$, and generally where ks are wave numbers of their respective wave equation.

Furthermore, in deriving for transmission probability, we need to apply boundary conditions to the three wave functions ψ_1, ψ_2 and ψ_3. At boundaries wave functions and its derivatives are equal.

i.e. at $r = 0$, $\psi_1(0) = \psi_2(0)$

$$A_1 + B_1 = A_2 + B_2 \quad (8)$$

$$\frac{\partial\psi_1(0)}{\partial r} = \frac{\partial\psi_2(0)}{\partial r}$$

$$ik_1A_1 - ik_1B_1 = -k_2A_2 + k_2B_2 \quad (9)$$

and at r = a, $\psi_2(a) = \psi_3(a)$

$$A_2e^{-k_2a} + B_2e^{k_2a} = A_1e^{ik_1a} \quad (10)$$

$$\frac{\partial\psi_2(a)}{\partial r} = \frac{\partial\psi_3(a)}{\partial r}$$

$$-k_2A_2e^{-k_2a} + k_2B_2e^{k_2a} = ik_1A_1e^{ik_1a} \quad (11)$$

Solving equation (8) and equation (9) simultaneously for A_1 and rearranging:

$$A_1 = \frac{1}{2}A_2\left(1 - \frac{k_2}{ik_1}\right) + \frac{1}{2}B_2\left(1 + \frac{k_2}{ik_1}\right) \quad (12)$$

Now again solving equation(10) and equation(11) simultaneously for both A_2 and B_2 one after another:

$$A_2 = \frac{1}{2}A_3\left(1 - \frac{ik_1}{k_2}\right)e^{(ik_1+k_2)a} \quad (13)$$

and

$$B_2 = \frac{1}{2}A_3\left(1 + \frac{ik_1}{k_2}\right)e^{(ik_1-k_2)a} \quad (14)$$

Now substituting equation(6) and equation(7) into equation(5), we get

$$A_1 = \frac{1}{2}\left[\frac{1}{2}A_3\left(1 - \frac{ik_1}{k_2}\right)\left(1 - \frac{k_2}{ik_1}\right)e^{(ik_1+k_2)}\right] + \frac{1}{2}\left[\frac{1}{2}A_3\left(1 + \frac{ik_1}{k_2}\right)\left(1 + \frac{k_2}{ik_1}\right)e^{(ik_1-k_2)a}\right]$$

Then,

$$\left|\frac{A_1}{A_3}\right| = \frac{1}{4}\left[\left(1 - \frac{ik_1}{k_2}\right)\left(1 - \frac{k_2}{ik_1}\right)e^{(ik_1+k_2)}\right] + \frac{1}{4}\left[\left(1 + \frac{ik_1}{k_2}\right)\left(1 + \frac{k_2}{ik_1}\right)e^{(ik_1-k_2)a}\right]$$

and

$$\left|\frac{A_1}{A_3}\right|^2 = \left(\frac{A_1}{A_3}\right)\left(\frac{A_1}{A_3}\right)^*$$

$$= \frac{1}{16}\left(1 + \frac{{k_1}^2}{{k_2}^2}\right)\left(1 + \frac{{k_2}^2}{{k_1}^2}\right)e^{2k_2a} + \frac{1}{16}\left(1 + \frac{{k_1}^2}{{k_2}^2}\right)\left(1 + \frac{{k_2}^2}{{k_1}^2}\right)e^{-2k_2a}$$

Here, we consider only the incident part the result and the second term can be omitted. Therefore,

$$\left|\frac{A_1}{A_3}\right|^2 = \frac{1}{16}\left(1+\frac{{k_1}^2}{{k_2}^2}\right)\left(1+\frac{{k_2}^2}{{k_1}^2}\right)e^{2k_2a}$$
$$= \frac{({k_1}^2+{k_2}^2)^2}{16{k_1}^2{k_2}^2}e^{-2k_2a}$$

Clearly, the transmission probability becomes taking its form:

$$T=\left|\frac{A_3}{A_1}\right|^2 = \frac{16{k_1}^2{k_2}^2}{({k_1}^2+{k_2}^2)^2}e^{-2k_2a} \quad (15)$$

Substituting for k_1 and k_2 into the coefficient of the power of T, then

$$T = 16\frac{\left[\sqrt{\frac{2m_\alpha E_\alpha}{\hbar^2}}\right]^2\left[\sqrt{\frac{2m_\alpha(V-E_\alpha)}{\hbar^2}}\right]^2}{\left[\left(\sqrt{\frac{2m_\alpha E_\alpha}{\hbar^2}}\right)^2+\left(\sqrt{\frac{2m_{\acute{a}}(V-E_\alpha)}{\hbar^2}}\right)^2\right]^2}e^{-2k_2a}$$

and rearranging

$$T = \left[16\frac{E_\alpha}{V_C}(1-\frac{E_\alpha}{V_C})\right]e^{-2k_2a} \quad (16)$$

For the value of Coulomb potential height, $V_C = 28\text{MeV}$ and the kinetic energy of α-particle from $^{238}_{92}U$, $E_\alpha = 4.198$ MeV, the result of $16\frac{E_\alpha}{V_C}\left(1-\frac{E_\alpha}{V_C}\right) = 16\times\frac{4.198}{28}\times(1-\frac{4.198}{28})$ ≈2.04. From equation (16), $2k_2a \gg 1$ and therefore, $e^{-k_2a} \ll 1$.Consequently, equation (16) can be approximated to:

$$T \approx e^{-2k_2a} \quad (17)$$

Therefore, transmission probability can be estimable at Coulomb potential height(the maximum Coulomb barrier) if the source of α-particle is taken to be Uranium-238. For $V(\text{at } r = a) = 28$ MeV,

$2k_2a \approx 2\frac{\sqrt{2m_\alpha(V_C-E_\alpha)}}{\hbar}\times 10^{-14}\text{m} \approx$
$2\frac{\sqrt{2\times4.002602u\times1.6605\times10^{-27}kg/u\times(28-4.198)MeV}}{1.054\times10^{-34}Js}\times 10^{-14}\text{m} \approx$
42.7..........................(18)

and the transmission probability at maximum coulomb barrier becomes

$$T \approx e^{-2k_2a} \approx e^{-42.7} \approx 10^{-19} = \frac{1}{10^{19}} \quad (19)$$

which is very small quantity.

IV. CONCLUSION

The value of transmission probability through Coulomb barrier(barrier height), which is estimated to be very small in equation (19), is largely implying that the probability of escaping of α- particle to outside world is incredibly small. The value of T is approximately 10^{-19} means that if α- particle assaults the potential barrier 10^{19} times the chance of its coming out is 1. It highly suggests that α- would not come out with continuous energy spectrum. The beams of α- particle would penetrate the Coulomb barrier quantum mechanically and come out in quanta where their energy depends only on the frequency and therefore α- decay does not contradict the principle of conservation of energy. Despite apparently the conservation of energy is violated in the vicinity of barrier and classically inaccessible, the emission of the α- particle by penetrating the barrier does not actually violate the conservation of energy quantum mechanically. In general energy is realistically always conserved in the process of α- decay.

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