

OPPOSITION-BASED LEARNING PARTICLE SWARM OPTIMIZATION OF RUNNING GAIT FOR HUMANOID ROBOT

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Abstract- This paper investigates the problem of running gait optimization for humanoid robot. In order to reduce energy consumption and guarantee the dynamic balance including both horizontal and vertical stability, a novel running gait generation based on opposition-based learning particle swarm optimization (PSO) is proposed which aims at high energy efficiency with better stability. In the proposed scheme of running gait generation, a population initiation policy based on domain knowledge is employed, which helps to guide searching direction guidance at the beginning. A population update mechanism based on opposition learning is proposed for speeding up the convergence and improving the diversity. Simulation results validate the proposed method.

Index terms: gait planning, Humanoid Robot, Yaw Moment, opposition learning, ZMP.

I. INTRODUCTION

In recent decades, humanoid biped robots have gained the increasing attention from scientific community for the ability to facilitate human lives and work for human beings in industrial and domestic surroundings. However, the problems of high energy consumption and slow movement limit the practical application of humanoid robots. Fully Passive robot [1, 2, 3] can generate natural gaits and good energy efficiency, but the problem of less robustness and slow movement are still an open question. Traditional ZMP-based methods can generate robust and versatile locomotion, while the high energy consumption limits its further application. How to generate the gait with high energy efficiency and stability has been a hot topic in the field of humanoid robot.

In view of the aforementioned problems, some researchers have proposed methods to get higher energy efficiency and better stability. Hyeok-Ki Shin [4] proposed a gait plan method based on three-mass inverted pendulum mode, which gets optimized gait parameters by minimizing joints torque, while this method didn't take the vertical balance into account.

Fujimoto Y[5] generated an energy-optimized trajectory which minimized the input energy in a cycle. And the computation burden would become heavy while the DOFs increase. Lingyun Hu[6] formulated the gait pattern generation into a multi-objective optimization problem and generated the optimal joints trajectories based on Spline-based estimation of distribution algorithm, which can deal with complex probability distribution function without a prior knowledge. However, the gait generated by this method is not natural and interpretation of transition probability models needs further study. Liu [7] proposed an energy-efficiency optimization method, which generated the biped gait while minimizing energy consumption in the ZMP stability domain. While it is easy to involve in minimal point for searching along the gradient direction and lack the consideration of vertical balance.

In order to overcome these problems mentioned above, a novel running gait generation based on opposition learning particle swarm is proposed. The main focus in this paper is to utilize opposition-based learning particle swarm to optimize the parameters of gait with the minimum energy consumption and obtain the desired joints trajectories, which guarantee both horizontal and vertical dynamic balance. In order to speed up convergence, a policy of initiating population based on domain knowledge and a population update mechanism are proposed.

II. PROBLEM STATE

ZMP is one of the most popular stability criteria. However, ZMP can only guarantee the horizontal moment balance according to the definition of ZMP [8,9,10]. The undesired moment of yaw axis with respect to the support leg, usually called yaw moment [11], would be generated for the forward movement, which is usually counteracted by the ground friction ground. When humanoid robot walk too fast and the yaw moment exceeds the maximum ground friction moment, the rotation would happen. Thus, the problem of the horizontal and vertical dynamic balance needs to be investigated.

According to [12], the yaw moment could be expressed as follow.

$$M_{z} = \sum_{i=1}^{n} m_{i} \ddot{y}_{i} (x_{i} - x_{zmp}) - \sum_{i=1}^{n} m_{i} \ddot{x}_{i} (y_{i} - y_{zmp})$$
(1)

where M_z represents the yaw moment, m_i is the mass of the *i*th link and (x_i, y_i) is the coordinate of the *i*th link. And the ZMP coordinates can be represented by equation (2), which ignores the inertia effect.

$$\begin{cases} x_{zmp} = \frac{\sum_{i=1}^{n} [m_i(\ddot{z}_i + g)x_i - m_i\ddot{x}_i z_i]}{\sum_{i=1}^{n} m_i(\ddot{z}_i + g)} \\ y_{zmp} = \frac{\sum_{i=1}^{n} [m_i(\ddot{z}_i + g)y_i - m_i\ddot{x}_i z_i]}{\sum_{i=1}^{n} m_i(\ddot{z}_i + g)} \end{cases}$$
(2)

where x_i , y_i and z_i represent the coordinate of the *i*th link, and g is gravity force. One dot and two dots represent first and second derivatives with respect to time.

According to [13], the position of mass play a great role in the coordinates of ZMP, thus, we can adjust the ZMP position through modify the mass position via control the hip joint. On the other hand, the parameters of the inclination ankle of trunk, step length and speed are strong related with the stability and energy efficiency. In order to optimize achieve good performance, the following running gait parameters are defined as follows.

$$X_i = [d_{xs}, d_{xe}, D_s, \theta_{trunk}, H_f, S_f]$$
(3)

where d_{xs} , d_{xe} denotes the distance between CTHJ(Center of two hips joints) and the ankle of support leg at the time of landing and leaving, respectively. D_s represents step length, θ_{trunk} expresses the angle between trunk and vertical line, H_f is the maximum height of rising and S_f means the speed.

The running gait could be divided into six phrases which are shown as Figure 1.

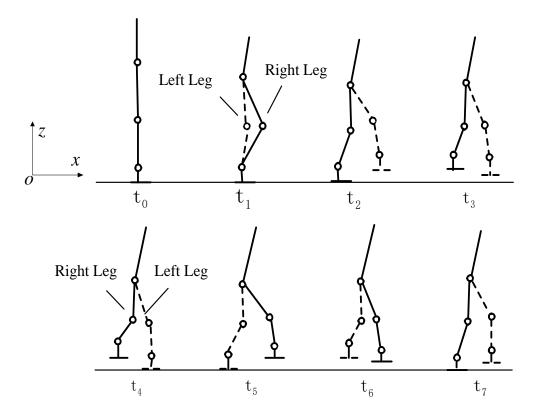


Figure 1. The schematic diagram of running gait in sagittal plane for humanoid robot

where dotted line denotes left leg and solid line represents right leg. The key states during running are represented by $t_0 - t_7$ for convenience.

The gait at every key state could be expressed as parametrization. The expression of hip joint trajectory could be represented as Figure 2.

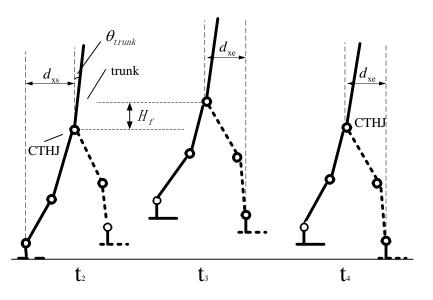


Figure 2. The diagram of hip position of x component at key moments

The x-axis expression of hip joint trajectory could be represented as equation (4).

$$f_{xhip}(t) = \begin{cases} L_{ab} + d_{xs}, & t = t_2 \\ L_{ab} + D_s/2 - d_{xe} - S_f \sqrt{\frac{2H_f}{g}}, & t = t_3 \\ L_{ab} + D_s/2 - d_{xe}, & t = t_4 \end{cases}$$
(4)

where L_{ab} denotes the x-axis distance between ankle joint and heel.

Thus the x-axis expression of hip joint can be formulated as below.

$$x_{hip} = f_{xhip}(d_{xs}, d_{xe}, D_s, H_f, S_f, t)$$
(5)

Using the similar method, knee and ankle joint trajectory of x-axis can be expressed in a parametric way.we formulate gait pattern generation

In the following section, the running pattern would be generated based on cubic spline at first, and then the trajectories would be optimized via opposition-based learning particle swarm. Suppose there are n+1 interpolation points $\{(t_0, f(t_0)), (t_1, f(t_1)), ..., (t_n, f(t_n))\}$, the interpolation function based on cubic spline could be expressed as follow.

$$S(t) = M_{i} \frac{(t_{i+1}-t)^{3}}{6h_{i}} + M_{i+1} \frac{(t-t_{i+1})^{3}}{6h_{i}} + [f(t_{i}) - \frac{M_{i}h_{i}^{2}}{6}]\frac{t_{i+1}-t}{h_{i}} + [f(t_{i+1}) - \frac{M_{i+1}h_{i}^{2}}{6}]\frac{t-t_{i}}{h_{i}}$$
(6)

Where t_i expresses the sample time, $f(t_i)$ denotes key moment pose value, $h_i = t_{i+1} - t_i$ denotes interpolation interval and s(t) is the pose value corresponding to the specific moment. According

to the equation (6), s(t) can be obtained once M_i is calculated successfully. As s(t) is derivable in mean square, the following equation can be obtained.

$$\mu_i M_{i-1} + 2M_i + \lambda_i M_{i+1} = d_i \tag{7}$$

According to the boundary conditions, the following equations hold on

$$\begin{bmatrix} 2 & 1 & & & \\ \lambda_{1} & 2 & \mu_{1} & & \\ & \lambda_{2} & 2 & \mu_{2} & & \\ & & \ddots & \ddots & \ddots & \\ & & & \lambda_{n-1} & 2 & \mu_{n-1} \\ & & & & 1 & 2 \end{bmatrix} \begin{bmatrix} M_{0} \\ M_{1} \\ M_{2} \\ \vdots \\ M_{n-1} \\ M_{n} \end{bmatrix} = \begin{bmatrix} d_{0} \\ d_{1} \\ d_{2} \\ \vdots \\ d_{n-1} \\ d_{n} \end{bmatrix}$$

$$\lambda = \frac{h_{i-1}}{2}$$

$$(8)$$

$$h_{i-1} + h_i$$

$$\mu_i = \frac{h_i}{h_{i-1} + h_i} \tag{9}$$

$$\begin{cases} d_{0} = \frac{6}{h_{0}} \left[\frac{f(t_{1}) - f(t_{0})}{h_{0}} - m_{0} \right] \\ d_{i} = \frac{6}{h_{i} + h_{i-1}} \left[\frac{f(t_{i+1}) - f(t_{i})}{h_{i}} - \frac{f(t_{i}) - f(t_{i-1})}{h_{i-1}} \right] \\ d_{n} = \frac{6}{h_{n-1}} \left[m_{n} - \frac{f(t_{n}) - f(t_{n-1})}{h_{n-1}} \right] \end{cases}$$
(10)

Substituting equation (8)-(10) into equation (8), M_i can be obtained. By this way, we can formulate running gait into the parameterized expression.

III. THE OPTIMIZATION OF GAIT PARAMETERS

In order to get the running gait with higher energy efficiency and both horizontal and vertical stability, this paper investigates the problem of a multi-objective optimization method based on particle swarm. Aiming at speeding up the convergence, a population initiation strategy based on domain knowledge and the update mechanism based on opposition learning are proposed.

a. Basic Concept of Opposition Learning

Opposition learning [14, 15] was first proposed by Tizhoosh in 2005. Different with traditional evolution algorithm, opposition-based learning evolution algorithm update the population by generating opposition entity to improve the convergence, which would be easier to find the optimal value when the optimal entity is far away from the initial entity.

Suppose the vector $X = (x_1, x_2, ..., x_n)$ is a candidate solution in a n-dimensional space with $x_i \in [a_i, b_i] \quad \forall i \in 1, 2, ..., n$ and the opposite entity of x_i is defined as below.

$$\hat{x} = a_i + b_i - x_i \tag{6}$$

where \hat{x} is the opposite entity of x_i . And the opposite vector of candidate solution can be

expressed as
$$\hat{X} = (\hat{x}_1, \hat{x}_2, ..., \hat{x}_n)$$
 (7)

b. The Strategy of Population Initiation

Compare with traditional GA, particle swarm optimization (PSO) convergence to the optimal solution based on the simple speed-position model, which avoids the complicated operation in traditional GA and remains the advantage of the global searching strategy. The outstanding memorizing feature enables PSO to obtain good robustness and performance without the prior knowledge of specific problem.

In this paper, a multi-objective optimization based on opposition learning particle swarm (named OBLPSO for convenience) is proposed, which uses the concepts of opposition learning and pareto dominance to determine the evolution direction of a particle.

Random population initialization is the most common strategy used in traditional GA, which could have a good diversity. However, the excessive randomization leads to the slow convergence, especially in the beginning of search. In view of this problem, an population initialization Strategy Based On Domain Knowledge is proposed, which is described as Figure 3.

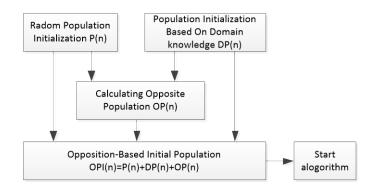


Figure 3. The diagram of hip position of x component at key moments

Aiming at to provide the guide of searching direction in the beginning, DP(n) are selected according to [16], which satisfy the ZMP criteria. The potential optimal solution can be formulated as $X_i = [d_{xs}, d_{xe}, D_s, \theta_{trunk}, H_f, S_f]$. In order to reduce the searching space, the physical restriction could be expressed as $x_i^- < x_i < x_i^+$, such as $\theta_{trunk}^- < \theta_{trunk} < \theta_{trunk}^+$.

c. The Construction of Objective Function

In order to get measure horizontal and vertical stability margin and energy efficiency, the first objective functions about horizontal stability is expressed as below.

$$J_{h} = a \sum_{i=1}^{n} r_{x}(i) + b \sum_{i=1}^{n} r_{y}(i)$$
(8)

where J_h denotes the horizontal stability margin, a, b represent weighted coefficient , a+b=1and $r_x(i)$, $r_y(i)$ are the distance of x-axis and y-axis between the center of foot and ZMP respectively. The schematic diagram of ZMP stability margin is shown as below.

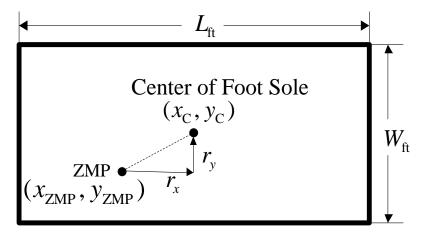


Figure 4. Schematic diagram of ZMP stability margin

The vertical stability margin is measured by J_{V} which is expressed as equation (9).

$$J_{V} = \left\| M_{z} \right\|_{2}^{2}$$
(9)

In order to evaluate the energy efficiency during a running cycle, the mean power rate of joint torque is shown as below.

$$J_{e} = \frac{1}{T} \sum_{i=1}^{n} \int_{0}^{t} |\tau_{i}(j)\dot{\theta}_{i}(j)| dt$$
(10)

Where J_e represents the mean power rate; τ_i means the torque of the *i*th joint and *T* is a running cycle.

Thus, the problem of running pattern generation has been formulated into the problem of multiobjective optimization show as below.

$$\begin{cases} \min & J = (J_T, J_H, J_V) \\ s.t. & g_i(x) \le 0, i = 1, 2, ..., m \end{cases}$$
(11)

where $g_i(x)$ is the physical restriction of the *i*th parameter.

d. The Population Update Mechanism Based On Opposition Learning

Particle swarm optimization is one of the most popular evolution algorithm , which has the advantages of good global search performance and parallel computation compared with traditional GA[17,18,19]. In traditional PSO[20,21,22] algorithm , every particle obtain the next better position according to the best global position and local position. In order to speed up the convergence of population, the population update mechanism based on opposition learning is proposed, which is described as below.

Algorithm 1 Opposition-Based Multiobjective Particle Swarm Optimization Algorithm 1: Population Initialization Pop(i) as described in Figure 3. 2: **for** i = 1; i<N; i++ **do** 3: $v_i = 0$, $p_i = 0$, $p_g = x_1$ and $F(x_i) = 0$ 4: end for 5: while $(BFV \leq VTR)$ and $(NFC \leq MAX_{NFC})$ do 6: for i = 1; i<N; i++ **do** 7: Evaluate the fitness of particle x_i using fitness function $F(x_i)$ **for** j = 1; $j < MAX_{dimension}$; j + + do8: Update velocity using equation (12) 9: 10: end for 11: Find the best particle and update p_{g} 12: end for

13: Select N_{op} particle with the lowest fitness 14: **if** random(0,1) > jcr**for** i = 1; i< N_{op}; i++ **do** 15: for j = 1; $j < MAX_{dimension}$; j ++ do16: 17: Generate the opposite entity using equation $\hat{x}_{i,j} = LB_j + UB_j - x_{i,j}$ 18: end for 19: end for end if 20: 21: end while

where N is the total number of particles, *BFV* denotes the best fitness value, *VTR* represents the value to reach, *NFC* means the maximum function call, p_g is the global best particle, $F(\bullet)$ is the fitness function.

In order to improve the population diversity, we update the population at a random possibility, which is exhibited in line 14. LB_j and UB_j represent the lowest and most upper value of the *j* dimension respectively.

 $v_i(t)$ is the velocity of the *i*th particle, which update as described equation.

$$v_{i,j}(t+1) = wv_{i,j}(t) + c_1 r_{i,j}(p_i(t) - x_{i,j}(t)) + c_2 r_{i,j}(p_g(t) - x_{i,j}(t))$$
(12)

$$x_{i,i}(t+1) = x_{i,i}(t) + v_{i,i}(t+1)$$
(13)

where *w* is the inertia weight, c_1 and c_2 are the acceleration coefficients, r_1 and r_2 are the random number between [0,1], p_i is the personal best position of particle i and p_g is the best particle in population.

IV. SIMULATION AND ANALYSIS

A series of experiments are conducted for the validation of the proposed method in our own simulation platform, which is based on MATLAB 7.12. In order to simulate the procedure of running, the running cycle is defined as 1.2s, the step length is set 0.1m, and the whole running procedure would last 4.8s which contain three kinds of gaits, which are respectively running start

gait, period running gait and running stop gait. All the experiments are conducted under the same environment including the same gait and configuration.

The configuration parameters of humanoid are shown in Tab.1.

| | Trunk | Thigh | Shank | Foot |
|-----------|-------|-------|-------|------|
| Length(m) | 0.28 | 0.14 | 0.11 | 0.04 |
| Mass(kg) | 1.38 | 0.533 | 0.423 | 0.20 |

Table 1. The Configuration Parameters of Humanoid Robot

In order to evaluate the running gait characters in different view, the whole running procedure are presented in Figure 5-6 which show the running gait in 2D and 3D. In Figure 5, the 3D running gait is shown where red solid lines and blue dotted lines represent right and left foot, leg and trunk respectively.

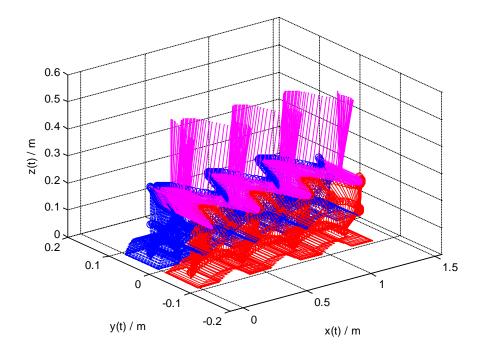


Figure 5. The 3D schematic diagram of running gait for humanoid robot

In Figure 6, the sagittal view of running gait is presented. In order to improve energy efficiency and obtain faster motion, the trunk forwards lean periodically during running.

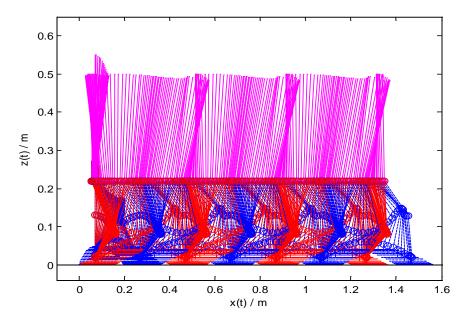


Figure 6. The sagittal view of running gait for humanoid robot

The hip and ankle trajectories are exhibited in Figure 7, in which dotted gray and blue solid lines denote θ_{ankle} and θ_{hip} trajectories.

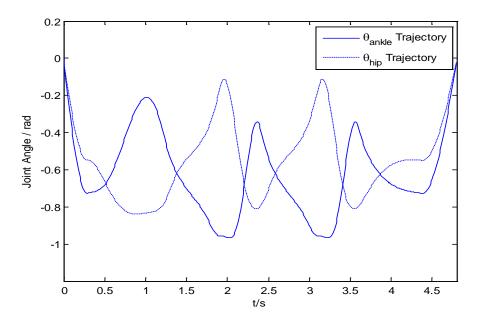


Figure 7. The angle trajectories of ankle and hip joint

ZMP is one of the most popular stability criteria, which can guarantee the horizontal dynamic balance. In order to evaluate the horizontal stability, ZMP response variations are presented in Figures 8-9. In Figure 8, the x-axis ZMP trajectory is plotted in red solid line. Black solid and dashed lines denote minimum and maximum x-axis ZMP boundary, respectively. Similarly, the y-axis ZMP trajectory is exhibited with blue color in Figure 9.

As shown in Figure 8-9, the x-axis and y-axis ZMP trajectory are within the ZMP margin, which ensure the horizontal balance.

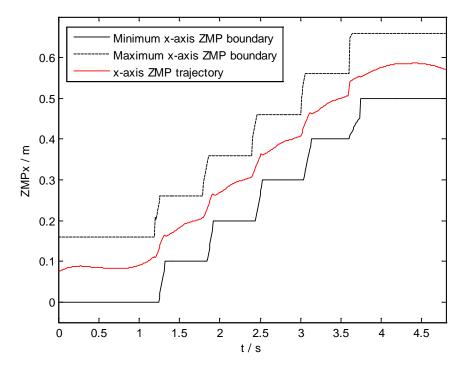


Figure 8. The x-axis ZMP trajectory

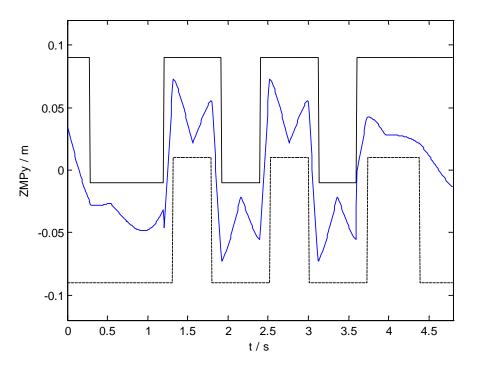


Figure 9. The y-axis ZMP trajectory

As previously explained, yaw moment indicates the vertical dynamic balance. In Figure 10, yaw moment response variations with and without optimization are plotted with dotted gray line and solid blue line, respectively. Dashed lines denote the maximum friction moment which can compensate yaw moment. When yaw moment exceeds the maximum friction moment, rotation would happen.

As displayed in Figure 10, yaw moment exceeds the maximum friction moment at t=1.33s, 1.82s, 1.93s, 2.41s, 2.53s, 3.01s, 3.13s in the case of no optimization employed. And the yaw moment are successfully compensated and guarantee the y-axis balance with the proposed method.

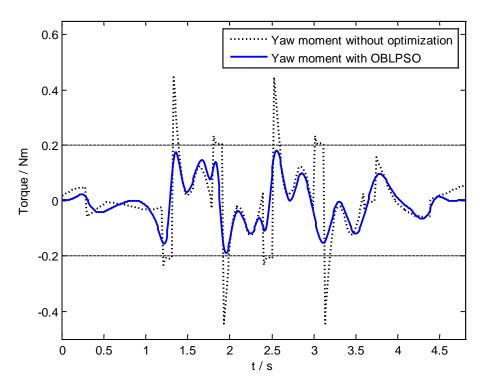


Figure 10. Yaw moment with and without optimization

In order to evaluate the performance difference between traditional GA and proposed OBLPSO, the average acceleration rate (AAR) is defined as equation (13).

$$AAR = \frac{1}{n} \sum_{i=1}^{n} \frac{NFC_{GA}^{i}}{NFC_{OLMOGA}^{i}}$$
(13)

where NFC represents the number of function call, *i* denotes the number of experiments and n = 60. When AAR > 1, it means the convergence of OBLPSO is faster. In the same simulation, the experiment are performed 60 times and the result is shown as below.

| | Number of Fall into local | Average | AAR |
|--------|---------------------------|---------|------|
| | optimal | NFC | |
| GA | 18 | 13.16 | 1.93 |
| OBLPSO | 0 | 6.82 | |

Table 2: Comparison of optimization results of two methods

V. CONCLUSIONS

In this paper, we present a new method to generate running pattern for humanoid robot based on opposition learning particle swarm optimization. To generate smooth running motion with high energy efficiency and stability, we formulate the problem of running pattern generation to the optimization of gait parameters, and an opposition-based learning PSO is employed. In order to speed up convergence, a population initiation policy based on domain knowledge is proposed. By this method, we generate reliable running pattern with good energy efficiency. Simulation results validate this proposed method.

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