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Two-to-Four Wire Hybrid Circuits in Telecommunications

Telephone lines are two-wire links that carry information via electrical signals bi-directionally. Thus, a telephone company subscriber not only hears his calling party, but can speak to him as well. Both signals are on the two-wire line simultaneously. A similar situation occurs at the telephone company central office. Obviously, the two signals should not interfere with one-another; such interference would render the other signal unintelligible or, at least, degraded in audio quality. On the customer premises, and in the telephone company central office as well, the in-going and out-going electrical signals are in separate circuits. These two signal "lines" must ultimately be joined to one two-wire telephone line. Hence, a two-to-four wire *hybrid circuit* is required in each location to receive the incoming signal with as little corruption as is possible by the outgoing signal. XDSL applications using two-to-four wire hybrids are subject to the same considerations. Signal discrimination capability assumes even greater importance here, because otherwise, fatal signal errors may result.

The two-to-four wire hybrid circuit enables simultaneous two way communications down a single pair of wires.

In this paper we will present several passive two-to-four wire hybrid circuits and endeavor to present a fairly comprehensive treatment of these devices. The circuit topologies will be presented using the simplest implementation of components and signal sources so as not to obscure the workings of the circuit. However, various other schematic representations will also be presented, so that if he comes upon a hybrid schematic diagram in the literature, the reader can recognize the category of hybrid with which he is dealing and identify component function by referring to the basic implementation. All transformer derivations will consider perfect coupling.

Usually, "transmitted" signifies that the signal is generated locally, and "received" signifies that the signal is generated remotely and received locally. However, use of the words "transmitted" and "received" can generate confusion. After all, a received signal at one end of a data link is a transmitted signal at the other end. Hence, we will use the phrase "*local* signal source" to indicate a signal generated locally and the phrase "*remote* signal source" to indicate a signal generated remotely, that is, at the other end of the link.

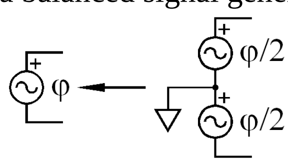
In this paper the following (passive) hybrids will be considered:

- Asymmetric Wheatstone Bridge Hybrid or Asymmetric Lattice Hybrid
- Symmetric Wheatstone Bridge Hybrid or Symmetric Lattice Hybrid
- Unbalanced Line One-Transformer Hybrid
- Balanced-Line One Transformer Hybrid
- Two-Transformer Hybrid

A Wheatstone bridge circuit is a lattice network. The phrases are synonymous. The names "Asymmetric Hybrid" and "Symmetric Hybrid" are used here, because, as far as the author is aware, there are no established names for these hybrid circuit topologies.

Conventions adopted for this paper:

1. Portray resistances and reactive impedances on schematic diagrams using resistor symbols.
2. φ_s = local signal source voltage (the signal is generated locally for transmission to the other end of the data link and is an input to the local hybrid). The signal itself is represented by the symbol T_x.
3. φ_g = remote signal source voltage, the voltage that is generated at the remote source and measured at that location in the circuit.
4. $\Delta\varphi$ = the signal voltage from the remote source that is *recovered* at the output of the local hybrid. In electronics literature, the signal that is received from the other end of the data link (from the remote generator) is usually represented by the symbol R_x, but sometimes the particular signal is ambiguous and not specified. A received signal might be measured at the input of the coupling transformer, at the input to the local hybrid, at the input or output of a gain stage, or other place. In this paper, R_x is the *name* of the local hybrid output signal whose voltage is $\Delta\varphi$.
5. In electrical schematic diagrams, the symbol  is usually used to represent a signal generator. By itself, the two terminals are ambiguous. One side of the signal generator is *usually* connected to circuit ground, and in this context the ground side is implicitly understood to be the reference side and the other side the voltage source, say $\varphi = \varphi_m \sin\omega t$. If we represent a balanced (push-pull) generator, we use two of these symbols connected in a series aiding configuration. But since it is possible to have two generators in series opposition, place a "+" label on the voltage source side (opposite to the reference side). Then we replace the single-ended signal generator by a balanced signal generator.



NOTE: In a hybrid schematic diagram, the signal source reference terminal will either be grounded (electrically tied to zero volt ground reference) or floating. Frequently, a floating signal source reference terminal is at *virtual* ground potential, because of the balanced bridge situation in the hybrid, but not always so.

6. For brevity, a two-to-four wire hybrid will be referred to simply as a "hybrid". This should introduce no confusion, because that is the only connotation in which this term is used in this paper.

Trans-Hybrid Loss

In dealing with hybrids it is useful to have appropriate measures of performance. One such measure is *trans-hybrid loss*. One of the two signals entering a hybrid is the locally generated signal, T_x, intended to be transmitted to a remote receiver. An ideal hybrid would impress this signal on the two-wire line, and none of this signal would appear on the R_x port, which port is intended to locally output the remote signal recovered from the two wire line. Unfortunately, in real devices, the recovered signal will be contaminated by a fraction of the locally generated and transmitted signal, T_x.

Let φ_s be the voltage of the locally generated signal, T_x, and $\Delta\varphi_s$ be a fractional portion of this same voltage that evades discrimination by the hybrid, and appears on the output port, R_x. Specifically, this fraction is...

$$\gamma = \frac{\Delta\varphi_s}{\varphi_s} \qquad (1)$$

In an ideal hybrid, $\gamma = 0$. In a real hybrid, we strive to have gamma (γ) as small as is possible. Gamma can be regarded as a voltage gain less than one. If we wish to use logarithmic measure (and we usually do), we take...

$$g(dB) = 20\log_{10}|\gamma| = 20\log_{10}\left|\frac{\Delta\varphi_s}{\varphi_s}\right| \qquad (2)$$

The logarithm of a number less than one is negative. The absolute value above is used, because γ may be negative. Smaller numbers imply larger negative logarithms. Since we are always dealing with numbers $\gamma < 1$, and hence, negative logarithmic values, we define *trans-hybrid loss* as...

$$\text{THL} = |g| = -20\log_{10}\left|\frac{\Delta\varphi_s}{\varphi_s}\right| = 20\log_{10}\left|\frac{\varphi_s}{\Delta\varphi_s}\right| \qquad (3)$$

In this way we can measure the performance of a hybrid in discriminating against the undesired T_x signal using positive numbers. Larger positive numbers indicate better performance; smaller positive numbers poorer performance.

The Role of Transformers in Telecom Hybrids

Several types of hybrid employ one or two transformers as basic components to implement the signal discrimination process characteristic of two-to-four wire hybrids. Nevertheless, transformers are also used in conjunction with non-transformer telecom hybrids for one or more of the following purposes:

- To provide DC electrical isolation of the electrical circuit from the twisted-pair line.
- To provide impedance matching of the circuit to the twisted-pair line.
- To provide voltage gain to the received signal.

A transformer hybrid can combine these capabilities with its hybrid function capability without requiring any extra components.

The Asymmetric Lattice Hybrid

In Figure 1 below, I say that this Wheatstone bridge configuration has an asymmetric *Lattice Hybrid circuit topology,* because the voltage generators are placed asymmetrically. Unlike a typical Wheatstone bridge, the right-hand-side voltage divider has a voltage source in its lower leg. There is no voltage source across electrical points C and D. Z_g is the impedance that the local circuit sees when looking into the twisted-pair line. Z_B is a "balance impedance" which *should be* equal to Z_g.

Assume zero source resistance of the local generator and high (infinite) input impedance of the difference amplifier; that is, let $R_s = 0$ and $R_L = \infty$.

But with this same circuit topology, the bridge resistors (impedances) can be assigned different functions, as in Figure 2, in which the balance impedance Z_B is placed in the lower left leg of the bridge. Figure 1 is sometimes drawn differently, as in Figure 3.

The difference amplifier (indicated by the op-amp symbol) is a load on the Wheatstone bridge, and may be represented by the symbol R_L . In this paper, the difference amplifier is assumed to have high input impedance, ideally infinite; that is, $R_L = \infty$. Therefore, there is no load across points C and D of Figure 1, and this Wheatstone bridge provides a voltage difference $\Delta\varphi$ between two unloaded voltage dividers.

Furthermore, assume that the local generator source resistance is zero; that is, $R_S = 0$. This is a realistic assumption in xDSL applications, wherein the generator will usually consist of two wide bandwidth low output impedance operational amplifiers (video amplifiers) in a balanced (push-pull) configuration.

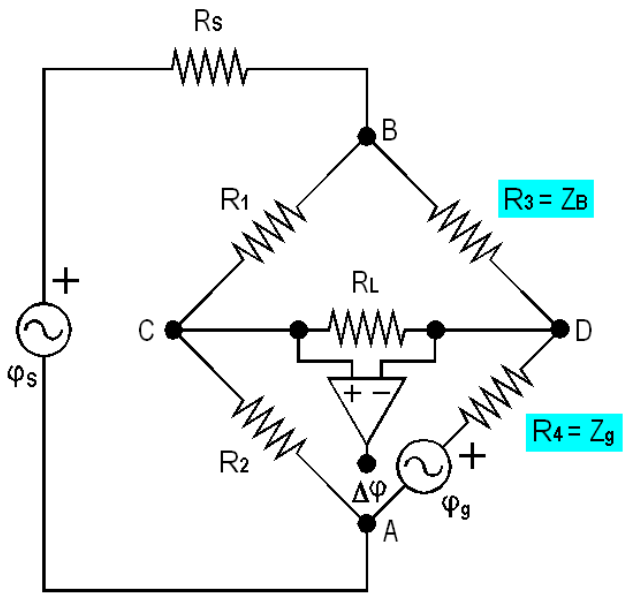


Figure 1. Asymmetric Lattice Hybrid Circuit Topology

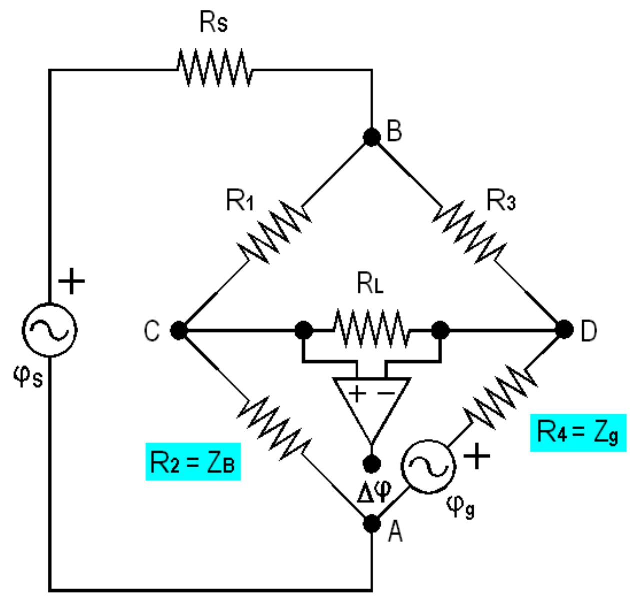
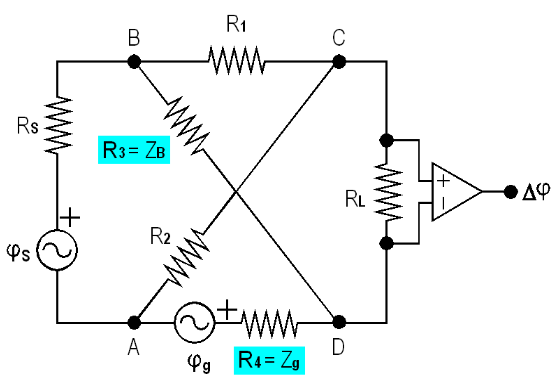


Figure 2. Asymmetric Lattice Hybrid Circuit Topology, a Variant Implementation.

In Figures 3 and 4 we see the same circuits redrawn in lattice representation. Notice that the only difference between Figures 1 & 2 and Figures 3 & 4 is the graphical representation, that is, in the way in which they are drawn.



Figures 3. Asymmetric Lattice Hybrid Circuit Topology.

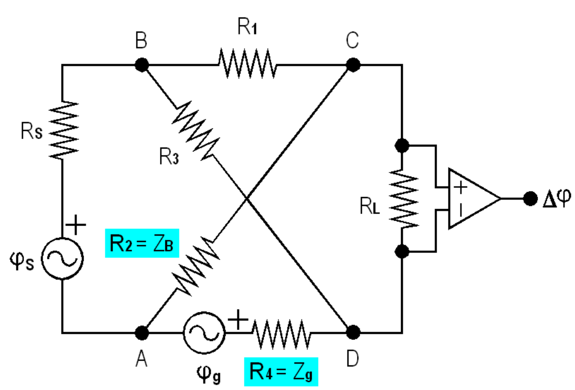


Figure 4. Asymmetric Lattice Hybrid Circuit Topology, a Variant Implementation.

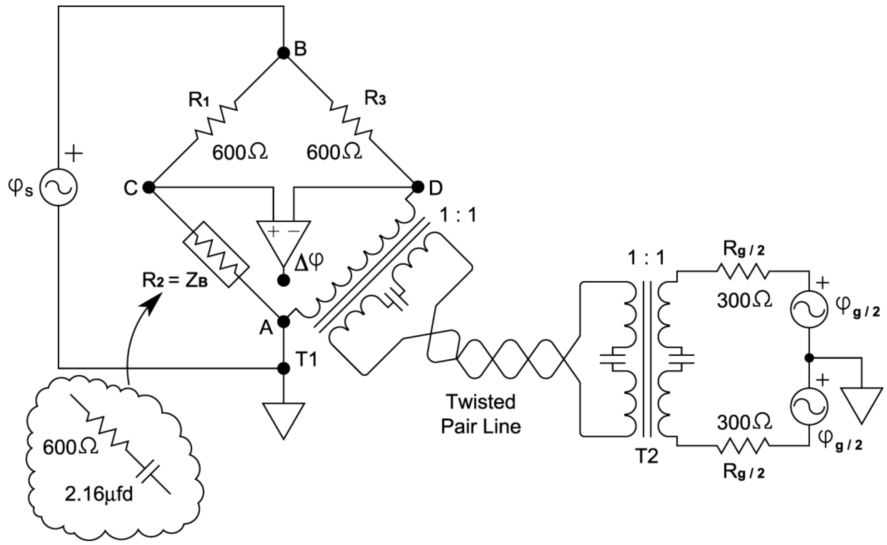


Figure 5. Further Representations of Circuits with Asymmetric Lattice Hybrid Topology.

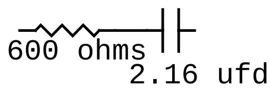
Balance

In Figure 5, we attempt to make the balance impedance Z_B (between B and D) the same as the impedance between D and ground due to the impedance R_g (or Z_g) to the right of it that is coupled through transformer T_1. The balance impedance Z_B *can* be moved to the lower left hand leg so that $R_2 = Z_B$ as in Figure 2 if desired. However, I prefer the first configuration, Figure 1, 3 or 5, because then, equal reactive components are in the same voltage divider, and thus, the voltage divider output is the same as if both voltage divider elements were resistors (like a compensated attenuator on an oscilloscope probe).

The balancing circuit Z_B is a key part of a hybrid.

The telephone company central office circuitry presents a source impedance into the telephone "transmission line" that can be represented, to a first order approximation, by the series RC network

Z_g =



This network is known as a "series compromise network".
Sometimes this resistance value is 900 ohms instead of 600 ohms, such as on the last telephone circuit of a network. The balancing circuit Z_B is a key part of a hybrid. Two-terminal balancing networks, Z_B, of greater complexity (known as "precision balance networks") can be constructed if greater signal discrimination is required. That is beyond the scope of this paper, and the reader is referred to reference [5].

The impedance between points D and A in Figure 5 is the *line impedance*, Z_{Line}, corrected for transformer turns ratio. The line impedance is the impedance looking into the twisted-pair cable of characteristic impedance Z_0 loaded at the remote site by Z_g. We would like to achieve perfect impedance matching of the twisted-pair transmission line and remote generator circuit; that is, we would like to have $Z_0 = Z_g$. For a perfect impedance match, a signal propagating from the local generator φ_s on the left to the remote generator φ_g on the right, there is no wave reflection at the remote generator. If, however, $Z_0 \neq Z_g$, the impedance at the local generator looking into the twisted-pair line is $Z_{line} = Z_0 \frac{Z_g + Z_0 \tanh \Gamma l}{Z_0 + Z_g \tanh \Gamma l}$, where Γ is the complex propagation constant and l is the length of the line.

Figure 6 is another typical rendition of the same Asymmetric Lattice Hybrid Circuit topology with two complementary hybrids in a balanced (push-pull) configuration.

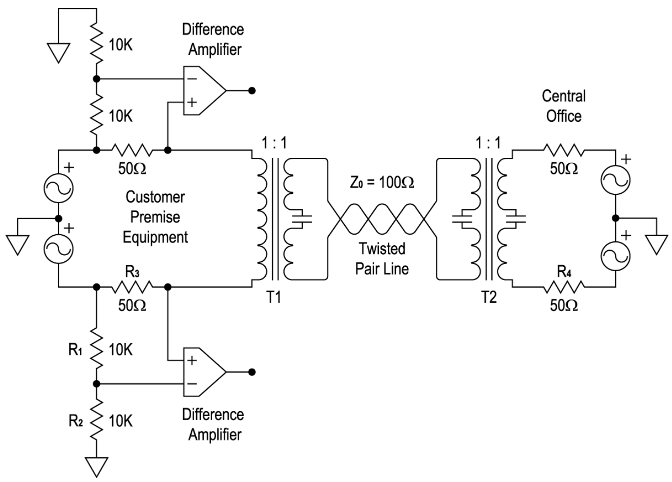


Figure 6. Complementary Asymmetric Lattice Hybrids in Balanced Configuration.

Asymmetric Lattice Hybrid Derivation

Reference: Figure 1. For voltage measurements, the reference point (GROUND) is point A. φ_C and φ_D are the respective voltages of points C and D.

Keep in mind that in this paper $R_S = 0$ and $R_L = \infty$.

$$\varphi_C = \frac{R_2}{R_1 + R_2} \varphi_s \tag{4}$$

$$\varphi_D = \frac{R_4}{R_3 + R_4} (\varphi_s - \varphi_g) \tag{5}$$

$$\Delta\varphi = \varphi_C - \varphi_D = \frac{R_2}{R_1 + R_2} \varphi_s - \frac{R_4}{R_3 + R_4} (\varphi_s - \varphi_g)$$

$$= \left(\frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4} \right) \varphi_s + \frac{R_4}{R_3 + R_4} \varphi_g \tag{6}$$

Object of the hybrid is to make the φ_s term on the right hand side of the equation equal to zero, so that...

$$\Delta\varphi = \frac{R_4}{R_3 + R_4} \varphi_g \tag{7}$$

To get this equation, rather than $\Delta\varphi = -\frac{R_4}{R_3+R_4}\varphi_g$, the non-inverting (+) input of the difference amplifier must be connected to point C rather than to point D.

To make the φ_s term on the right hand side of the Equation 3 equal to zero, it is necessary that

$$\frac{R_2}{R_1+R_2} = \frac{R_4}{R_3+R_4} \quad .\Rightarrow. \quad \frac{R_2}{R_1} = \frac{R_4}{R_3} \tag{8}$$

For a hybrid installed in the telephone network, R_4 is reactive, $R_4 = Z_g$, and the nature of this reactive circuit Zg is established by the telephone company. For balancing out this reactive impedance, R_2 must be a *balance network* of operational impedance Z_B such that $Z_B = Z_g$. Therefore, $R_2 = Z_B$.

$$\frac{Z_B}{R_1} = \frac{Z_g}{R_3}, \quad \text{or} \quad Z_B = \frac{R_1}{R_3} Z_g \tag{9}$$

End of Derivation.

The Symmetric Lattice Hybrid

This circuit is also a Wheatstone bridge, but differs from the asymmetric lattice hybrid in several ways.

1. The symmetric lattice hybrid has the second generator in the center and in series with the Wheatstone bridge load. As a consequence, the two generators φ_s and φ_g are electrically located somewhat symmetrically.
2. Two opposite arms of the bridge in the symmetric lattice hybrid have strings of two resistors in series rather than only one resistor. The center junction of each resistor string is tapped and across these two junctions an *unloaded* differential amplifier is placed (that is, a virtually ideal differential amplifier with virtually infinite input impedance).

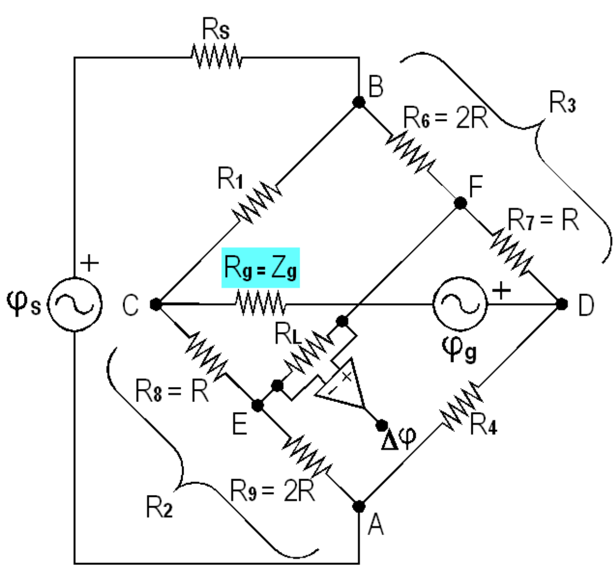


Figure 7. Symmetric Lattice Hybrid Circuit Topology.

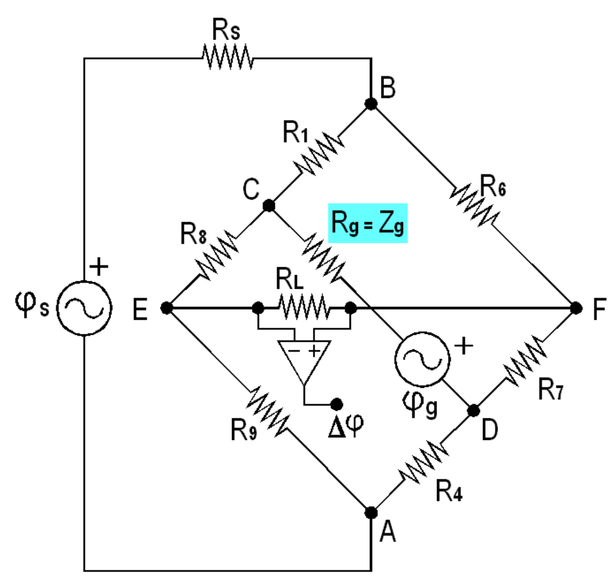


Figure 8. Alternate Schematic Rendition of Symmetric Lattice Hybrid Circuit Topology.

Figures 7 and 8 are electrically identical with the exception that the schematic diagrams are simply drawn differently.
Just as we drew a circuit representing the asymmetric lattice hybrid with twisted-pair cable, so we do here for a symmetric lattice hybrid circuit.

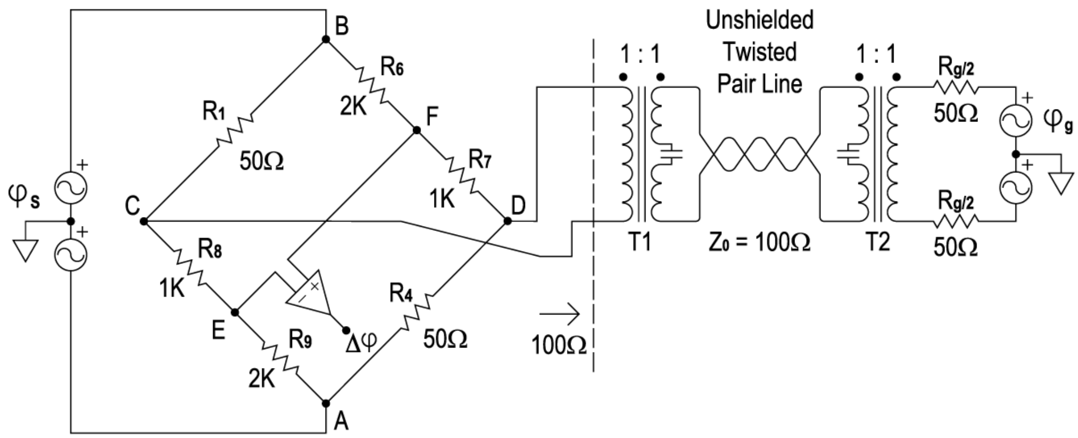


Figure 9. Circuit with Symmetric Lattice Hybrid Topology.

Figure 10 (below) is yet another way that this same balanced hybrid might be schematically portrayed.

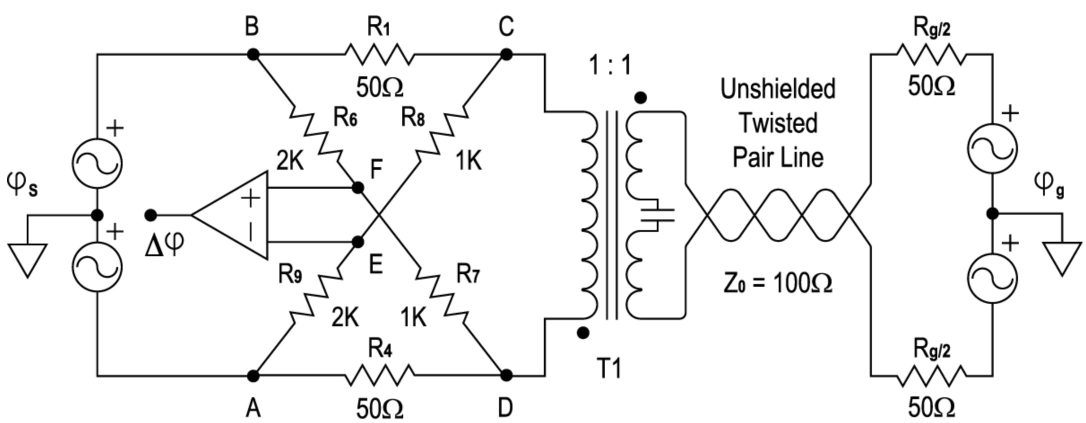


Figure 10. Circuit with Symmetric Lattice Hybrid Topology, Alternate Rendering.

Symmetric Lattice Hybrid Derivation

Reference: Figure 7.

There is a combination of resistor values R_1, R_4, R_6, R_7, R_8, R_9 in Figure 7 that will provide complete signal rejection of the locally generated signal φ_s in $\Delta\varphi$, the recovered remotely generated signal. More on this will be presented shortly. Notice that R_1 and R_4 have been replaced by a resistor of the same value, r, and that R_2 and R3 each have been replaced by a series string of resistor strings $R_2 = R_8 + R_9 = R + 2R$ and $R_3 = R_6 + R_7 = 2R + R$. Consider Figure 11 below, which is a simplified version of Figure 7 with only relevant information shown.

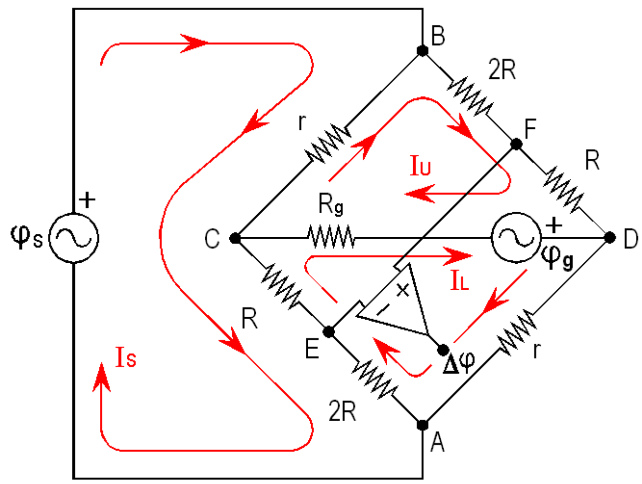


Figure 11. Loop Currents in the Symmetric Lattice Hybrid.

In Figure 11 we use Kirchoff Voltage Law (KVL) to write three equations to describe the circuit. We could also have employed flow-graphs here and at the outset, designated currents in each resistor. However, that would complicate the derivation, because the number of currents would be greater than three in the analysis. In the derivation below, although the matrix is only a 3 x 3 matrix, it is very easy to make a mistake and forget to copy an expression in doing this derivation by hand. Hence, it is a good idea to use a general mathematics computer program with symbolic mathematics capability to evaluate the determinant and inverse and to copy and paste terms.

Loop S:	$(r+3R)I_s \;+$	$(-r)I_U \;+$	$(-3R)I_L \;=$	φ_s
Loop U:	$(-r)I_s \;+$	$(r+3R+R_g)I_U \;+$	$(-R_g)I_L \;=$	$-\varphi_g$
Loop L:	$(-3R)I_s \;+$	$(-R_g)I_U \;+$	$(r+3R+R_g)I_L \;=$	φ_g

(10)

or in matrix notation,

$$\begin{pmatrix} r+3R & -r & -3R \\ -r & r+3R+R_g & -R_g \\ -3R & -R_g & r+3R+R_g \end{pmatrix} \begin{pmatrix} I_s \\ I_U \\ I_L \end{pmatrix} = \begin{pmatrix} \varphi_s \\ -\varphi_g \\ \varphi_g \end{pmatrix} \tag{11}$$

representing the matrix equation

$$\mathsf{R} \cdot I = \varphi \tag{12}$$

Solving for the current vector, I,

$$I = \mathsf{R}^{-1} \cdot \varphi \tag{13}$$

Remember that for any non-singular 3 x 3 matrix

$$\mathsf{R} = \begin{pmatrix} \mathsf{R}_{11} & \mathsf{R}_{12} & \mathsf{R}_{13} \\ \mathsf{R}_{21} & \mathsf{R}_{22} & \mathsf{R}_{23} \\ \mathsf{R}_{31} & \mathsf{R}_{32} & \mathsf{R}_{33} \end{pmatrix} \tag{14}$$

the inverse matrix is

$$\mathsf{R}^{-1} = \frac{1}{\Delta}\begin{pmatrix} \mathsf{R}_{22}\mathsf{R}_{33} - \mathsf{R}_{23}\mathsf{R}_{32} & \mathsf{R}_{13}\mathsf{R}_{32} - \mathsf{R}_{12}\mathsf{R}_{33} & \mathsf{R}_{12}\mathsf{R}_{23} - \mathsf{R}_{13}\mathsf{R}_{22} \\ \mathsf{R}_{23}\mathsf{R}_{31} - \mathsf{R}_{21}\mathsf{R}_{33} & \mathsf{R}_{11}\mathsf{R}_{33} - \mathsf{R}_{13}\mathsf{R}_{31} & \mathsf{R}_{13}\mathsf{R}_{21} - \mathsf{R}_{11}\mathsf{R}_{23} \\ \mathsf{R}_{21}\mathsf{R}_{32} - \mathsf{R}_{22}\mathsf{R}_{31} & \mathsf{R}_{12}\mathsf{R}_{31} - \mathsf{R}_{11}\mathsf{R}_{32} & \mathsf{R}_{11}\mathsf{R}_{22} - \mathsf{R}_{12}\mathsf{R}_{21} \end{pmatrix} \tag{15}$$

Hence, using the values of matrix elements in Equation 11,

$$\mathsf{R}^{-1} = \frac{1}{\Delta}\begin{pmatrix} (r+3R+2R_g)(r+3R) & (r+R_g)(r+3R) & (R_g+3R)(r+3R) \\ (r+R_g)(r+3R) & r^2+6rR+rR_g+3RR_g & rR_g+3RR_g+3rR \\ (R_g+3R)(r+3R) & rR_g+3RR_g+3rR & 6rR+rR_g+9R^2+3RR_g \end{pmatrix} \tag{16}$$

and

$$\Delta = \det\mathsf{R} = \begin{vmatrix} r+3R & -r & -3R \\ -r & r+3R+R_g & -R_g \\ -3R & -R_g & r+3R+R_g \end{vmatrix}$$

$$= 6r^2R + r^2R_g + 18rR^2 + 6rRR_g + 9R^2R_g$$

$$= (r+3R)[6rR + R_g(r+3R)] \tag{17}$$

$$\begin{pmatrix} I_s \\ I_U \\ I_L \end{pmatrix} = \frac{1}{\Delta}\begin{pmatrix} (r+3R+2R_g)(r+3R) & (r+R_g)(r+3R) & (R_g+3R)(r+3R) \\ (r+R_g)(r+3R) & r^2+6rR+rR_g+3RR_g & rR_g+3RR_g+3rR \\ (R_g+3R)(r+3R) & rR_g+3RR_g+3rR & 6rR+rR_g+9R^2+3RR_g \end{pmatrix}\begin{pmatrix} \varphi_s \\ -\varphi_g \\ \varphi_g \end{pmatrix} \tag{18}$$

from which

$$I_s = \frac{(3R+r+2R_g)\varphi_s + (3R-r)\varphi_g}{6rR + R_g(3R+r)} \tag{19}$$

$$I_U = \frac{(r+R_g)\varphi_s+\frac{3R-r}{3R+r}r\varphi_g}{6rR+R_g(3R+r)} \tag{20}$$

$$I_L = \frac{(R_g+3R)\varphi_s+3R\varphi_g}{6rR+R_g(3R+r)} \tag{21}$$

$$\Delta\varphi = \varphi_F - \varphi_E = (I_L r + I_U R) - (I_s - I_L)2R$$

$$= -I_s 2R + I_U R + (2R + r)I_L \tag{22}$$

$$\Delta\varphi = -\frac{(3R+r+2R_g)\varphi_s+(3R-r)\varphi_g}{6rR+R_g(3R+r)}2R + \frac{(r+R_g)\varphi_s-r\varphi_g}{6rR+R_g(3R+r)}R + (2R+r)\frac{(R_g+3R)\varphi_s+3R\varphi_g}{6rR+R_g(3R+r)} \tag{23}$$

or, after the above equation is simplified,

$$\boxed{\Delta\varphi = \frac{[2rR-(R-r)R_g]\varphi_s+4rR\varphi_g}{6rR+R_g(3R+r)}} \tag{24}$$

where φ_s is the output voltage from the driver of the local generator, Tx, φ_g is the output voltage from the remote generator driver, and $\Delta\varphi$ is the voltage output of the remote generator recovered from the local hybrid.

Ideally we want the coefficient of φ_s in the equation above to be equal to zero, indicative of total rejection of that signal. The condition for this situation is that the coefficient of φ_s be equal to zero; that is, that $2rR - (R-r)R_g = 0$, or

$$\boxed{R = \frac{rR_g}{R_g-2r}} \tag{25}$$

If Equation 25 is substituted into equation 24, we find that the recovered signal output voltage is

$$\boxed{\Delta\varphi = \frac{r}{r+R_g}\varphi_g} \tag{26}$$

R_g is the impedance that the line presents to the hybrid. Sometimes setting the resistance r to this value is not feasible, because r is usually chosen to be very small for good power transfer (small I-r drop) from the local generator to the telephone transmission line. Usually there is a transformer between the line and the hybrid for impedance matching purposes and for gain.

End of Derivation.

The Transformer Hybrid Circuit

Transformer hybrids can be divided into one-transformer hybrids and two-transformer hybrids. There are two types of one-transformer hybrids, and these are shown below in Figures 12 and 13. One-transformer hybrids have three windings, or two-windings with one winding having a center-tap.

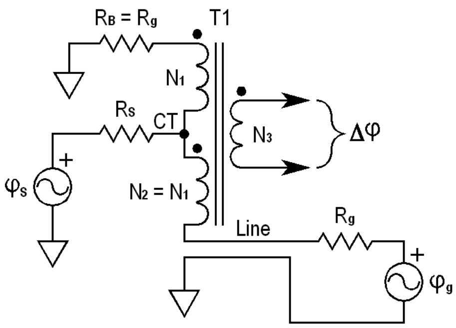


Figure 12. Unbalanced-Line One-Transformer Hybrid.

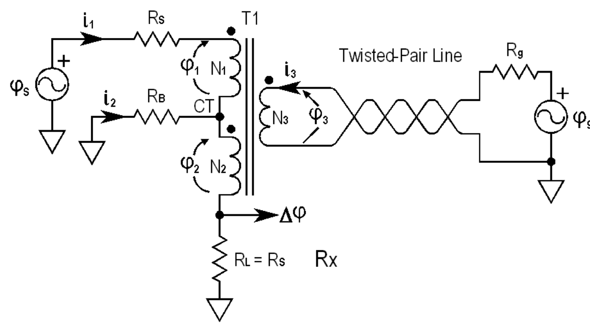


Figure 13. Balanced Line One-Transformer Hybrid.

Unbalanced-Line One-Transformer Hybrid

Consider Figure 12 and the transformer T_1 with a split primary winding N_1 and N_2, and a secondary winding N_3 which may be a single winding or a split winding. $N_2 = N_1$; Let N_1, N_2, and N_3 represent both, the names of the windings and the numbers of turns on the windings. We designate N_1 and N_2 as the primary windings, because φ_s , the voltage of the local generator, and φ_g , the voltage from the remote generator, are both impressed on these coils. Coil N_3, the secondary coil, is not connected to any active voltage sources.

In the 2 primary coils, the local generator φ_s creates opposing magnetic fluxes, but the remote generator φ_g creates aiding magnetic fluxes.

The local signal source generates a voltage φ_s that creates a current that flows through R_s and then enters the center-tap of transformer T_1. The voltage at the center-tap, CT, relative to ground ▽ is impressed across winding N_1 at the *non-dotted* end of the coil, and across the winding N_2 (= N_1) at the *dotted* end of the coil. Hence, resulting currents in the two primary coil windings create opposing magnetic fluxes in the core of transformer T_1. This differential flux generates an Emf in secondary coil N_3. The load resistance (impedance) on coil N_2 is roughly R_g (or Z_g) and the load resistance (impedance) on coil N_1 is R_B (or Z_B) , but R_B is chosen to be equal to R_g; consequently, the counteracting magnetic fluxes cancel one another, with the result that the voltage $\Delta\varphi$ induced in secondary coil N_2 as a result of φ_s is (ideally) equal to zero.

Now consider the signal φ_g generated by the remote signal source in Figure 12. This voltage generates a current that flows through the line to the left and up through windings N_2 and N_1 of transformer T_1. The magnetic fluxes from these windings aid one another, and thus, induce in winding N_2 a non-zero Emf $\Delta\varphi$, the signal R_x, recovered from the remote signal source.

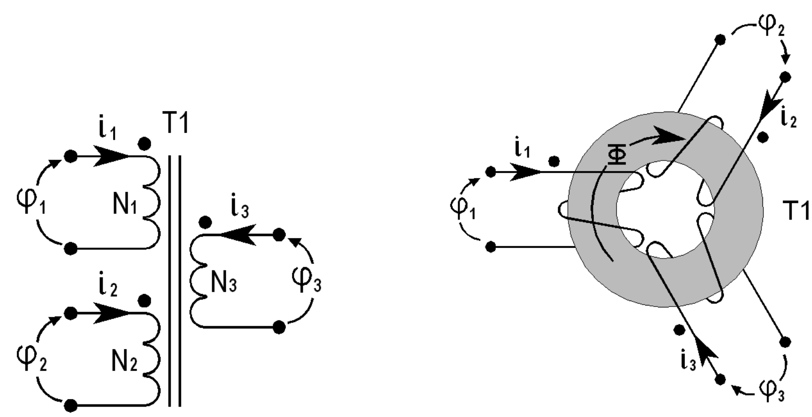
The unbalanced-line one-transformer hybrid is not suitable for use in xDSL or other broadband applications using the telephone lines, because the telephone line is a *balanced* twisted-pair line and this one is single ended. Its application is usually limited, perhaps, to hand-held telephone test sets used by telephone company service personnel.

Balanced-Line One-Transformer Hybrid

The balanced-line one-transformer hybrid of Figure 13 is somewhat more complicated than the unbalanced-line one-transformer hybrid, and includes one more resistor in its most basic version. Rules for using this hybrid: Choose $N_2 = N_1$ and $R_L = R_s$. If $N_3 = 2N_1$, then $R_B = R_g/4$, but keep in mind that R_B and R_g are reactive components. Usually $N_3 = 2N_1$, $N_3 = \sqrt{2}*N_1$, or $N_3 = N_1$. However, as will be shown in the following derivation, letting $N_3 = \sqrt{2}*N_1$ results in the largest possible recovered signal amplitude.

Balanced-line One-Transformer Hybrid Derivation

Consider Figure 14 below, which is a three-winding transformer. Additionally, the transformer is separately pictured as a circular toroid merely to display the symmetry of the windings and of the resultant equations that describe the transformer's electrical operation.



(a) Usual Representation (b) Toroid Representation

Figure 14. General Three-Winding Transformer.

$$\begin{array}{|c|c|}\hline \varphi_1 = & i_1(L_{11}D) + i_2(L_{12}D) + i_3(L_{13}D) \\ \hline \varphi_2 = & i_1(L_{21}D) + i_2(L_{22}D) + i_3(L_{23}D) \\ \hline \varphi_3 = & i_1(L_{31}D) + i_2(L_{32}D) + i_3(L_{33}D) \\ \hline \end{array} \qquad (27)$$

Equations (27) constitute an electrical description of a three-winding transformer such as the one in Figure 14. $D = \frac{d}{dt}$ is the time differential operator. Typically, for a phasor signal $e^{i\omega t}$, we make a replacement operation, $D \leftarrow i\omega$, but that is not necessary here.

$$\begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix} = \begin{pmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} D \qquad (28)$$

L_{11}, L_{22}, and L_{33} are the bulk inductances of the individual coil windings. In schematic diagrams these would typically be represented by symbols such as L_1, L_2, and L_3 respectively. However, these parameters appear in the derivation as diagonal elements of a 3 x 3 matrix, and for that reason are given the double subscripts. L_{12}, L_{13}, and all of the other L_{ij}s with $i \neq j$ represent mutual inductances between various coil windings. In schematic diagrams these would typically be represented by symbols such as M_{21}, M_{31}, and so on, but as matrix elements, we use $L_{ij} = M_{ji}$.
If we attach resistors R_s , R_g, R_B, and the generators of Figure 13 and apply Kirchoff's voltage law (KVL) around the three resistor-generator-transformer circuits, we obtain...

$$\begin{aligned} \varphi_1 &= \varphi_s - i_1 R_s - (i_1 - i_2) R_B \\ \varphi_2 &= 0 - i_2 R_s - (i_2 - i_1) R_B \\ \varphi_3 &= \varphi_g - i_3 R_g \end{aligned} \qquad (29)$$

Equations (28) and (29) can be combined into one matrix equation,

$$\begin{pmatrix} \varphi_s \\ 0 \\ \varphi_g \end{pmatrix} = \begin{pmatrix} R_s + R_B + L_{11}D & -R_B + L_{12}D & L_{13}D \\ -R_B + L_{21}D & R_s + R_B + L_{22}D & L_{23}D \\ L_{31}D & L_{32}D & R_g + L_{33}D \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} \qquad (30)$$

However, $L_{ij} \propto N_i N_j$ for all i and j, and if we normalize all inductances to the bulk inductance L_{11} of coil winding N_1, we have...

$$\left\{ \begin{aligned} L_{11} &= L_{11} \\ L_{12} &= \tfrac{N_2}{N_1} L_{11} \\ L_{13} &= \tfrac{N_3}{N_1} L_{11} \end{aligned} \right. \quad \begin{aligned} L_{21} &= \tfrac{N_2}{N_1} L_{11} \\ L_{22} &= \left(\tfrac{N_2}{N_1}\right)^2 L_{11} \\ L_{23} &= \tfrac{N_2}{N_1}\tfrac{N_3}{N_1} L_{11} \end{aligned} \quad \begin{aligned} L_{31} &= \tfrac{N_3}{N_1} L_{11} \\ L_{32} &= \tfrac{N_3}{N_1}\tfrac{N_2}{N_1} L_{11} \\ L_{33} &= \left(\tfrac{N_3}{N_1}\right)^2 \end{aligned} \qquad (31)$$

Furthermore, create a center-tapped transformer model by joining terminals of windings N_1 and N_2 , set $N_2 = N_1$, and define turns ratio $N \overset{D}{=} \frac{N_3}{N_1}$. The matrix equation becomes...

$$\begin{pmatrix} \varphi_s \\ 0 \\ \varphi_g \end{pmatrix} = \begin{pmatrix} R_s + R_B + L_{11}D & -R_B + L_{11}D & NL_{11}D \\ -R_B + L_{11}D & R_s + R_B + L_{11}D & NL_{11}D \\ NL_{11}D & NL_{11}D & R_g + N^2L_{11}D \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} \quad (32)$$

- Multiply both sides of this equation by the inverse of the matrix to determine the currents flowing in the network.
- In computing the inverse of the above matrix, factor $L_{11}D$ out of the adjugate matrix and out of the determinant.
- Assume that L_{11} is very large. Take the limit of that equation as L_{11} approaches infinity.

Then,

$$i_1 = \frac{\left[N^2(R_s+R_B)+R_g\right]\varphi_s - N(R_s+2R_B)\varphi_g}{\left(N^2R_s+2R_g\right)(R_s+2R_B)} \quad (33)$$

$$i_2 = \frac{\left(N^2R_B-R_g\right)\varphi_s - N(R_s+2R_B)\varphi_g}{\left(N^2R_s+2R_g\right)(R_s+2R_B)} \quad (34)$$

$$i_3 = \frac{-N\varphi_s+2\varphi_g}{N^2R_s+2R_g} \quad (35)$$

The recovered signal from the remote source is

$$\Delta\varphi = i_2R_s = \frac{\left(N^2R_B-R_g\right)\varphi_s - N(R_s+2R_B)\varphi_g}{\left(N^2R_s+2R_g\right)(R_s+2R_B)}R_s = \Delta\varphi_s - \Delta\varphi_g \quad (36)$$

where

$$\Delta\varphi_s = \frac{\left(N^2R_B-R_g\right)R_s}{\left(N^2R_s+2R_g\right)(R_s+2R_B)}\varphi_s \quad (37)$$

and

$$\Delta\varphi_g = \frac{NR_s}{N^2R_s+2R_g}\varphi_g \quad (38)$$

Notice that, in general, the recovered signal $\Delta\varphi$ contains portions of both, φ_s, the voltage of the local generator, and φ_g, the voltage of the remote generator. These are labelled $\Delta\varphi_s$ and $\Delta\varphi_g$ respectively. For perfect rejection of the local signal, the coefficient of φ_s in Equations (36) and (37) must be zero; that is, $R_BN^2 - R_g = 0$.

$$\boxed{R_B = \frac{1}{N^2}R_g = \left(\frac{N_1}{N_3}\right)^2 R_g = \frac{L_{11}}{L_{33}}R_g} \qquad (39)$$

Then, the recovered voltage is...

$$\Delta\varphi = \frac{-NR_s}{N^2R_s+2R_g}\varphi_g \qquad (40)$$

The balanced-line one-transformer hybrid is usually configured with $R_s = R_g$. then,

$$\boxed{G = \frac{\Delta\varphi}{\varphi_g} = \frac{-N}{N^2+2}} \qquad (41)$$

From this equation it is evident that the relative magnitude of the recovered remote signal varies with the transformer turns ratio. Of course, for a given voltage φ_g on the remote generator, we would like to recover as large a signal $\Delta\varphi$ as is possible. Figure 15 below is a plot of Equation (41), G versus N.

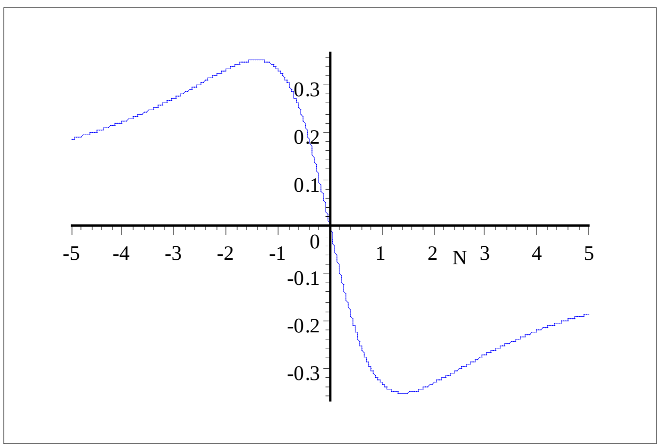


Figure 15. Plot of $G = \frac{-N}{N^2+2}$

The maximum amplitude of the recovered remote signal voltage occurs when

$$\frac{dG}{dN} = \frac{(N^2+2)(-1)-(-N)(2N)}{(N^2+2)^2} = 0 \quad .\Rightarrow. \quad N = \frac{N_3}{N_1} = \sqrt{2} = 1.414$$

This is the value for turns ratio N to give the largest magnitude of G, and hence, of $\Delta\varphi$. Then,

$$G = \frac{\Delta\varphi}{\varphi_g} = \frac{-N}{N^2+2} = \frac{-\sqrt{2}}{\left(\sqrt{2}\right)^2+2} = -\frac{\sqrt{2}}{4} = -0.3535$$

The Two-Transformer Hybrid

The two-transformer hybrid is portrayed below in Figures 16.

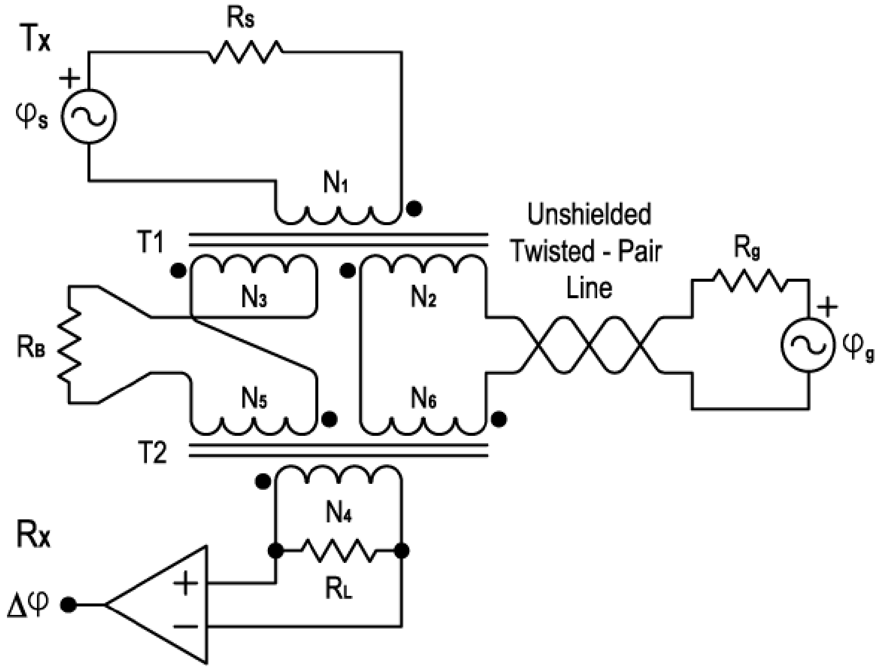


Figure 16. Two-Transformer Hybrid Topology.

Signal Name Review

First, a review of the signal names used in this paper:

T_x is the *name* of the locally generated signal intended to be transmitted, and may have different voltages depending upon where it is observed in the local circuit.

The voltage of the locally generated signal T_x is φ_s , measured directly across the local (ideal) voltage source generator.

R_x is the *name* of the remotely generated signal intended to be locally received, and may have different voltages depending upon where it is observed in the local circuit.

The voltage of the remotely generated signal R_x is $\Delta\varphi$, measured at the output of the hybrid difference amplifier relative to local reference (ground). The output of the difference amplifier is the *recovered remote signal*.

As in all hybrid circuits, the two-transformer hybrid should ideally reject all components of the locally transmitted signal in the recovered remote signal. Prior to deriving a mathematical description of the two-transformer hybrid, an intuitive explanation will be provided here.

An Intuitive Approach

Refer to Figure 16. First of all, observe that the hybrid circuit is not symmetrical. Local generator φ_s is connected to only one transformer, T_1, whereas the remote generator φ_g is connected, via the unshielded twisted-pair line, to both transformers, T_1 and T_2. Furthermore, the transformer connections to the balance impedance, R_B, are undotted-to-undotted, whereas the transformer connections to the remote generator are undotted-to-dotted. (The "dots" refer to the polarity dots on the transformers). Let phase(N) mean the phase of the signal at the dotted end of winding N relative to the dotted end of N_1.

Consider the transformer path N_1-N_3-N_4-N_6. Phase(N_6) = phase (N_1), because the non-dotted ends of all the windings coincide.
Consider the transformer path N_1-N_2-N_5-N_6. Phase(N_6) = phase (N_1) +180°, because the non-dotted end of N_2 is connected to the dotted end of N_5. The path segment N_2-N_5 reverses the phase 180°.

Consequently, the two signals from φ_s that reach N_6 via the two transformer paths are 180° out of phase with one another and tend to cancel each other. Since $N_2 = N_3 = N_4 = N_5$, the voltages induced in N_6 along the two different paths do (ideally) cancel one another.

Consider the transformer path N_1-N_2. Phase (N_2) = phase (N_1).
Along transformer path N_1-N_3-N_4-N_5 , phase(N_5) = phase (N_1), because the non-dotted end of N_3 is connected to the non-dotted end of N_4. N_2 and N_5 are connected in series, dotted end (positve) to non-dotted end (negative), like batteries connected in series. Consequently, there is a net non-zero voltage developed across the line.

Two-Transformer Hybrid Derivation

For clarity, the derivation will proceed in small steps. Consider, in general, a three-winding transformer. A two-winding transformer with one of the windings center-tapped is a three-winding transformer having two of the windings connected together.

The schematic diagram of a general three-winding transformer was presented in two ways in Figure 14, conventionally and as a toroid in order to better display the symmetry of the windings and voltages, and to make more obvious the direction of magnetic flux through the core.
To extend this general transformer circuit somewhat further, consider Figure 17 below.

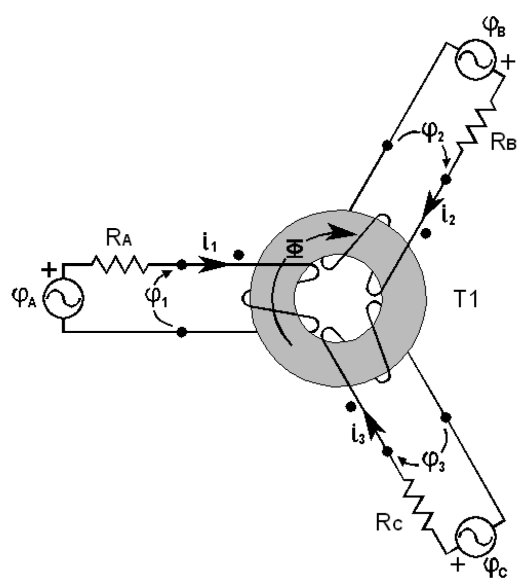


Figure 17. General Three-Winding Transformer with Generators and Load Resistors.

In Figure 17, generators and load resistors have been added to each of the three arms, with the positive side of each generator connected to a dotted terminal on the transformer. The general circuit still retains its symmetry. The following equation is representative of this network. All of the terms in the matrix are positive.

$$\begin{pmatrix} \varphi_A \\ \varphi_B \\ \varphi_C \end{pmatrix} = \begin{pmatrix} R_A + L_{11}D & L_{12}D & L_{13}D \\ L_{21}D & R_B + L_{22}D & L_{23}D \\ L_{31}D & L_{32}D & R_C + L_{33}D \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} \qquad (42)$$

or equivalently,

$$\begin{aligned} \varphi_A &= i_1(R_A + L_{11}D) + i_2(L_{12}D) + i_3(L_{13}D) \\ \varphi_B &= i_1(R_B + L_{21}D) + i_2(L_{22}D) + i_3(L_{23}D) \\ \varphi_C &= i_1(R_C + L_{31}D) + i_2(L_{32}D) + i_3(L_{33}D) \end{aligned}$$

In Equation (42) do not confuse the resistor R_B with the balance impedance R_B. To avoid confusion, the resistor R_B of Figure 17 will subsequently be designated as R_B(EQ 42). Again, $D = \frac{d}{dt}$ is the time differential operator. Replicate Equation (42) with other subscripts to represent the second transformer, T_2. Then,

$$\begin{pmatrix} \varphi_D \\ \varphi_E \\ \varphi_F \end{pmatrix} = \begin{pmatrix} R_D + L_{44}D & L_{45}D & L_{46}D \\ L_{54}D & R_E + L_{55}D & L_{56}D \\ L_{64}D & L_{65}D & R_F + L_{66}D \end{pmatrix} \begin{pmatrix} i_4 \\ i_5 \\ i_6 \end{pmatrix} \qquad (43)$$

In the two-transformer hybrid of Figure 16, two of these transformers are combined as shown below in Figure 18.

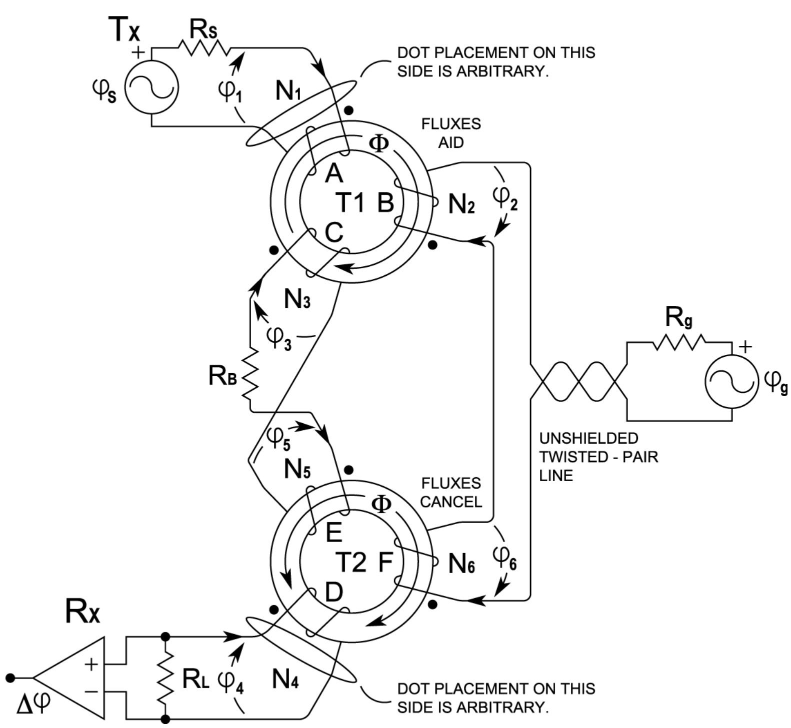


Figure 18. Two-Transformer Hybrid in Stylized Toroidal Representation.

There are four independent network currents: i_1, i_2, i_3, i_4, two non-zero voltage sources, φ_s, φ_g, and the output voltage, $\Delta\varphi$, which is the remotely generated signal that is locally recovered. The following six equations are basic descriptions of the circuit which augment the transformer equations.

$$\varphi_s = \varphi_1 + i_1 R_s \tag{44}$$

$$\varphi_g = \varphi_6 + \varphi_2 + i_2 R_g \tag{45}$$

$$0 = -\varphi_5 + \varphi_3 + i_3 R_B \tag{46}$$

$$0 = \varphi_4 + i_4 R_L \tag{47}$$

$$i_5 = -i_3 \tag{48}$$

$$i_6 = i_2 \tag{49}$$

Turns ratios of transformers T_1 and T_2 are...

$$\begin{aligned} N &= \frac{\varphi_2}{\varphi_1} = \frac{\varphi_6}{\varphi_4} = \frac{N_2}{N_1} = \frac{N_6}{N_4}, \text{ e.g., } \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \\ M &= \frac{\varphi_3}{\varphi_1} = \frac{\varphi_5}{\varphi_4} = \frac{N_3}{N_1} = \frac{N_5}{N_4}, \text{ e.g., } \frac{1/2}{\sqrt{2}} = \frac{\sqrt{2}}{4} \end{aligned} \tag{50}$$

Recall that $L_{ij} \propto N_i N_j$ for all i and j, and if we normalize all inductances to the bulk inductance L_{11} of coil winding N_1, in a manner similar to what we have done before, we have...

$$\left\{ \begin{array}{l|l|l} L_{11} = L_{44} = 1 \cdot L_{11} & L_{21} = L_{64} = NL_{11} & L_{31} = L_{54} = ML_{11} \\ L_{12} = L_{46} = NL_{11} & L_{22} = L_{66} = N^2 L_{11} & L_{32} = L_{56} = MNL_{11} \\ L_{13} = L_{45} = ML_{11} & L_{23} = L_{65} = NML_{11} & L_{33} = L_{55} = M^2 L_{11} \end{array} \right. \tag{51}$$

Then,

φ_1	=	$i_1(L_{11}D)+i_2(L_{12}D)+i_3(L_{13}D) = i_1(L_{11}D)+i_2(NL_{11}D)+i_3(ML_{11}D)$
φ_2	=	$i_1(L_{21}D)+i_2(L_{22}D)+i_3(L_{23}D) = i_1(NL_{11}D)+i_2\left(N^2L_{11}D\right)+i_3(MNL_{11}D)$
φ_3	=	$i_1(L_{31}D)+i_2(L_{32}D)+i_3(L_{33}D) = i_1(ML_{11}D)+i_2(MNL_{11}D)+i_3(M^2L_{11}D)$
φ_4	=	$i_4(L_{44}D)+i_5(L_{45}D)+i_6(L_{46}D) = i_4(L_{11}D)+(-i_3)(ML_{11}D)+i_2(NL_{11}D)$
φ_5	=	$i_4(L_{54}D)+i_5(L_{55}D)+i_6(L_{56}D) = i_4(ML_{11}D)+(-i_3)\left(M^2L_{11}D\right)+i_2(MNL_{11}D)$
φ_6	=	$i_4(L_{64}D)+i_5(L_{65}D)+i_6(L_{66}D) = i_4(NL_{11}D)+(-i_3)(MNL_{11}D)+i_2(N^2L_{11}D)$

(52)

or simply,

$$\varphi_1 = \frac{\varphi_2}{N} = \frac{\varphi_3}{M} = L_{11}D(i_1+i_2N+i_3M) \tag{53}$$

$$\varphi_4 = \frac{\varphi_5}{M} = \frac{\varphi_6}{N} = L_{11}D(i_2N - i_3M+i_4) \tag{54}$$

With this information in mind, consider Equations (44) through (47) and (53) and (54):

Equation (44)

$$\varphi_s = \varphi_1 + i_1R_s \Rightarrow \varphi_s = L_{11}D(i_1+i_2N+i_3M)+i_1R_s \Rightarrow$$

$$\boxed{\varphi_s = i_1(L_{11}D+R_s)+i_2NL_{11}D+i_3ML_{11}D}$$

Equation (45)

$$\varphi_g = \varphi_6+\varphi_2+i_2R_g \Rightarrow$$

$$\varphi_g = NL_{11}D(i_2N-i_3M+i_4)+NL_{11}D(i_1+i_2N+i_3M)+i_2R_g$$

$$\boxed{\varphi_g = i_1NL_{11}D+i_2\left(2N^2L_{11}D+R_g\right)+i_4NL_{11}D}$$

Equation (46)

$$0 = -\varphi_5+\varphi_3+i_3R_B = 0 \quad \Rightarrow$$

$$0 = -ML_{11}D(i_2N-i_3M+i_4)+ML_{11}D(i_1+i_2N+i_3M)+i_3R_B = 0 \quad \Rightarrow$$

$$0 = i_1(ML_{11}D)+i_3\left(R_B+2M^2L_{11}D\right)-i_4ML_{11}D = 0$$

Therefore,

$$\boxed{0 = i_1(ML_{11}D)+0\cdot i_2+\left(2M^2L_{11}D+R_B\right)i_3-i_4(ML_{11}D)}$$

Equation (47)

$$0 = \varphi_4+i_4R_L \Rightarrow L_{11}D\left(i_2N - i_3M+i_4\right)+i_4R_L = 0 \quad \Rightarrow$$

$$\boxed{0 = 0\cdot i_1+i_2(NL_{11}D)-i_3(ML_{11}D)+i_4(L_{11}D+R_L)}$$

Write these four resultant equations in the form of a 4x4 symmetric matrix.

$$\begin{pmatrix} \varphi_s \\ \varphi_g \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} L_{11}D + R_s & NL_{11}D & ML_{11}D & 0 \\ NL_{11}D & 2N^2L_{11}D + R_g & 0 & NL_{11}D \\ ML_{11}D & 0 & 2M^2L_{11}D + R_B & -ML_{11}D \\ 0 & NL_{11}D & -ML_{11}D & L_{11}D + R_L \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{pmatrix} \tag{55}$$

which is a symmetric matrix.

or, $$\varphi = Z \bullet I$$

where $\varphi = \begin{pmatrix} \varphi_s \\ \varphi_g \\ 0 \\ 0 \end{pmatrix}$, $I = \begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{pmatrix}$, and $Z = \begin{pmatrix} L_{11}D + R_s & NL_{11}D & ML_{11}D & 0 \\ NL_{11}D & 2N^2L_{11}D + R_g & 0 & NL_{11}D \\ ML_{11}D & 0 & 2M^2L_{11}D + R_B & -ML_{11}D \\ 0 & NL_{11}D & -ML_{11}D & L_{11}D + R_L \end{pmatrix}$

a symmetric matrix. Then,

$$I = Z^{-1}\varphi \quad \text{where} \quad Z^{-1} = \frac{adj(Z)}{\Delta}$$

Z^{-1} is the inverse matrix, adj(Z) is the adjugate matrix, and Δ = det (Z) is the determinant of matrix Z.

Equation (55) can be regarded as the comprehensive equation of the two-transformer hybrid.

Performing these calculations by hand is messy and very error-prone, due to the algebraic complexity of the resulting terms in both, the determinant and the adjugate. When performing such derivations and calculations, I advise using a general mathematics computer program that has symbolic algebra capability such as Waterloo Maple, Macsyma, or Mathematica. Even so, the strings of algebraic expressions generated are very long.

However, there is a way around this difficulty.

Factor out $L_{11}^2D^2$ from the determinant and from the adjugate to cancel them out. Some of the terms in these two will include factors of the form $\frac{1}{L_{11}D}$ or $\frac{1}{L_{11}^2D^2}$. Since we are dealing with an ideal transformer, L_{11} will be very large, so those terms can be set to zero. When this is done, we find that...

$$\begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{pmatrix} = \frac{1}{\Delta} \begin{pmatrix} M^2(4N^2R_L + R_g) + N^2R_B & -N(2M^2R_L + R_B) & -M(2N^2R_L + R_g) & N^2R_B - M^2R_g \\ -N(2M^2R_L + R_B) & M^2(R_L + R_s) + R_B & -NM(R_s - R_L) & -N(2M^2R_s + R_B) \\ -M(2N^2R_L + R_g) & -NM(R_s - R_L) & N^2(R_L + R_s) + R_g & M(2N^2R_s + R_g) \\ N^2R_B - M^2R_g & -N(2M^2R_s + R_B) & M(2N^2R_s + R_g) & M^2(4N^2R_s + R_g) + N^2R_B \end{pmatrix} \tag{56}$$

where

$$\det A = \Delta = 4N^2M^2R_sR_L + \left(N^2R_B + M^2R_g\right)(R_s + R_L) + R_gR_B \tag{57}$$

Therefore,

$$i_4 = \frac{\left(N^2R_B - M^2R_g\right)\varphi_s - N\left(2M^2R_s + R_B\right)\varphi_g}{4N^2M^2R_sR_L + \left(N^2R_B + M^2R_g\right)(R_s + R_L) + R_gR_B}$$

and the remotely generated voltage recovered in the local hybrid is

$$\Delta\varphi = i_4R_L = \frac{\left(N^2R_B - M^2R_g\right)\varphi_s - N\left(2M^2R_s + R_B\right)\varphi_g}{4N^2M^2R_sR_L + \left(N^2R_B + M^2R_g\right)(R_s + R_L) + R_gR_B}R_L$$

$$= \frac{\left(N^2R_B - M^2R_g\right)\varphi_s - N\left(2M^2R_s + R_B\right)\varphi_g}{4N^2M^2R_s + \left(N^2R_B + M^2R_g\right)\left(1 + \frac{R_s}{R_L}\right) + \frac{R_gR_B}{R_L}} \tag{58}$$

where $N = \frac{N_2}{N_1} = \frac{N_6}{N_4}$ and $M = \frac{N_3}{N_1} = \frac{N_5}{N_4}$.

If

$$N^2R_B = M^2R_g, \text{ that is, if } R_B = \left(\frac{M}{N}\right)^2 R_g \tag{59}$$

then the hybrid will have perfect discrimination against the locally generated signal φ_s . Substituting this equation into Equation (58) and simplifying, we find that the recovered signal is

$$\Delta\varphi = \frac{-N\varphi_g}{2N^2 + \frac{R_g}{R_L}} \tag{60}$$

Equations (58), (59) and (60) are boxed to call attention to them as the basic equations for use in designing two-transformer hybrids.

If $R_B = \left(\frac{M}{N}\right)^2 R_g$ and $R_L \to \infty$ (the usual situation with a difference amplifier), then...

$$\Delta\varphi = i_4R_L = -\frac{\varphi_g}{2N} \tag{61}$$

End of Derivation.

Example 1

Reference Equation (61). Suppose that N = M = ½.

Then $\Delta\varphi = -\varphi_g$; that is, if we consider the network from remote generator to difference amplifier, there is no voltage loss.

Example2

Suppose a two-transformer hybrid has turns-ratios of $N = \frac{N_2}{N_1} = \frac{N_6}{N_4} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ and $M = \frac{N_3}{N_1} = \frac{N_5}{N_4} = \frac{1/2}{\sqrt{2}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$. Further suppose that $\varphi_s = \varphi_g = 10\ VPk$.

The impedance looking into the hybrid is $R_g = 100\,\Omega$. To match this impedance and to eliminate components of the locally generated signal from the hybrid output, the value of R_B must be $R_B = \left(\frac{M}{N}\right)^2 R_g = \frac{1}{4}(100\,\Omega) = 25\ \Omega$.

Substituting these numbers into Equation (60), we find that $\Delta\varphi = -3.5355\ VPk$, the voltage of the recovered signal. It contains only a portion of signal φ_g . The signal φ_s has been totally eliminated. This second example was chosen merely to show how the equations are used. Usually in a two-transformer hybrid, the turns ratios would be selected to provide either no insertion loss (perhaps gain) and impedance matching.

A Few Observations and Comparisons of Passive Hybrids

- Because they are constructed of resistors, which are power-dissipating components, the insertion loss of a lattice hybrid is usually inferior to the insertion loss of a transformer hybrid.

The asymmetric lattice hybrid circuit topology has several advantages over the symmetric lattice hybrid topology.

- It is physically simpler; that is, it has fewer components.
- It is mathematically more tractable (easier to analyze).

The balanced-line one-transformer hybrid has these advantages:

- The two-wire port is balanced with respect to ground.
- The transformer provides DC isolation of the local circuit from the two-wire line.

The balanced-line one-transformer hybrid has these disadvantages:

- The local signal ports (four-wire ports) are single-ended; that is, they are not balanced with respect to ground.
- There is a 3 dB insertion loss.

The unbalanced-line one-transformer hybrid has limited usefulness in xDSL applications, because its output port to a two-wire line connection is single-ended rather than balanced.

The two-transformer hybrid has these advantages:

- The transformers provide DC isolation of the local circuit from the two-wire line.
- It is used with a balanced two-wire line.
- It can have zero (or better) insertion loss.

The two-transformer hybrid has this disadvantage:

- It requires two transformers rather than one or none.

BIBLIOGRAPHY

[1] Rauschmayer, Dennis J.: ADSL/V*DSL Principles;* A Practical and Precise Study of Symmetric Digital Subscriber Lines and Very High Speed Digital Subscriber Lines; Macmillan Technical Publishing, 1999. ISBN 1-57870-015-9.

[2] Starr, Cioffi and Silverman: *Understanding Digital Subscriber Line Technology*; Prentice Hall PTR, 1999. ISBN 0-13-780545-4.

[3] Chen, Walter Y.: *DSL Simulation Techniques and Standards Development for Digital Subscriber Line Systems;* Macmillan Technical Publishing, 1998. ISBN 1-57870-017-5.

[4] Grossner, Nathan R.: *Transformers for Electronic Circuits*; 2nd Edition; McGraw-Hill Book Company, 1983. ISBN 0-07-024979-2.

[5] Gayler, Winston D.: *Telephone Voice Transmission, Standards and Measurements*. Prentice-Hall, 1989. ISBN 0-13-9027769.

[6] Reeve, Whitham D.: *Subscriber Loop Signaling and Transmission Handbook: Analog*. IEEE, 1992. ISBN 0-87942-274-2.
Wiley, (Soft cover) ISBN 0-7803-4746-3.

End

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