Low Complexity Channel Shortening for Discrete Multitone Modulation Systems

Van Minh Nguyen* and Stefano Tomasin†
Mathematical and Algorithmic Sciences Laboratory, Huawei Technologies, Boulogne-Billancourt 92100, France
* vanminh.nguyen@huawei.com, † tomasin@dei.unipd.it

***Abstract*—In discrete multitone (DMT) modulation systems, the channel duration can be longer than the cyclic prefix, yielding both inter-block and inter-carrier interference. In order to avoid these effects, time-domain equalization (TEQ) techniques are usually employed before DMT demodulation. This letter introduces a modification of the iterative minimum delay spread (MDS) method proposed by Lopez-Valcarce in [1] to adapt the TEQ coefficients to the channel. The proposed solution is based on a) the approximation to the solution of a set of equation, and b) a new reference time selection. The resulting scheme requires only a single matrix inversion at the last iteration rather than one per iteration, achieving a significantly lower implementation complexity than iterative MDS, while performing similarly in terms of spectral efficiency.**

***Index Terms*—Digital Communications, Discrete Multitone, Time-Domain Equalization, Subscriber Loops.**

I. Introduction

Discrete multitone (DMT) is used as the modulation technique in different digital subscriber line (xDSL) systems, allowing the conversion of dispersive channels into a set of parallel channels, at the cost of reduced spectral efficiency due to the transmission of the cyclic prefix (CP). In order to allow for a shorter CP while keeping the benefits of DMT, a channel shortening technique, also called time-domain equalization (TEQ), is typically adopted (we refer the reader to [2] for a survey on the topic). This technique makes use of a filter on the time-domain received signal that compensates for the effects of a shorter CP. In particular, the resulting channel as the cascade of the channel and the TEQ filter is effectively shorter than the CP length. Therefore, at the expense of an increased receiver complexity, inter-block interference (IBI) and inter-carrier interference (ICI) are reduced. However, system performance depends closely on the TEQ design.

Various solutions for the design of the channel shortening filter have been proposed in the literature [2]. Among those, the iterative minimum delay spread (MDS) method of [1] always converges, while computing one matrix inversion per iteration, thus yielding significant computational complexity and memory requirement. All other methods require the computation of eigenvectors of channel-dependent matrices, resulting again in a high computational complexity typically of the order of N^3, where N is in the order of the total length of the TEQ filter and the channel impulse response [3]. The method proposed by Chow *et al.* in [4] reduces the complexity to the order of $N \log_2 N$ by using an iterative solution based on discrete Fourier transform (DFT) instead of matrix inversion. It is, however, well known that Chow's method (CM) suffers from the unstable convergence problem that is even more critical than the computational complexity.

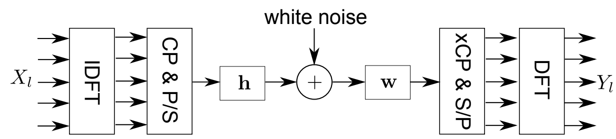

Fig. 1. DMT model. P/S: parallel to serial, S/P: serial to parallel, CP: add CP, xCP: remove CP.

In this paper, we propose a modification of the iterative MDS to adapt the TEQ filter coefficients to the channel. The proposed solution is based on *a*) a modification of the original linear equation system that must be solved at each iteration, and *b*) a new reference time selection. Thereby, we can avoid the matrix inversion at each iteration and only need a single matrix inversion at the end of the iterative process to obtain the shortening filter from the optimized effective channel response. The resulting scheme, denoted *fast* MDS (FMDS), has a significantly reduced implementation complexity with respect to MDS while performing similarly in terms of spectral efficiency.

Notation: We use italic letters for scalars, boldface small letters for vectors ($\mathbf{x}$), and boldface capital letters for matrices ($\mathbf{X}$). Also, $(\cdot)^T$ denotes transpose, $\mathrm{diag}(\mathbf{x})$ denotes the diagonal matrix whose main diagonal is $\mathbf{x}$. For vector $\mathbf{x}$, $\mathbf{x}(i)$ denotes the i-th entry of $\mathbf{x}$, and $\mathbf{x}(\mathcal{I})$ denotes the vector composed of elements of $\mathbf{x}$ specified by index set $\mathcal{I}$. Similarly, for matrix $\mathbf{A}$, $a_{i,j}$ denotes its element at i-th row and j-th column. $||\mathbf{x}||$ is the norm-2 of $\mathbf{x}$. $\mathbf{x} \odot \mathbf{y}$, and $\mathbf{x} \oslash \mathbf{y}$ are the element-wise product and element-wise division, respectively.

II. Background

A. System Description

With reference to Fig. 1, we consider a discrete multitone (DMT) transmission over a dispersive channel with sampled-time impulse response $\mathbf{h} = [\mathbf{h}(0), \ldots, \mathbf{h}(L_\mathrm{h}-1)]^T$. The DFT size of the DMT is N_DFT and the CP comprises ν samples. The sampled received signal is filtered with a TEQ with impulse response $\mathbf{w} = [\mathbf{w}(0), \ldots, \mathbf{w}(L_\mathrm{w}-1)]^T$. The effective channel $\mathbf{b}$, as the convolution of $\mathbf{h}$ and $\mathbf{w}$, is a vector of length $L_\mathrm{b} = L_\mathrm{h} + L_\mathrm{w} - 1$. Define $\mathbf{H}$ as the rectangular $L_\mathrm{b} \times L_\mathrm{w}$ Toeplitz matrix with the first row $[\mathbf{h}(0), 0, \cdots, 0]$ and the first

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column $[\mathbf{h}(0), \mathbf{h}(1), \cdots, \mathbf{h}(L_\mathrm{h}-1), 0, \cdots, 0]^T$, then we have $\mathbf{b} = \mathbf{H}\mathbf{w}$. For notational convenience, we also denote by $\mathbf{w}_i$, and $\mathbf{b}_i$ the TEQ filter and the effective channel, respectively, at iteration $i \geq 0$, and define $\bar{n}_i$ to be the center-of-mass (CoM) of $\mathbf{b}_i$. In addition, denote $\mathbf{C} = \mathbf{H}^T\mathbf{H}$. Lastly, for a given scalar k, we define

$$\mathbf{v}_k = [-k, 1-k, \cdots, L_\mathrm{b} - 1 - k]^T, \quad (1)$$

$$\mathbf{V}_k = \mathrm{diag}(\mathbf{v}_k). \quad (2)$$

B. The Iterative MDS Method

The iterative method proposed by [1] combines the principle of *minimum delay spread* proposed by [5] to shorten the channel and a tracking of the CoM of the effective channel, which is shortened over iterations. In particular, [1] used the inverse power method to iteratively solve the optimization problem, which is formulated as a single Rayleigh quotient problem so as to avoid the computation of eigenvectors. As our proposed solution for TEQ design is based on the method of [1], it is briefly summarized here.

Initialization. Set $\mathbf{w}_0 = [1/\sqrt{c_{1,1}}, 0, \ldots, 0]^T$, $\mathbf{A}_0 = \mathbf{H}^T\mathbf{V}^2_{\bar{n}_0}\mathbf{H}$, and $\mathbf{B}_0 = \mathbf{H}^T\mathbf{V}_{\bar{n}_0}\mathbf{H}$.

Iteration. For iteration $i > 0$, the TEQ filter $\mathbf{w}_i$ is obtained by first solving into $\tilde{\mathbf{w}}_i$ the equation

$$\mathbf{A}_{i-1}\tilde{\mathbf{w}}_i = \mathbf{C}\mathbf{w}_{i-1}, \quad (3)$$

and then normalizing $\tilde{\mathbf{w}}_i$ so that $\mathbf{b}_i$ has unit power, i.e.,

$$\mathbf{w}_i = (\tilde{\mathbf{w}}_i^T\mathbf{C}\tilde{\mathbf{w}}_i)^{-1/2}\tilde{\mathbf{w}}_i\,.$$

Note that solving (3) requires the inversion of the $L_\mathrm{w} \times L_\mathrm{w}$ matrix $\mathbf{A}_{i-1}$. Parameters are updated as follows

$$\begin{aligned}\delta_i &:= \bar{n}_i - \bar{n}_{i-1} = \mathbf{w}_i^T\mathbf{B}_{i-1}\mathbf{w}_i,\\ \mathbf{A}_i &= \mathbf{A}_{i-1} + \delta_i^2\mathbf{C} - 2\delta_i\mathbf{B}_{i-1}, \quad (4)\\ \mathbf{B}_i &= \mathbf{B}_{i-1} - \delta_i\mathbf{C}.\end{aligned}$$

Termination. The algorithm terminates when it reaches a predefined number of iterations.

III. Proposed Solution

Although MDS is one of the solutions that have the lowest complexity after Chow's method, see [3], it requires matrix operations and especially one linear system solution per iteration to solve (3). Matrix operations are still quite complex and yield problems of numerical stability in microprocessors that prevent their application in most cases. In the specific context of xDSL, the large number of taps of the channel and of the filter is translated into a large size of the matrices to be handled, thus worsening the problem. Therefore, we propose the *Fast* MDS solution, which eliminates matrix operations in the iterative process apart for one matrix inversion in the last iteration.

A. Reference Time

A first difference of FMDS with respect to MDS is that it uses a different reference time. We aim at squeezing the channel around the maximum energy part of the channel, thus enhancing convergence stability and signal to interference ratio. To this end, we choose as reference time the tap of the equivalent channel at current iteration $\mathbf{b}_i$ having the maximum power within the maximum energy window of size of the CP. Formally, let $\mathcal{S}_i$ be the window of the size ν given as

$$\mathcal{S}_i := (n^*, n^*+1, \cdots, n^*+\nu-1), \quad (5)$$

with n^* chosen to maximize the energy of effective channel at the i-th iteration $\mathbf{b}_i$ on $\mathcal{S}_i$, i.e.,

$$n^* := \underset{n=0,\cdots,L_\mathrm{b}-\nu}{\mathrm{argmax}} \sum_{j=0}^{\nu-1} \mathbf{b}_i(n+j)^2. \quad (6)$$

Note that the windowed $\mathbf{b}_i$ captures the equivalent channel that we would like to be seen by the DMT system, so that the energy of the residual interference is minimized.

Denote by κ_i the new reference time, which is the index in $\mathcal{S}_i$ of the tap having the maximum power, i.e.,

$$\kappa_i = \underset{j\in\mathcal{S}_i}{\mathrm{argmax}}\, \mathbf{b}_i(j)^2. \quad (7)$$

This choice has been compared with other approaches and has turned out to provide the best performance.

B. The FMDS Algorithm

By merging the set of equations (4) and considering the new reference time, we obtain

$$\mathbf{A}_i = \mathbf{H}^T\mathbf{D}_i\mathbf{H}, \quad (8)$$

where $\mathbf{D}_i = \mathbf{V}^2_{\kappa_i}$ and we also note that the main diagonal of $\mathbf{D}_i$ is given by

$$\mathbf{d}_i = \mathbf{v}_{\kappa_i} \odot \mathbf{v}_{\kappa_i}, \quad (9)$$

which does not require matrix operations.

The second difference of FMDS with respect to MDS is the update of the TEQ filter. Noting that $\mathbf{b}_i = \mathbf{H}\mathbf{w}_i$, from (3) we obtain the following approximation

$$\begin{aligned}\mathbf{A}_{i-1}\mathbf{w}_i = \mathbf{C}\mathbf{w}_{i-1} &\overset{(*)}{\Rightarrow} \mathbf{H}^T\mathbf{D}_{i-1}\mathbf{b}_i = \mathbf{H}^T\mathbf{b}_{i-1}\\ &\Leftrightarrow \mathbf{H}^T(\mathbf{D}_{i-1}\mathbf{b}_i - \mathbf{b}_{i-1}) = \mathbf{0}, \quad (10)\end{aligned}$$

where $(*)$ is one-side derivation because we are moving from the linear equation system defined for $\mathbf{w}_i$ to another one defined for $\mathbf{b}_i$. Thereby, vector $\mathbf{D}_{i-1}\mathbf{b}_i - \mathbf{b}_{i-1}$ belongs to the null space of matrix $\mathbf{H}^T$. In particular, we consider as solution of (10)

$$\mathbf{D}_{i-1}\mathbf{b}_i - \mathbf{b}_{i-1} = \mathbf{0}. \quad (11)$$

Note that this is not the only solution of the equation, as any vector in the null space of $\mathbf{H}^T$ would be a solution. As a result of this approximation, we can easily compute $\mathbf{b}_i$ by a simple element-wise division since $\mathbf{D}_{i-1}$ is diagonal. Thus, the matrix inversion in each iteration is eliminated. We note in particular

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that $\mathbf{d}_{i-1}(\kappa_{i-1}) = 0$ by construction. Thus, for the element-wise division, we set $\mathbf{d}_{i-1}(\kappa_{i-1}) := 1$, which physically means that the reference channel tap is kept unchanged. Note that these approximations will lead in general FMDS and MDS to different solutions.

The iterative algorithm using (11) is as follows:

- *Initialization*. Set $\mathbf{b}_0 = [\mathbf{h}(0), \ldots, \mathbf{h}(L_\mathrm{h}-1), 0, \ldots, 0]^T$.
- *Iteration*. For iteration $i > 0$, compute the reference time κ_{i-1} of $\mathbf{b}_{i-1}$, and $\mathbf{v}_{\kappa_{i-1}}$ according to (1) and (2), then $\mathbf{d}_{i-1}$ according to (9) and set $\mathbf{d}_{i-1}(\kappa_{i-1}) = 1$. After that, $\mathbf{b}_i$ is computed and normalized to unit power as follows $\mathbf{b}_i = \mathbf{b}_{i-1} \oslash \mathbf{d}_{i-1}$, and $\mathbf{b}_i \leftarrow \mathbf{b}_i/||\mathbf{b}_i||$.
- *Termination*. When a specified number of iterations is reached, the algorithm stops and computes the TEQ filter as $\mathbf{w} = \mathbf{C}^{-1}\mathbf{H}^T\mathbf{b}_i$.

We can see that the effective channel at iteration i is obtained as $\mathbf{b}_i(\kappa_{i-1}) = \mathbf{b}_{i-1}(\kappa_{i-1})$ and $\mathbf{b}_i(n) = \mathbf{b}_{i-1}(n)/(n-\kappa_{i-1})^2$ for $n \neq \kappa_{i-1}$. Hence, the farther is a tap from the reference one, the more quickly its power is decreased. Therefore, the effective channel contracts around the reference tap after each iteration, and up to a number of iterations a large number of taps vanish, resulting in a short effective channel. The more iterations are performed, the shorter is the effective channel, resulting in the convergence to the effective channel with few taps.

IV. Performance Assessment

Typical xDSL parameters and loops lengths have been considered with DFT size $N_\mathrm{DFT} = 128$ and CP length $\nu = 8$. The upstream (i.e., from subscriber to the center) has been considered which is transmitted on the 6th to 31st subcarriers of the bandwidth. As per ADSL standard [6], basically the channel response is only available in the transmission band. This partial channel response (PCR) results in an estimated time response that does not reflect the true channel, see examples in Fig. 2. However, acquiring the full channel response (FCR) is only possible during the initialization phase with special efforts. To assess the robustness of the TEQ design against the channel response, both PCR and FCR scenarios will be evaluated in the sequel in which the corresponding channel response is used as the input to the TEQ design.

With reference to Fig. 1, let X_l and Y_l denote, respectively, the transmitted symbol with unit power and the received symbol on subcarrier l. The noise plus interference power on subcarrier l is estimated as

$$\sigma_l^2 = \mathbb{E}\{||Y_l - X_l B_l||^2\},$$

where $[B_0, B_1, \ldots, B_{N_\mathrm{DFT}-1}]^T$ is the DFT of $\mathbf{b}$. The signal to interference plus noise ratio (SINR) estimate is thus obtained as $\mathrm{SINR}_l = ||B_l||^2/\sigma_l^2$. The bit rate (in kbps) used as the performance metric is given by

$$\mathcal{C} = \beta \sum_{l=6}^{31} \log_2(1 + \mathrm{SINR}_l),$$

where $\beta = 4.3125$ kHz is the subcarrier bandwidth.

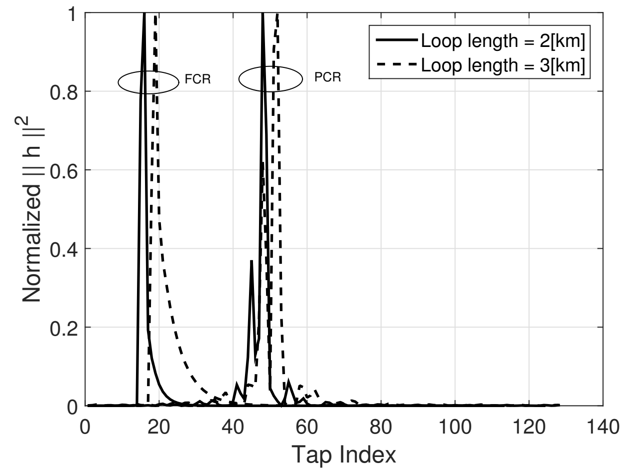

Fig. 2. Channel impulse response in the FCR and PCR scenarios generated with the standard ABCD parameter model for different loop lengths.

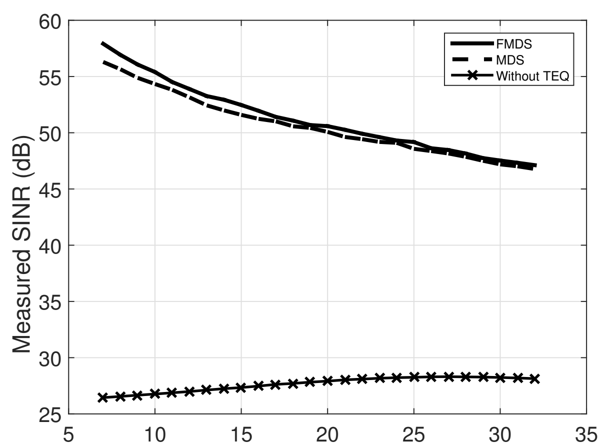

Fig. 3. Measured SINR. FCR, loop of 3,000 m, $\mathrm{SNR} = 100$ dB.

A. Performance Evaluation

Fig. 3 shows as an example of the measured SINR per subcarrier for an ADSL loop of length of 3,000 m with signal to noise ratio (SNR) of 100 dB. Here, we can see that for this negligible-noise scenario, IBI is the dominant performance impairment. In this case, TEQ is particularly important. Indeed, from the same figure we can observe that both MDS and FMDS have very close performance and significantly increase the resulting SINR.

Further, Fig.s 4 and 5 show the performance of the two solutions versus the SNR for loop lengths of 2,000 m, while Fig.s 6 and 7 show the performance for loop lengths of 3,000 m. For each loop length, we plot the bit rate for the FCR and PCR scenarios. From the figures, we observe that MDS and FMDS perform very closely for FCR, while we observe a small gap when the PCR is used.

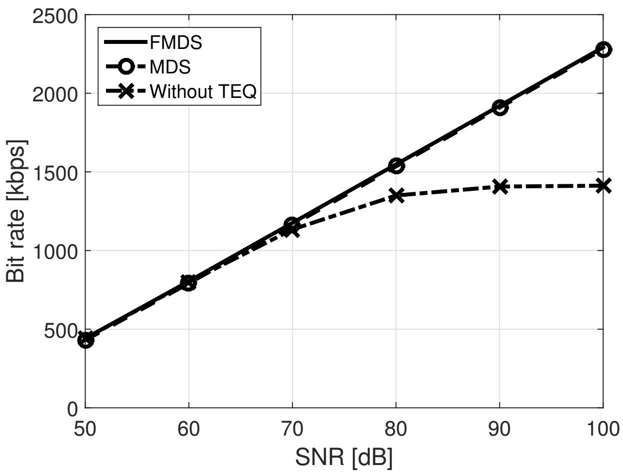

Fig. 4. Performance comparison with loop length of 2,000 m. FCR.

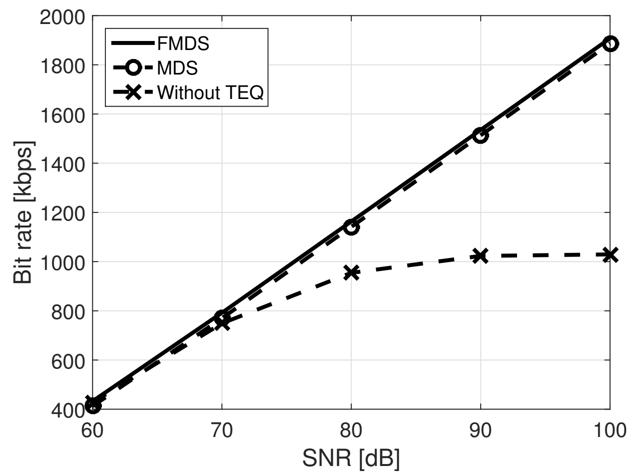

Fig. 6. Performance comparison with loop length of 3,000 m. FCR.

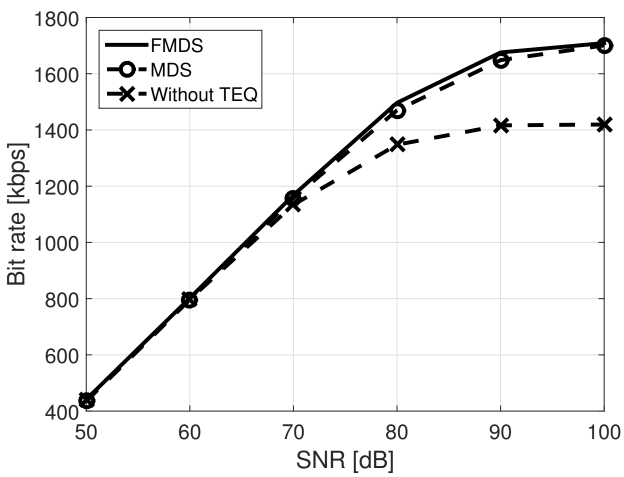

Fig. 5. Performance comparison with loop length of 2,000 m. PCR.

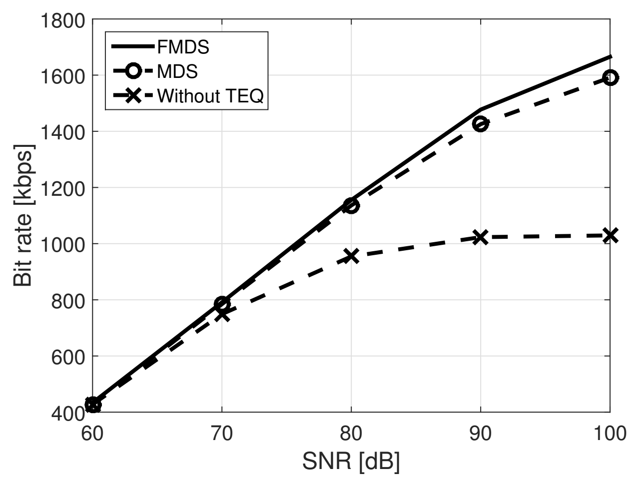

Fig. 7. Performance comparison with loop length of 3,000 m. PCR.

B. Complexity Comparison

Complexity is expressed in terms of the number of multiplication-and-accumulations (MAC), and for simplicity we assume that division has the same complexity of multiplication. With FMDS, reference time computation is in the order of $\nu L_{\rm b}$, and inversion of $\mathbf{C}$ in the final step is $3L_{\rm w}^2$ instead of $(2/3)L_{\rm w}^3$ considering its special structure. Then, its complexity is obtained as

$$C_{\rm FMDS} = \underbrace{(4+\nu)L_{\rm b}I_{\rm FMDS}}_{\text{iteration}} + \underbrace{L_{\rm b}L_{\rm w} + 4L_{\rm w}^2 + 0.5L_{\rm b}L_{\rm w}^2}_{\text{final }\mathbf{w}},$$

where $I_{\rm FMDS}$ is the number of iterations.

For comparison purpose, we also provide here the complexity of CM and MDS by refining the coarse estimation in [3]. Let $I_{\rm CM}$ and $I_{\rm MDS}$ be the number of iterations used for CM and MDS, respectively, we have

$$C_{\rm CM} = \underbrace{3N_{\rm DFT}}_{\text{initialization}} + \underbrace{(9N_{\rm DFT} + 4N_{\rm DFT}\log_2 N_{\rm DFT})I_{\rm CM}}_{\text{iteration}},$$

$$C_{\rm MDS} = \underbrace{(2.5L_{\rm w}^2 + 2L_{\rm w})L_{\rm b}}_{\text{initialization}} + \underbrace{[(2/3)L_{\rm w}^3 + 10L_{\rm w}^2]I_{\rm MDS}}_{\text{iteration}},$$

where basically, in each iteration CM requires 4 DFT operations, while MDS is more costly due to inversion of $\mathbf{A}_{i-1}$ and updates in (4).

Table I shows the complexity of the discussed methods. Note that the relative high number of iterations for CM is what is used in practice to seek the convergence even for problematic channels, while FMDS requires slightly longer filter w.r.t. MDS to compensate for the approximation effects.

We observe that FMDS reduces the design complexity of about two orders of magnitudes w.r.t. MDS. Indeed, thanks to the lower number of required iterations to achieve convergence, it yields also a complexity reduction of about 15% w.r.t. to CM. Moreover, since FMDS uses element-wise operations

TABLE I
COMPLEXITY EVALUATION

Method	Setting	MAC normalized to CM
CM	$L_{\mathrm{w}} = 120$, $I_{\mathrm{CM}} = 460$	1.000
FMDS	$L_{\mathrm{w}} = 120$, $I_{\mathrm{FMDS}} = 5$	0.865
MDS	$L_{\mathrm{w}} = 90$, $I_{\mathrm{MDS}} = 20$	7.250

and only a single matrix operation in comparison to I_{MDS} matrix inversions and various matrix operations in MDS, FMDS requires much less memory storage than MDS. Thus, we conclude that FMDS is much more suited than MDS for implementation into a chip.

V. CONCLUSIONS

We have proposed the FMDS algorithm for the design of channel shortening filters. FMDS turns out to require only one matrix operation while having only element-wise MAC during the iterative process. When comparing FMDS with existing solutions, we conclude that it significantly outperforms current low-complexity solutions, achieving bit rate close to MDS with two order of magnitude less complexity.

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