



## Extended Kalman Filtering and Pathloss modeling for Shadow Power Parameter Estimation in Mobile Wireless Communications

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*Abstract- In this paper accurate estimation of parameters, higher order state space prediction methods and Extended Kalman filter (EKF) for modeling shadow power in wireless mobile communications are developed. Path-loss parameter estimation models are compared and evaluated. Shadow power estimation methods in wireless cellular communications are very important for use in power control of mobile device and base station. The methods are validated and compared to existing methods, Kalman Filter (KF) with Gaussian and Non-Gaussian noise environments. These methods provide better parameter estimation and are more accurate in most realistic situations. EKF can estimate the model channel parameters and predict states in state-space.*

**Index terms:** Extended Kalman Filter; Fading Channel, Handoff, Kalman Filter, local mean, multipath, power estimation, shadowing, state space, Path-Loss, Parameter Estimation.

## I. INTRODUCTION

There has been a rapid growth in the last couple of decades in wireless mobile communications thus creating a need for research. New and cheaper wireless devices and services have emerged due to advantages in Digital signal processing (DSP), Radio frequency (RF) circuit fabrication and large scale deployment of communication networks.

Performance is critical in wireless cellular communications and can be to a large degree affected by fading [1]. Wireless communication fading is defined as the fluctuation in attenuation of a signal over a specific transmission medium. Fading can vary depending on geographical location and frequency in time. Fading can be a result of multipath propagation or shadowing.

Shadowing is described as the effect of the power fluctuation of the received power due to objects obstructing the propagation path between the transmitter and receiver [1-3].

High performance shadow/fading power estimation methods are very important for use in power control of mobile device and base station handoff coordination. There are two main causes of fading between a mobile station (MS) and a base station (BS) [1-3]. One is multipath propagation loss, where the received signal strength fluctuates due to multiple paths, and shadowing (Local Mean), where the transmitted signal is lost through physical phenomena, such as absorption, refraction (Figure 1), scattering and diffraction. Shadowing is caused by obstacles, such as buildings or trees along the path of a signal from the base station (BS) to the mobile station [1-3]. The amplitude and phase of the transmitted signal will change as the carrier frequency of a signal is being varied [3].

For mobile users, frequently occurring fading dips will cause unnecessary and capacity degrading, retransmissions. To achieve a high throughput over fading channels, adaptive methods for adjustment of (e.g. the modulation alphabet, and the coding complexity) can be used [10-12]. All these techniques require accurate shadow power estimation and prediction to combat time-variability.

Weighted sample average estimators of local mean power, are currently used by many wireless communication system providers [10].

Window based estimators work best under the assumptions that the shadow power process is constant over the duration of the averaging window[1]. In reality shadow power varies with time due to fading, which causes deterioration of these estimates as the window size increases beyond a certain value. The Kalman Filter (KF) algorithm has been used for discrete linear systems. KF is an optimal recursive estimator. Estimate errors are minimized by the Mean Squared Error (MSE). Wiener-Kolmogorov filter was the predecessor that Kalman filter[2]. While KF can be applied to linear systems is not a good solution for systems with nonlinearities. EKF Techniques have been proposed to modify KF to be applied to nonlinear systems. For example, EKF has been proposed by linearizing estimated state variables through Jacobian matrices [2]. However, EKF may not be a good choice in system with high nonlinearity, or systems that are very difficult to calculate their Jacobian matrices.

State space models provide systematic quality channel approximation. Low-order, high-quality models are of interest because they hold the prospect of requiring fewer parameters for their descriptions and consequently an improved adaptation rate. This paper has been organized as follows. Section I is an introduction. Section II is explaining the Kalman filter theory used. Section III is the method of Extended Kalman Filter (EKF) used. Section IV Multipath and Non-Linear Kalman Filter 2nd order. Section V Measurements, simulations and results are given. Section VI Path-loss parameter estimation . Section VII Conclusion and future work.

Further, statistical methods for parameter estimation of linear models in dynamic mobile communication systems have been developed; the estimation of both states and parameters of nonlinear dynamic systems remains also challenging and is being addressed in this paper.

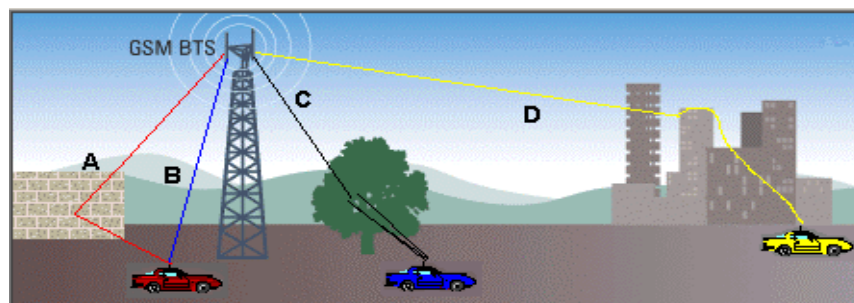


Figure 1: MS/BS Fading/shadowing effect.



Figure 2: Doppler shift effect (Left) , mobile wireless antenna (Right)

## II. Kalman Filter Theory

The method of Kalman filtering also known as linear quadratic estimator has been used for shadow power estimation in wireless communication [1-3]. The algorithm was originally developed for systems with the assumption of system model linearity.

Kalman filter works in a two-step process. In the prediction step, produces estimates of the current state variable, along with their uncertainties. The result of the next measurement corrupted with certain amount of error which includes random noise is observed and these estimates are updated via a weighted average. The weighted average has more weight toward the applied estimates with higher certainty. Due to the recursive nature of the algorithm, it can be performed in real time using only the present input measurements and the previously calculated state and its uncertainty matrix.

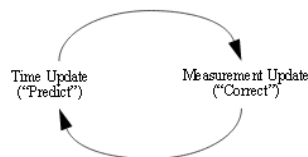


Figure 3. Kalman Predict/Correction steps.

Kalman filtering is a repeated process of time updating projecting the current state estimate ahead in time and measurement updating that adjusts the projected estimate by an actual measurement at that time. The equations below show the two updates:

$$\hat{x}_k = A_k \hat{x}_{k-1} + B_k u_k + w_k \quad (1)$$

where:

$x_k$  is the state vector containing the terms of interest for the system

$A_k$  is the state transition matrix model that is applied to the previous state  $x_{k-1}$

$B_k$  is the control input matrix model that is applied to the control vector  $u_k$  on the state vector.

$w_k$  is the vector that contains the process noise that is assumed to be drawn from a zero mean normal distribution with covariance  $Q$ .

$$p(w) \sim N(0, Q) \quad (2)$$

(2) Project the error covariance ahead:

$$P_k^- = A_k P_{k-1} A_k^T + Q \quad (3)$$

The second step is the Update:

(1) Compute the KF gain:

$$K_k = P_k^- H^T (H P_k^- H^T + R)^{-1} \quad (4)$$

where  $H$  is the measurement vector of the measurement  $z_k$  of the true state space:

$$z_k = H_k x_k + v_k \quad (5)$$

$v_k$  is the vector measurement noise that is assumed to have a zero mean Gaussian white noise with covariance  $R$ .

(2) Update estimate with measurement  $z_k$  :

$$\hat{x}_k = \hat{x}_k^- + K_k (z_k - H\hat{x}_k^-) \quad (6)$$

(3) Update the error covariance:

$$P_k = (1 - K_k H) P_k^- \quad (7)$$

The following assumptions are used when applying Kalman filter to shadow power in mobile communications:

- 1) The Shadow process  $S(n)$  is constant of the average window.
- 2) The multipath process  $H(nT_s)$  is independent and identically distributed, and independent of shadow process  $S(nT_s)$
- 3) The shadow process  $S(n)$  represents the 1<sup>st</sup> order Autoregressive model.

First order autoregressive model model for shadow process is shown below:

$$c_s(t) = \sigma_s^2 \exp\left(\frac{-v|\tau|}{X_c}\right), \text{ where } X_c = -\frac{D}{\ln(\varepsilon_D)} \geq 0 \quad (8)$$

$\sigma_s$  is the shadow variance

$X_c$  is the effective correlation distance

$\varepsilon_D$  is the correlation coefficient of the shadow process

$D$  is the distance

$V$  is the magnitude of the mobile velocity

The specific 1<sup>st</sup> order kalman filtering application is described below:

Prediction Steps:

The state ahead in equation (9).

$$\hat{S}(n | n-1) = \alpha_1 \hat{S}(n-1 | n-1) \quad (9)$$

The error covariance ahead in equation (10).

$$\begin{aligned} M(n | n-1) &= E \left[ (S(n) - \hat{S}(n | n-1))^2 \right] \\ &= \alpha_1^2 M(n | n-1) + \sigma_\varphi^2 \end{aligned} \quad (10)$$

Kalman Gain:

$$K(n) = \frac{M(n | n-1)}{\sigma_H^2 + M(n | n-1)} \quad (11)$$

where  $\sigma_H^2$  is noise due to multipath

Update Steps:

Updates the estimate using P(n) the measured value

$$\hat{S}(n | n) = \hat{S}(n | n-1) + K(n)(P(n) - \hat{S}(n | n-1)) \quad (12)$$

$$\begin{aligned} M(n | n) &= E \left[ (S(n) - \hat{S}(n | n))^2 \right] \\ &= (1 - K(n))M(n | n-1) \end{aligned} \quad (13)$$

The error covariance updates is shown in equation (13).

### III. Extended Kalman Filter Theory

The Extended Kalman Filter (EKF) is the nonlinear extension of Kalman Filter (KF). EKF is therefore suitable due to take into account the non-linearity's of the shadow power system model [23 -25]. EKF is a well-known method and standard that has been considered in the theory of

nonlinear state estimation [28]. KF and EKF are known to be recursive data processing algorithms that estimate current mean and covariance. EKF is reprocessing data at every time step without the need of storing previous measurements. The state distribution along with the mean and the covariance are being propagated analytically using a first order linearization. The predicted state estimation  $\hat{x}_k^-$  for a linearized nonlinear process is expressed as follows:

$$\hat{x}_k^- = \mathbf{J}_x \hat{x}_{k-1} + \mathbf{J}_u \mathbf{u}_{k-1} \quad (14)$$

The following expression is representing the error covariance  $P_k^-$  of the predicted state estimation:

$$\hat{x}_k^- = \mathbf{J}_x \hat{x}_{k-1} + \mathbf{J}_u \mathbf{u}_{k-1} \quad (15)$$

where  $Q_{k-1}$  is the process noise, and  $J_f(x_{k-1}^-)$ ,  $J_f^T(x_{k-1}^-)$  are the Jacobian matrix and its transpose respectively. As it can be seen below  $J_f$  is the Jacobian matrix with partial derivative of all the state estimates:

$$\mathbf{J}_x = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \dots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \dots & \frac{\partial f_n}{\partial x_n} \end{pmatrix} \quad (16)$$

$$\mathbf{J}_y = \begin{pmatrix} \frac{\partial f_1}{\partial y_1} & \frac{\partial f_1}{\partial y_2} & \dots & \frac{\partial f_1}{\partial y_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial y_1} & \frac{\partial f_n}{\partial y_2} & \dots & \frac{\partial f_n}{\partial y_n} \end{pmatrix} \quad (17)$$

Jacobian  $J_x$ ,  $J_y$  and matrices are shown in 16, 17 where measurement matrix  $H_k$  is the Jacobian, of  $h(x_k)$ .

EKF was applied to our system implemented in matlab as illustrated in the block diagram in Figure 6.



### V. Multipath 2nd order Kalman Filter

A non-Linear EKF 2<sup>nd</sup> order model is based on:

Prediction step:

$$\begin{bmatrix} \hat{S}_{1(k|k-1)} \\ \hat{S}_{2(k|k-2)} \end{bmatrix} = J_x \begin{bmatrix} \hat{S}_{1(k|k-1)} \\ \hat{S}_{2(k|k-2)} \end{bmatrix} = \begin{bmatrix} a_{11} & 0 \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \hat{S}_{1(k|k-1)} \\ \hat{S}_{2(k|k-1)} \end{bmatrix} \quad (18)$$

$$P_{(k|k-1)} = J[P_{(k|k-1)}]J^T + Q_k \quad (19)$$

**Kalman Gain:**

$$K_{(n)} = (P_{(k|k-1)} H^T (P_{(k|k-1)} H^T + \sigma_H^2)^{-1}) \quad (20)$$

**UPDATE STEPS:**

$$\begin{bmatrix} \hat{S}_{1(k|k)} \\ \hat{S}_{2(k|k)} \end{bmatrix} = \begin{bmatrix} \hat{S}_{1(k|k-1)} \\ \hat{S}_{2(k|k-2)} \end{bmatrix} + K \left( P_{(k)} - H \begin{bmatrix} \hat{S}_{1(k|k-1)} \\ \hat{S}_{1(k|k-1)} \end{bmatrix} \right) \quad (21)$$

$a_{ij}$

Shown below is a piecewise linear representation used for in the state jacobian linearization.

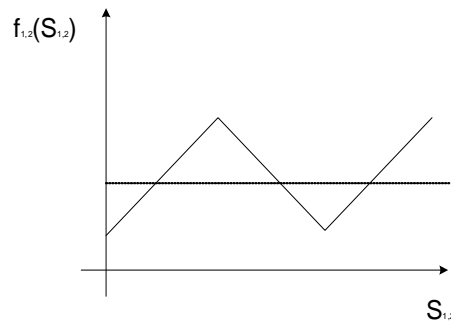


Figure 4. linear as coefficient constant.

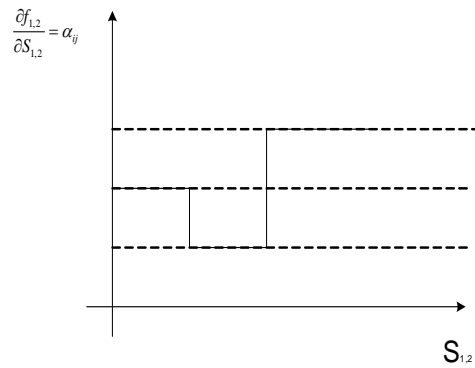


Figure 5. Nonlinear as coefficient piecewise constant.

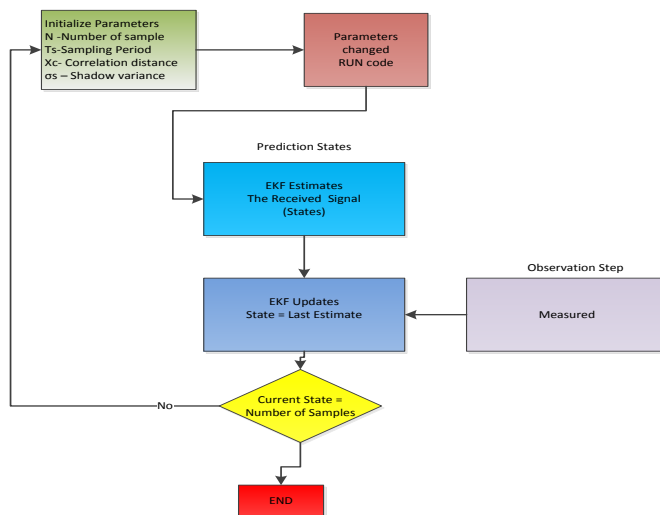


Figure 6. Block diagram of EKF simulation code.

## VI. Path-loss parameter estimation

Path loss in wireless communications is defined as the difference between the transmitter power and the receiver power. The units are in decibels (dB).

Received power is being represented as signal level attenuation that is a result of free space propagation and various physical phenomena's for example, reflection, diffraction and scattering as in figure 7 [23].

$S(t)$  of power  $P_t$  is transmitted through a given channel, then the received signal  $r(t)$  of power  $P_r$  is averaged over any random variations due to shadowing [23].

Linear path loss of a channel is defined as the ratio of the transmitted power to the received power as it can be seen in the equation below:

$$P_L = \frac{P_t}{P_r} \quad (22)$$

$$P_L dB = 10 \log_{10} \frac{P_t}{P_r} dB \quad (23)$$

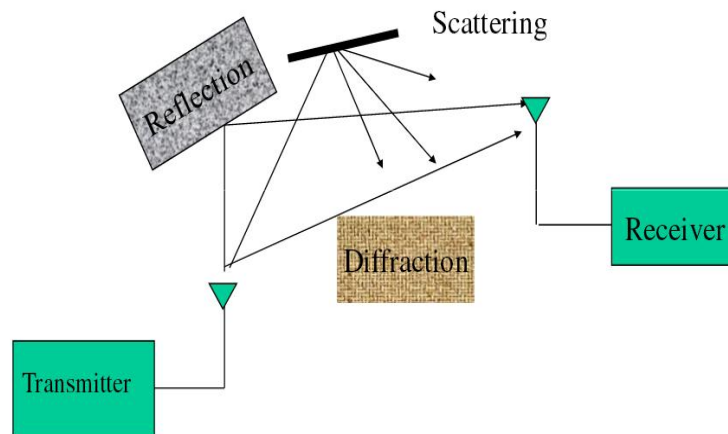


Figure 7. Path loss as a result of physical phenomena.

Several simulation scenarios' were used in our system to illustrate the effects of path loss.

There are two main scenarios that were looked at. One is the Urban (figure 8, 9) setting where there are tall buildings, not many trees or vegetation, major streets, vehicles. The other setting is where residential homes are a relative spread out from each other, major streets exist, tall trees and vegetation is present (figure 10, 11).

These two environment settings play a big role in path-loss.



Figure 8. Urban environment, tall buildings, visible streets, little vegetation

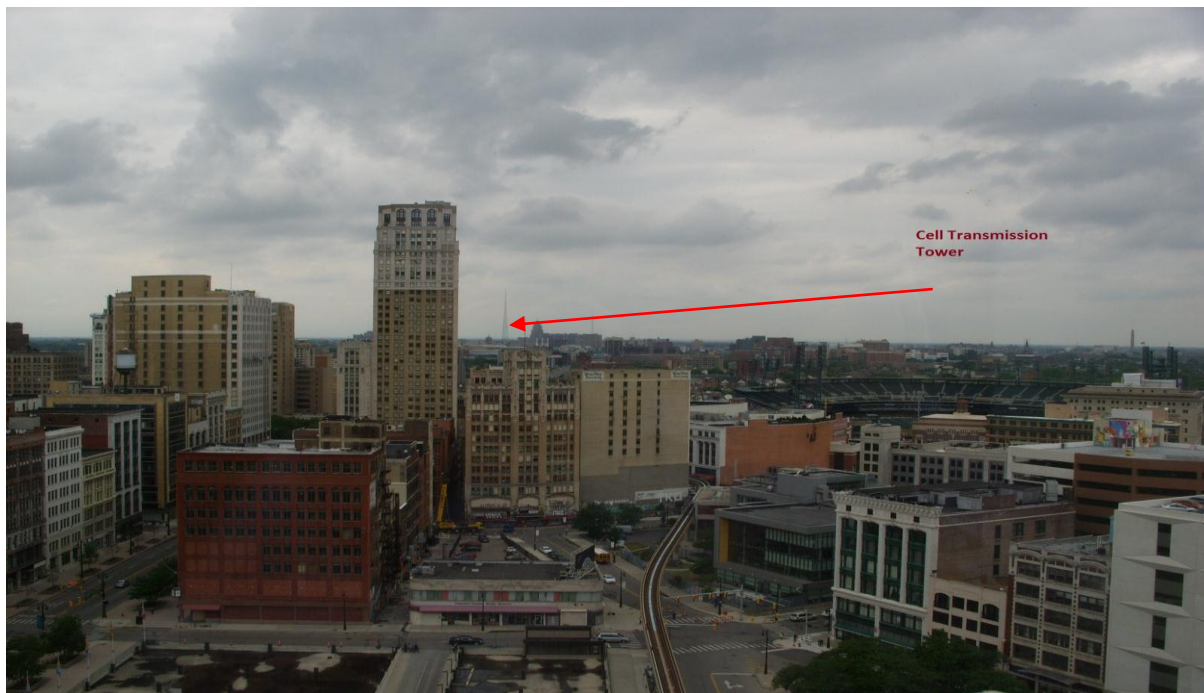


Figure 9. Urban environment, tall buildings, visible streets, little vegetation, base station (wireless transmitting tower)



Figure 10. Suburban environment, tall trees, visible streets, vegetation, base station (wireless transmitting tower)



Figure 11. Suburban environment, tall trees, visible streets, cars, vegetation, base station (wireless transmitting tower)

#### 4. Simulation and Results

Several simulations were executed. After examining simulations results shown in figures 12,13,14,15,16,17,18,19,20. Clearly the EKF is performing as expected. Simulation of various path-loss scenarios were also ran as demonstrated in figures 21,22,23,24.

It can be shown also that EKF performs satisfactory within the range of -8dB to 8dB. Non-Gaussian noise distributions were included as well as zero-mean Gaussian distributions.

Even though the computational complexity of EKF is higher than the KF results are satisfactory. The assumption made when using KF is that the shadow process is driven by non-Gaussian white noise.

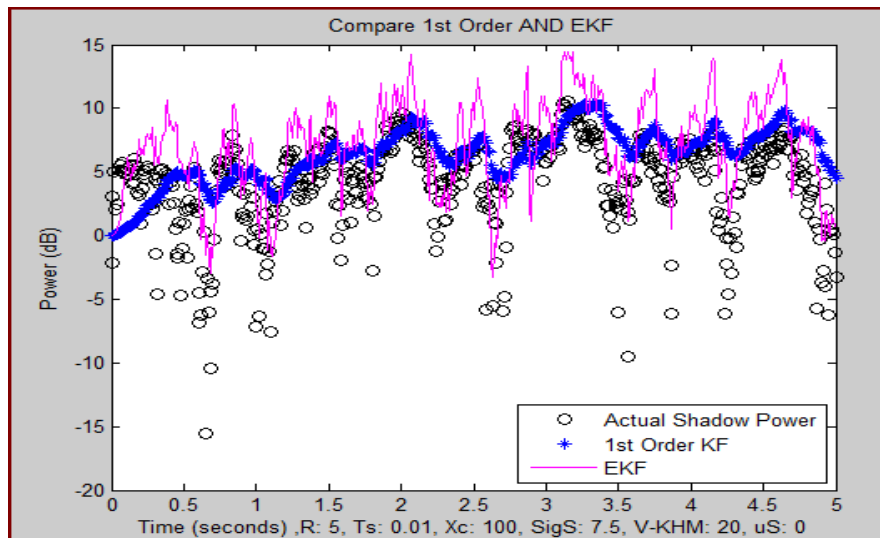


Figure 12. EKF of Shadow Power estimation 20km/h, Xc 100m.

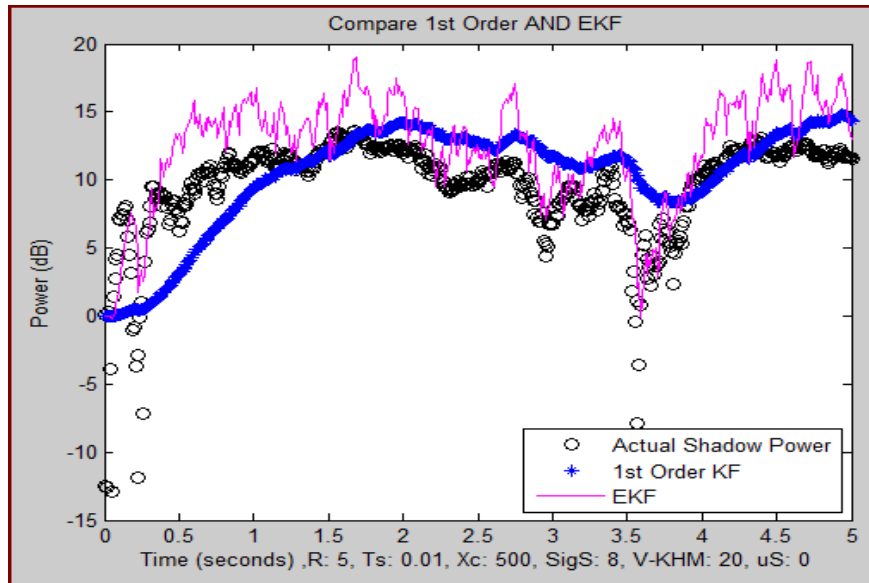


Figure 13. Simulink estimation results 8db average.

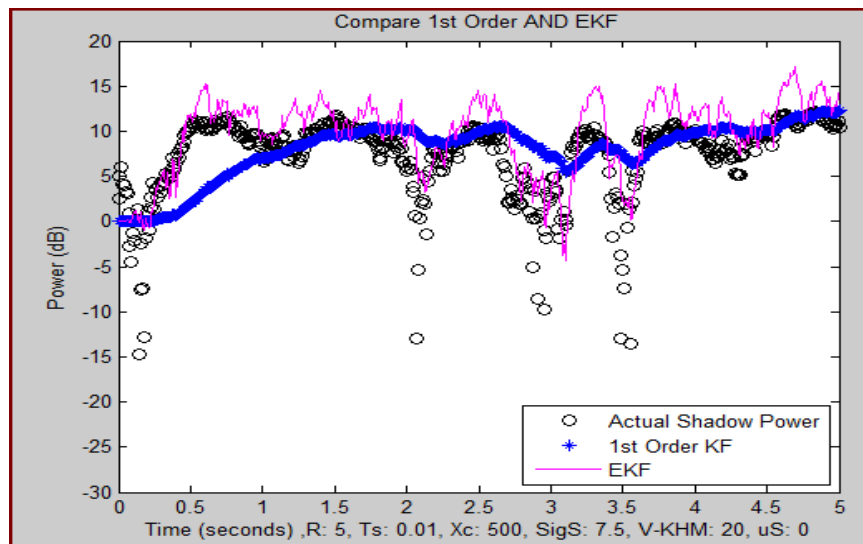


Figure 14. Compare 1<sup>st</sup> order KF with actual Shadow Power.

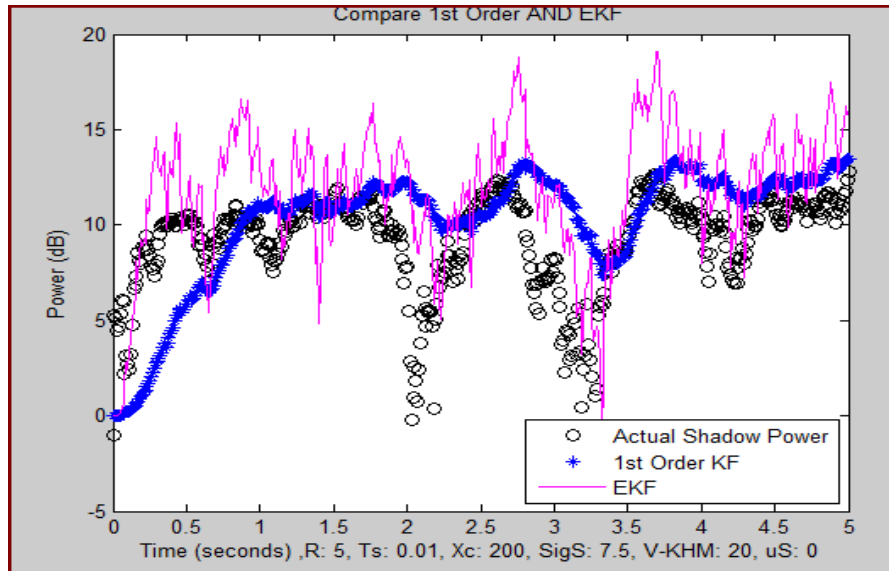


Figure 15. EKF of Shadow Power at low speeds range 20km/h, Xc[200]m.

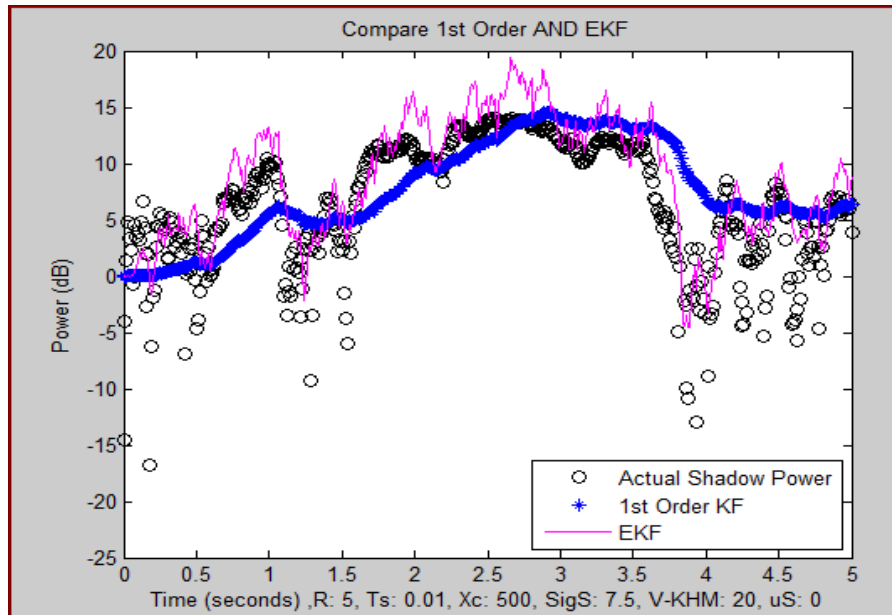


Figure 16. EKF of Shadow Power at low speeds range 20km/h, Xc[500]m.



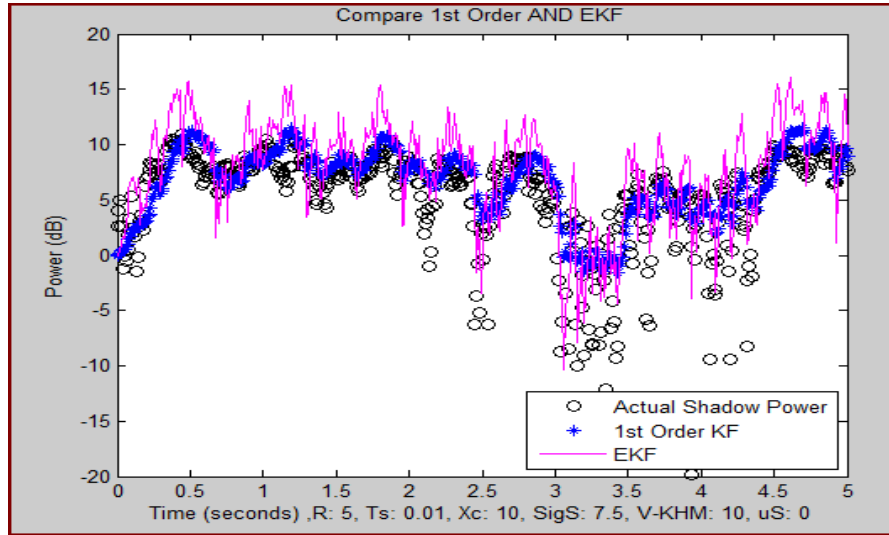


Figure 17. EKF of Shadow Power at low speeds range 10km/h, Xc[10]m.

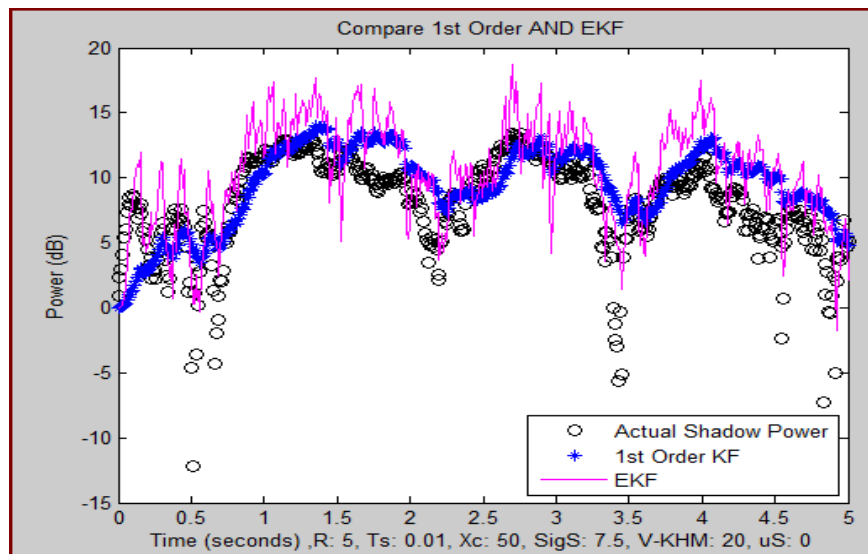


Figure 18. EKF of Shadow Power at high speeds distance 20km/h, Xc[50]m.

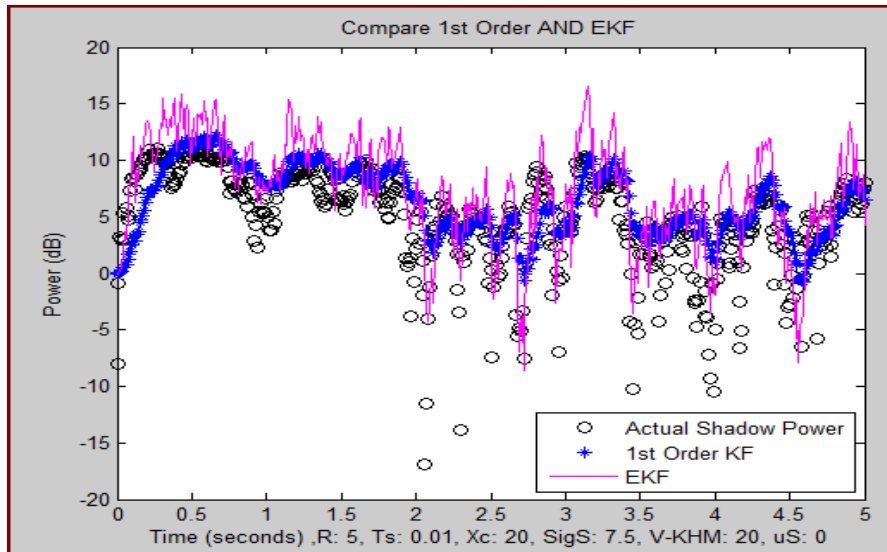


Figure 19. EKF of Shadow Power at low speeds Time sample [.05 .5]sec

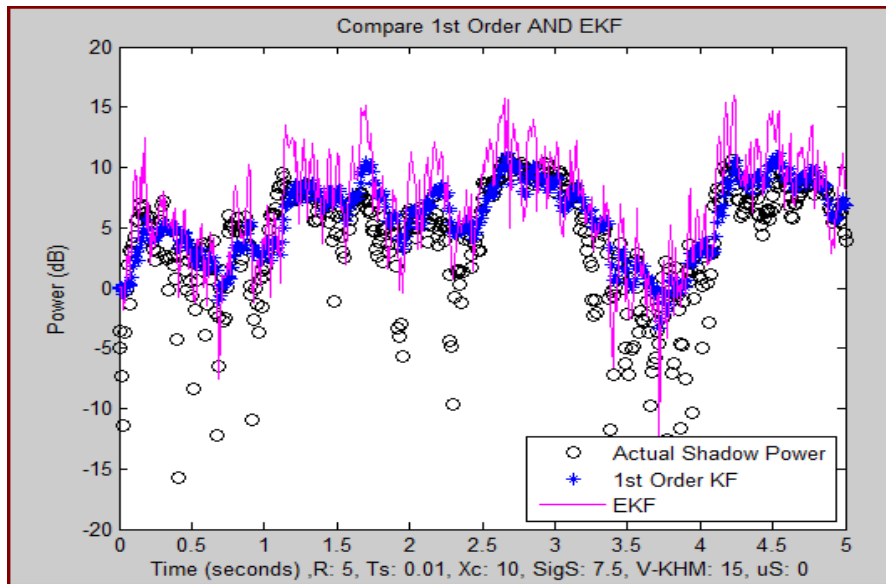


Figure 20. EKF of Shadow Power Velocity [15] Km/h.

Path-Loss model simulation comparisons

The figures 21,22,23,24 respectively show the attenuation of channel propagation. Simulating urban environment the path loss is affected by distance between transmitter and receiver. There is a change in behavior when moving from an urban (Figure 9) and suburban terrain (Figure 10). Simulations were also performed in a suburban environment (Figure 25, 26, 27, 28).

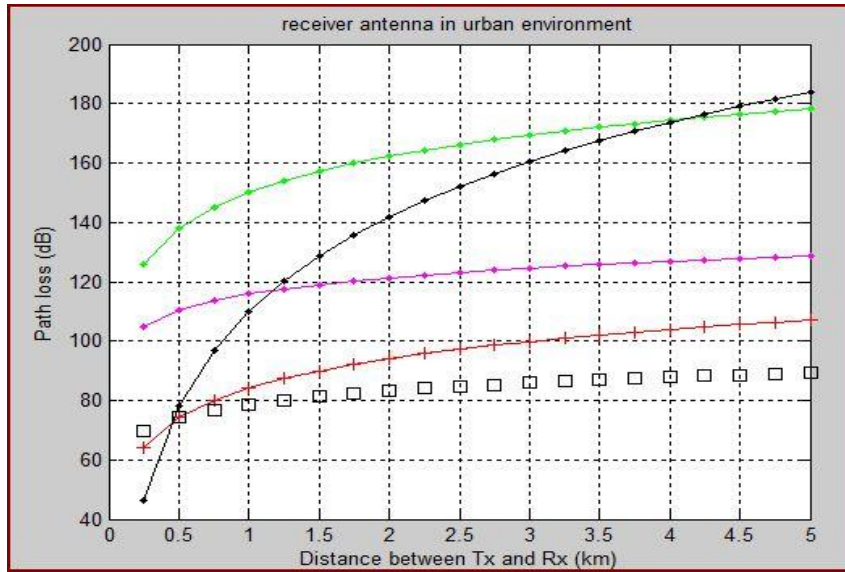


Figure 21. Path-loss at distance between Transmitter and Receiver, 5km.

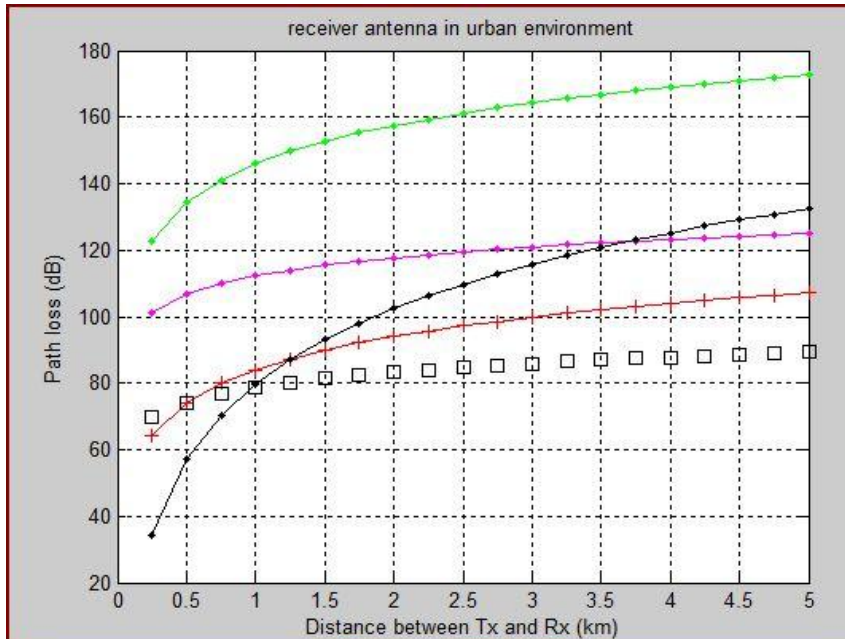


Figure 22. Path-loss at a distance of 10 Km

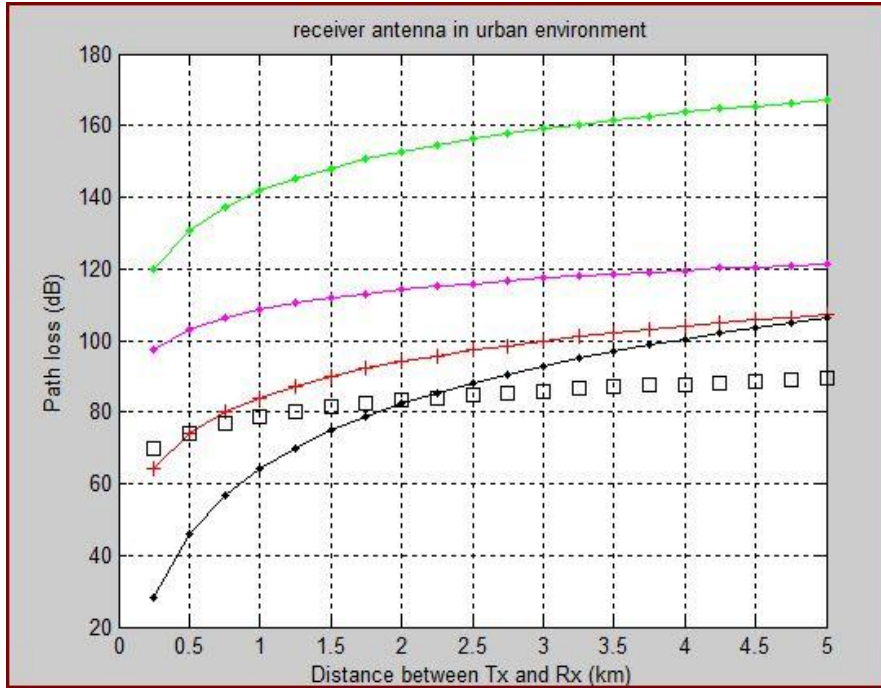


Figure 23. Path-loss at a distance of 15 Km

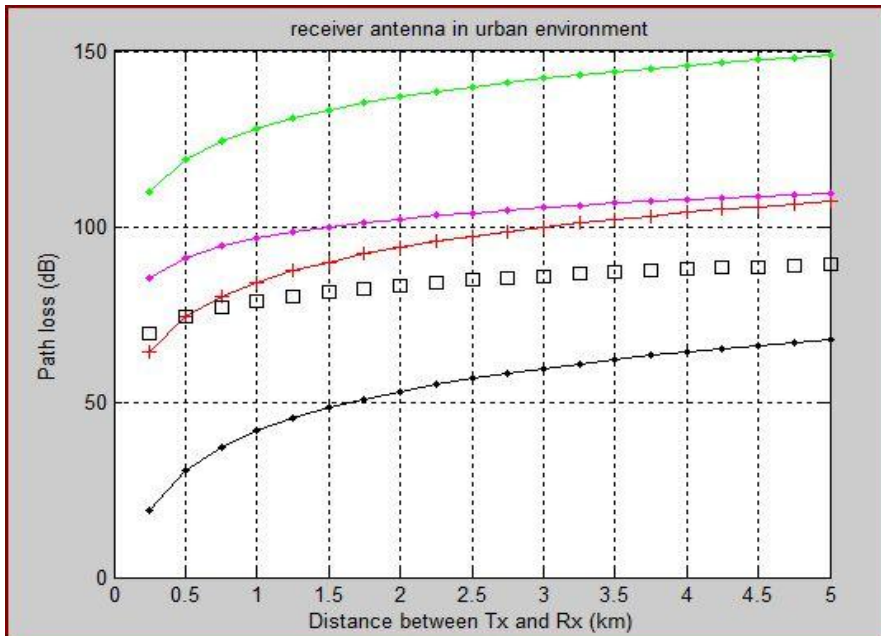


Figure 24. Path-loss at a distance of 20 Km

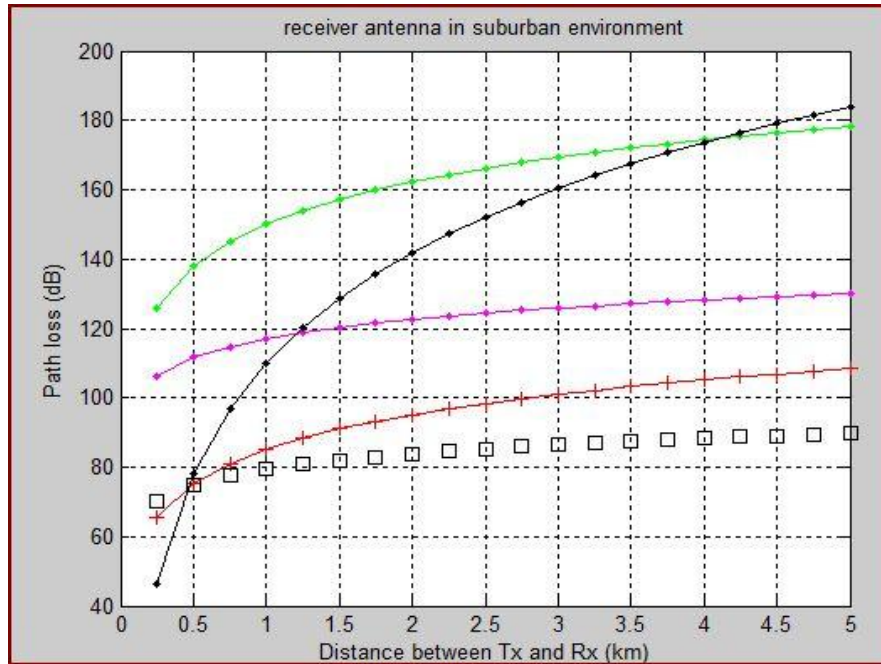


Figure 25. Path-loss (suburban)at a distance of 5 Km

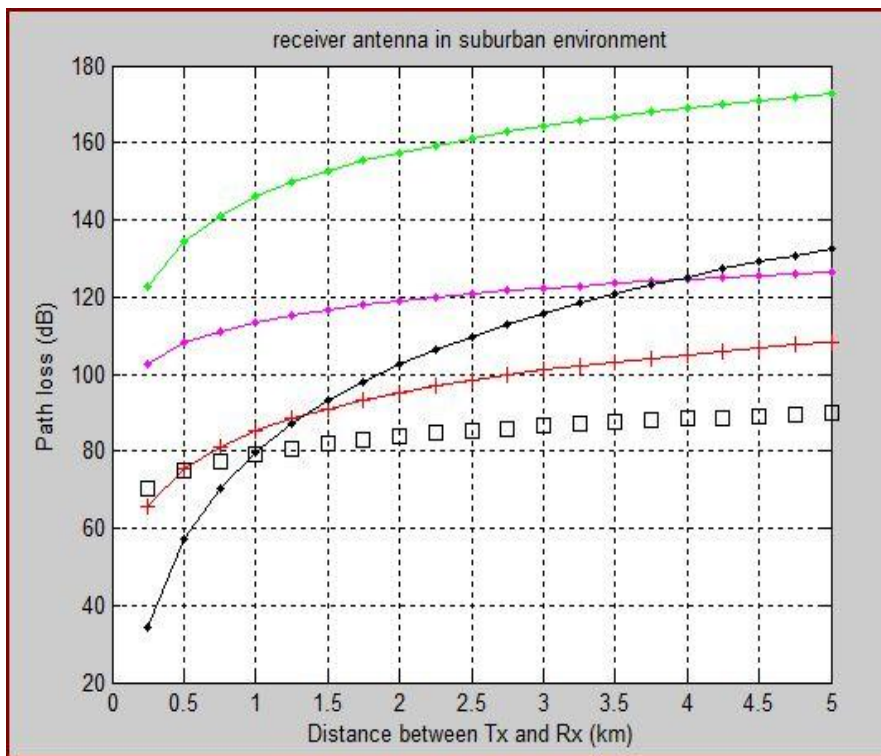


Figure 26. Path-loss (suburban)at a distance of 10 Km

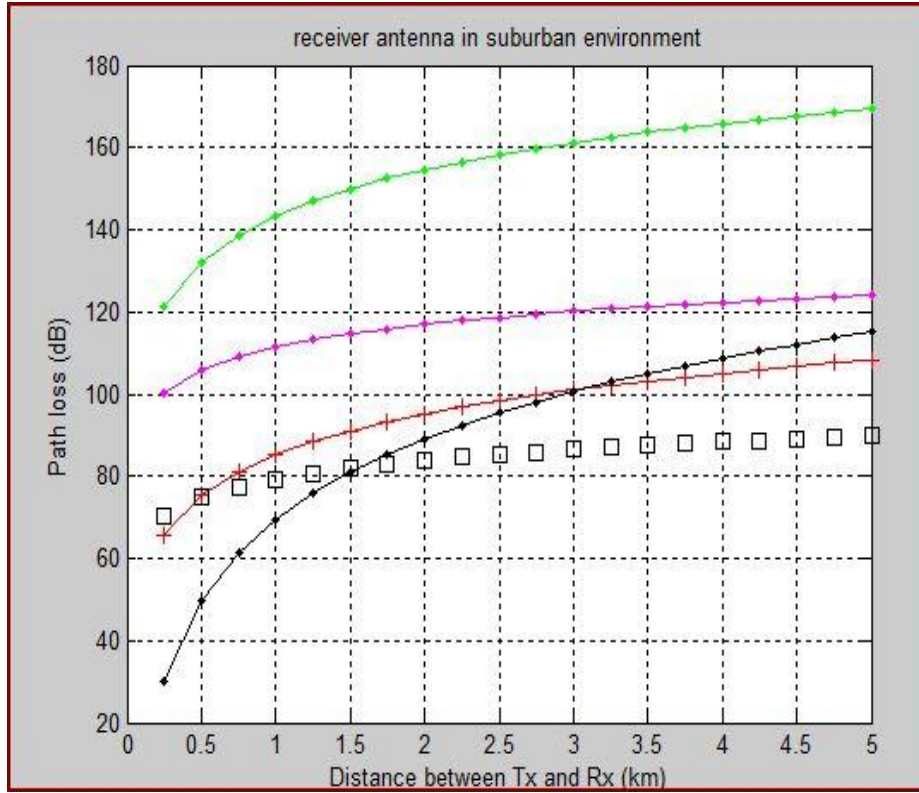


Figure 27. Path-loss (suburban)at a distance of 15 Km

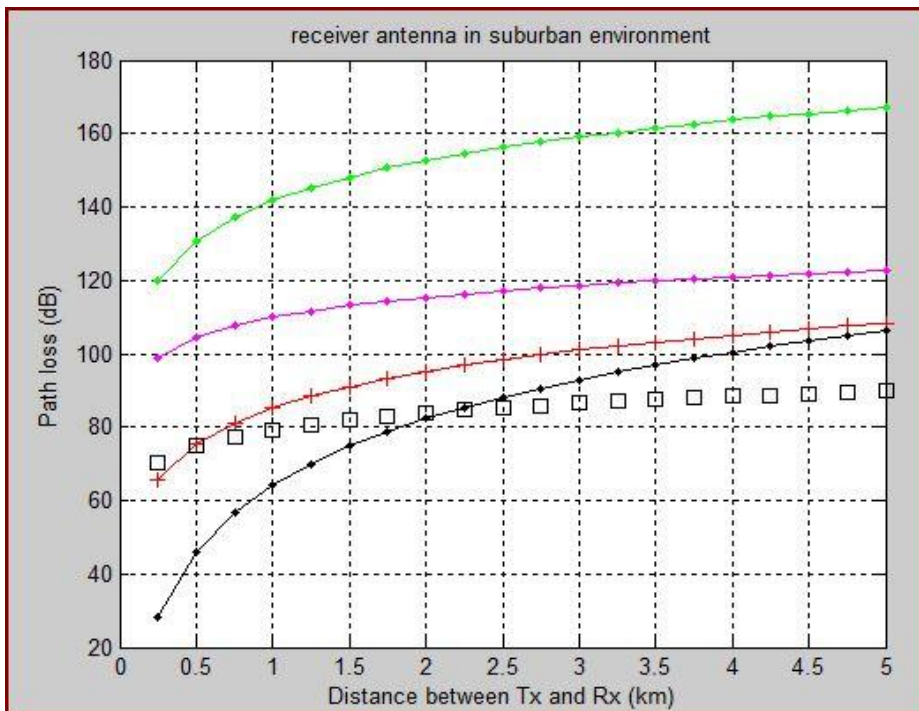


Figure 28. Path-loss (suburban)at a distance of 20 Km

## VII. Conclusion and Future work

In this paper, EKF method has been proposed to optimize the shadow power state estimation [1]. Simulation results show that the incoming signal is tracked in a satisfactory manner. Increasing the shadow power variance has direct affect in increasing the noise level as seen in the estimate. In a suburban scenario, the shadow model coefficient, can be regarded as constants for a wide range of velocities due to the fairly large  $X_c$ . The results also show that this method is more efficient when implemented in both multipath affected signals. EKF performs significantly better than KF while preserving their structures. Channel parameters have been changed throughout to simulate conditions of typical urban areas as well as rural ones. Path loss simulations were also performed to compare and illustrate the results of phenomena's have on wireless communication signals. Data used for simulation was obtained from cell phone android app (Figure 29,30,31).

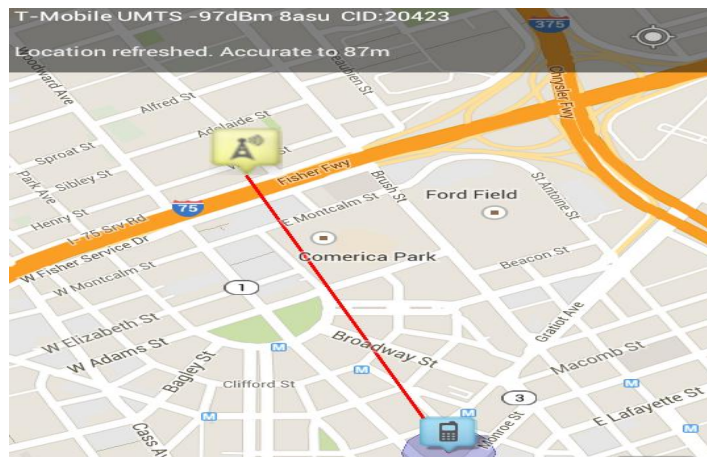


Figure 29. Data was obtain for simulation purposes (cell phone Android app )

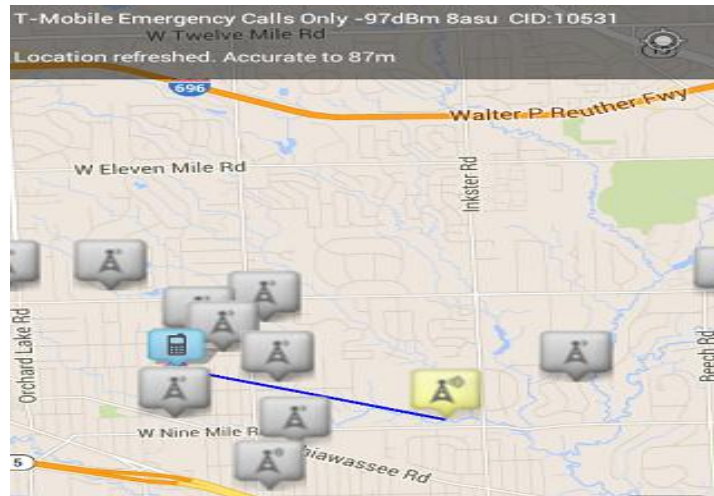


Figure 30. Data was obtain for simulation purposes (cell phone Android app )

-101dBm	UMTS	NONE	100%
<b>PHONE</b>			
Number			
Phone Type			GSM
IMEI/ESN			
Manufacturer			HTC
Model			HTC Sensation 4G
SIM SN		89300100080305403783	
SIM State			STATE_READY
Software Version			02
Sub ID			202011313103591
Signal Strength			-101 dBm
Bit Error Rate			-1
EVDO			-1 dBm
Ec/Io			-1
SNR			-1

Figure 31. Data was obtain for simulation purposes (cell phone Android app )



## REFERENCES

- [1] Tao Jiang, Nicholas D. Sidiropoulos and Georgios B. Giannakis “Kalman Filtering for Power Estimation in Mobile Communication”, January 2003.
- [2] M. Ali and M. Zohdy, "Interactive Kalman Filtering for Differential and Gaussian Frequency Shift Keying Modulation with Application in Bluetooth," *Journal of Signal and Information Processing*, Vol. 3 No. 1, 2012.
- [3] T. K. Dakhlallah, M. Zohdy, “Type-2 Fuzzy Kalman Hybrid Application for Dynamic Security Monitoring Systems based on Multiple Sensor Fusion” *International Journal on Smart and Intelligent Systems*, Dec 1, 2011.
- [4] A. Ribeiro, G. B. Giannakis, and S. I. Roumeliotis, “SOI-KF: Distributed Kalman filtering with low-cost communications using the sign of innovations,” *IEEE Trans. Signal Processing*, vol. 54, no. 12, pp. 4782–4795, Dec.2006.
- [5] C. Tepedelenliog˘ lu, N. D. Sidiropoulos, and G. B. Giannakis, “Median filtering for power estimation in mobile communication systems,” in *Proc. 3rd IEEE Signal Processing Workshop on Signal Processing Advances in Wireless Communications*, Taiwan, Mar. 20–23, 2001.
- [6] H. Zhu, I. D. Schizas, and G. B. Giannakis, “Power efficient dimensionality reduction for distributed channel-aware Kalman tracking using WSNs,” *IEEE Trans. Signal Processing*, vol. 57, no. 8, pp. 3193–3207, Aug. 2009.
- [7] C. Tepedelenliog˘ lu, A. Abdi, G. B. Giannakis, and M. Kaveh, “Estimation of doppler spread and signal strength in mobile communications with applications to handoff and adaptive transmission,” in *Wireless Communications and Mobile Computing*. New York: Wiley, Apr./June 2001.
- [8] M. Ali and M. Zohdy, “Unscented Kalman Filtering for Continuous Phase Frequency Shift Keying Equalization,” *Proceedings of the International Conference on Information and Industrial Electronics*, Chengdu, 14-15 January 2011.
- [9] Y. Liang, R. Ying, P. Liu, “Model Based Application Level Middleware for Design of Wireless Smart City” *Proceedings of the International Journal On Smart Sensing and Intelligent Systems* , vol. 6. No 3, 2013.
- [10] S. Edward Jero, A. Balaji Ganesh, and T. K. Radhakrishnan, "Implementation of A Simple

Wireless Sensor Node for the Detection of Gaseous Substances Leakage", International Journal on Smart Sensing and Intelligent Systems, vol. 4, no. 3, pp. 482-495, 2011.

[11] T.Jayakumar, C. Babu Rao, John Philip, C. K .Mukhopadhyay, J. Jayapandian, and C. Pandian, "Sensors for Monitoring Components, Systems and Processes", International Journal on Smart Sensing and Intelligent Systems, vol. 3, no. 1, pp. 61-74, 2010.

[12] Harb A.M., Zohdy M.A. Synchronization of two chaotic systems as applied in communication systems, submitted to Int. Journal of Nonlinear Dynamics, Aug.,2001

[13] G. Welch, G. Bishop, "An Introduction to the Kalman Filter,"UNC-CH Computer science Technical Report (2005).

[14] Nsour, Ahmad R | Zohdy, Mohamed A "One and two dimensional self organized learning applied to global positioning system (GPS) data. WSEAS Transactions on Information Science and Applications. Vol. 3, no. Dec. 2006.

[15]R. E. Kalman, "A New Approach to Linear Filtering and Prediction Problems," Research Institute for Advanced Study, Baltimore, 1960.

[16]W. Tam and F. C. M. Lau, "Analysis of power control and its imperfections in CDMA cellular systems," IEEE Trans. Veh. Technol., vol.48, Sept. 1999.

[17] M. Akar and U. Mitra, "Variations on optimal and suboptimal handoff control for wireless communication systems," IEEE J. Select. Areas Communication., vol. 19, June 2001.

[18] J. Proakis and M. Salehi, "Digital Communications," 5<sup>th</sup> Edition, McGraw-Hill Higher Education, New York, 2008.

[19] G. P. Pollini, "Trends in handover design," IEEE Communication. Mag., vol.34, pp. 82–90, Mar. 1996.

[20] G. L. Stüber, Principles of Mobile Communication. Norwell, MA: Kluwer, 1996.

[21] D. Labarre, E. Grivel, Y. Berthoumieu, E. Todini and M. Najim, "Consistent Estimation of Autoregressive Pa- rameters from Noisy Observation Based on Two Interacting Kalman Filters," Signal Processing, Vol. 86, No. 10, 2006.

[22] R. Vijayan and J. M. Holtzman, "A model for analyzing handoff algorithms," IEEE Trans. Veh. Technol., vol. 42, Aug. 1993.

[23] A. J. Goldsmith and S. G. Chua, "Variable-rate variable-power MQAM for fading channels," IEEE Tran. Communication., vol. 45, Oct. 1997.

- [24] S. Wei and D. L. Goeckel, "Adaptive signaling based on measurements with statistical uncertainty," in Proc. Conf. Rec. 33rd Asilomar Conf. Signals, Systems, and Computers, vol. 1, 1999.
- [25] A. Duel-Hallen, S. Hu, and H. Hallen, "Long-range prediction of fading signals," IEEE Signal Processing Mag., vol. 17, pp. 62–75, May 2000.
- [26] L. Hanzo and J. Stefanov, "The pan-European digital cellular mobile radio system—Known as GSM," in Mobile Radio Communications, R. Steel, Ed. New York: IEEE Press, 1994.
- [27] Motorola, Inc., Final Text for PACS Licensed Air Interface (TAG 3) J-STD014, June 1995.
- [28] C. Tepedelenlioglu, N. D. Sidiropoulos, and G. B. Giannakis, "Median filtering for power estimation in mobile communication systems," in Proc. 3rd IEEE Signal Processing Workshop on Signal Processing Advances in Wireless Communications, Taiwan, Mar. 20–23, 2001, pp. 229–231.
- [29] D. Wong and D. C. Cox, "Estimating local mean signal power level in a Rayleigh fading environment," IEEE Trans. Veh. Technol., vol. 48, pp. 956–959, May 1999.
- [30] A. J. Goldsmith, L. J. Greenstein, and G. J. Foschini, "Error statistics of real-time power measurements in cellular channels with multipath and shadowing," IEEE Trans. Veh. Technol., vol. 43, pp. 439–446, Aug. 1994.
- [31] W. C. Jakes, Microwave Mobile Communications. New York: IEEE Press, 1974.
- [32] A. Chockalingam, P. Dietrich, L. B. Milstein, and R. R. Rao, "Performance of closed-loop power control in DS-CDMA cellular systems," IEEE Trans. Veh. Technol., vol. 47, pp. 774–789, Aug. 1998.
- [33] D. Giancristofaro, "Correlation model for shadow fading in mobile radio channels," Electron. Lett., vol. 32, no. 11, pp. 958–959, May 1996.
- [34] OpenSignal App "<http://www.opensignal.com>" android app.
- [35] J. Paduart, J. Schoukens, R. "Nonlinear State Space Modeling of Multivariable Systems", 2005.
- [36] S. Julier and J. Uhlmann, "Unscented filtering and nonlinear estimation," Proc. IEEE, vol. 92, no. 3, pp. 401–422, Mar. 2004.